Stellar Physics (Radiative Processes): Homework V

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due in class on June 11, 2014

1. **Stokes parameters:**

    1.1 Show that the quantity $Q^2 + U^2$ is invariant under rotation of the axis along the propagation of an electromagnetic wave. Explain the underlying physics for the result.

    1.2 Show that $I_x = (I + Q)/2$ and $I_y = (I - Q)/2$, where I_x, and I_y are the intensities of an electromagnetic wave contributed respectively from the components along x-axis and y-axis of the polarization plane as illustrated in Figure 2.4 in Rybicki and Lightman.

2. **Thomson scattering and Stokes parameters:** Consider the process of Thomson scattering. Orient the linear polarization vector $\hat{\epsilon}$ for an outgoing scattered light with intensity I such that $\hat{\epsilon}_1$ is normal to the scattering plane and $\hat{\epsilon}_2$ lies in the scattering plane. Likewise, orient the linear polarization vector $\hat{\epsilon}'$ of an incident light with intensity I' in the same way.

    2.1 show that
    $$\frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{8\pi} \left|\hat{\epsilon}' \cdot \hat{\epsilon}\right|^2 , \qquad (1)$$
    where σ_T is the Thomson cross section.

    2.2 Assume the incident light is totally unpolarized, i.e. $I'_x = I'_y = I'/2$. Derive the I, Q and U parameters in terms of I', σ_T, and the angle θ between the incident and scattered light beams.

3. **Thomson scattering from an ionized cloud:**

    3.1 Rybicki & Lightman: Problem 3.4

    3.2 Follow up the problem. Consider the cloud sightly deviates from spherical symmetry such that the intensity of the incident light from the luminous object is given by $I'(\phi) = I_0 + I_2(3\cos^2\phi - 1)$ with $I_2 \ll I_0$, where ϕ is the azimuthal angle around the axis between the cloud center and the observer[1]. If the cloud is unresolved, what is the net polarization Π observed?

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[1]It is apparent that the angular distribution of I' is expanded in terms of the Legendre polynomial and I_2/I_0 should be related to the "flatness" (i.e. quadrupole moment) of the cloud.

4. **Cyclotron radiation & Scattering efficiency of a dielectric sphere:**

   4.1 Rybicki & Lightman: Problem 3.2

   4.2 Rybicki & Lightman: Problem 3.5. In the problem, the relation between the polarization $\mathbf{P}$ of a dielectric sphere and the induced dipole moment $\mathbf{d}$, $\mathbf{d} = (4\pi a^3/3)\mathbf{P}$, can be found for instance in Chap. 4 of Jackson. The exercise provides an illustrative example about the interaction of a dust grain with radiation.

5. **Thermal Bremsstrahlung:**

   5.1 Rybicky & Lightman: problem 5.1

   5.2 Rybicky & Lightman: problem 5.2. This is a standard problem applying to a viralized hot gas in a cluster of galaxies. The effective optical depth taking into account of both absorption and scattering is not taught in class, but you can find the derivation in Chap. 1 of Rybicki & Lightman.

   5.3 Follow up Problem 5.2. The f-f emission should lead to the gravitational contraction of the cloud unless there is a heating source. Assume the radius of the X-ray cloud $R = 10^{24}$ cm. Estimate the contraction timescale of the cloud.

   5.4 Follow up Problem 5.2 again. Draw a plot to compare the Bremsstrahlung spectrum with the blackbody spectrum for the temperature associated with the cloud.

6. **"Wien displacement law" for the thermal dust emission:** Given the absorption efficiency $Q_{abs} \propto \nu^\beta$, derive the expression for the wavelength λ_{max} at which the maximum thermal dust emission occurs at the dust temperature $= T_d$.