Stellar Physics: Homework I

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due in class on Mar 21th, 2014

1. **Stellar structures & virial theorem:** This is an exercise from Chapter 1 in the book "Stellar Interiors" by Hansen & Kawaler. A userful (albeit not terribly realistic) model for a star may be obtained by assuming that the density is a linear function of radius. Thus assume that $\rho(r) = \rho_c[1 - r/R]$, where ρ_c is the central density and R is the total radius where zero boundary conditions, $p(R) = T(R) = 0$, apply.

   1.1 Find an expression for the central density in terms of R and M.

   1.2 Use the equation of hydrostatic equilibrium and zero boundary conditions to find pressure as a function of radius. Your answer will be of the form $p(r) = p_c \times$ (polynomial in r/R). What is p_c in terms of M and R? (It should be proportional to GM^2/R^4 by the dimensional analysis.) Express p_c numerically with M and R in solar units.

   1.3 In this model, what is the central temperature T_c? (Assume an ideal gas). Compare this result to that obtained for the constant-density model, which you can find in the book or you can derive by yourself. Why is the central pressure higher for the linear model whereas the central temperature is lower?

   1.4 Verify that the virial theorem is satisfied and write down an explicit expression for the gravitational potential energy.

2. **Rotational energy:** In class, when I derive the global energy equation for a star in the hydrostatic equilibrium, the change of the rotational energy is missing because stellar spin is ignored. However, we know that stars are spinning based on observational evidence such as photometric variability, chromospheric line variability, or line width.

   2.1 Write down the centrifugal acceleration $\mathbf{a_{cf}}$ and then derive the centrifugal potential V_{cf} from the equation $\mathbf{a_{cf}} = -\nabla V_{cf}$. Note that the acceleration and hence the centrifugal potential should be functions of r and θ, meaning that the spherical symmetry is slightly violated due to rotational deformation.

   2.2 Write down the momentum equation in the hydrostatic equilibrium and show that the change of the rotational energy $\dot{E}_{rot}$ appears in the global energy equation for a star.

2.3 Roughly estimate $\dot{E}_{rot}/\dot{E}_g$ using the typical values for a classic T Tauri star, i.e. $R = 2R_\odot$, $M = M_\odot$, spin period is 6 days. And you can assume the interior structure of the star is given from the linear-density profile from the last problem. Is the rotation energy important in this case?

3. **bloated hot Jupiter:** Hot Jupiters are Jupiter-mass exoplanets away from their parent stars within about 0.1 AU. Transit surveys reveal that their sizes are larger than predicted by interior models. Without any internal heating, a Jupiter-mass planet should have contracted to about one Jupiter mass after a few billion years. Therefore, some astronomers postulate that there is an internal heating to slow down their gravitational contraction.

   3.1 Estimate the Kelvin-Helmholtz and dynamical timescales of our Jupiter. Look up the physical properties of Jupiter from the internet.

   3.2 Estimate how much an internal heating is required to make our Jupiter double its size. Use $\gamma = 2$.

4. **Turbulent energy transport:** In an ideal scenario, the radiation zone of a star is quiescent because there is no thermal convection. However, thermal convection is just one type of turbulence. Assume that the radiation zone is slightly turbulent driven by other types of mechanism, e.g., convection flows “overshooting” into the radiation zone, differential rotation, nonlinear tidal waves excited by a companion star, etc. What is the direction of the energy flux transported by the turbulence in the radiation zone, i.e., outward or inward? Explain your result.