Galactic Dynamics: Homework II

due in class on Mar 21st, 2008

1. **Surface density for a flat rotation curve:** Mestel (1963) introduces a mathematical model for a gravitational potential generated by an infinitesimally thin disk[1]:

$$\mathcal{V}(r,\theta) = V^2 \left(\ln \frac{r}{r_{ref}} + \ln \frac{1 + |\cos\theta|}{2} \right), \tag{1}$$

   where V and r_{ref} are constants. Eq(1) is written in spherical coordinates (r, θ, ϕ).

   1.1 Applying Poisson's equation in spherical coordinates to eq(1), show that the Mestel disk has zero density everywhere except $\theta = \pi/2$.

   1.2 I have shown in class how to integrate Poisson's equation vertically to obtain the surface density of a infinitesimally thin disk in linearized equations. Adopt the same integration strategy and prove that

$$\sigma(r) = \int \rho dz = \frac{V^2}{2\pi G r}, \tag{2}$$

   where σ is the surface density (hint: what is dz in terms of spherical coordinates?).

   1.3 Show that the constant V is just the rotation velocity of the disk if we neglect the radial gradient of gas pressure.

2. **Spiral density waves:** Use the flat rotation curve $r\Omega(r) = V$ =250 km/s and the surface density profile shown in eq(2) for a disk galaxy. Consider that the disk has a 2-arm spiral pattern with the pattern speed $\Omega_p = 12.5$ (km/s)/kpc.

   2.1 Find the radii of the inner Lindbald, the outer Lindblad, and the corotation resonances.

   2.2 Let $Q = \sqrt{2}$, find the location of the Q-barriers.

   2.3 Find the sound speed and estimate the corresponding disk temperature assuming that the disk is totally composed of hydrogen atoms.

   2.4 Find the radial wave number k_r as a function of Ω_p and r from the dispersion relation for short trailing waves.

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[1]This model for such a disk galaxy is called a Mestel disk. See Page 76 in Binnedy & Tremaine for more details. You do not need to read Page 76 to work out the homework problem though.

2.5 Integrate k_r to find the phase term $\Phi(r)$.

2.6 Does the answer in Prob 2.5 indicate that the spiral is perfectly logarithmic (i.e. the pitch angle does not vary with r)? If not, which part of the spiral has the shape much more like a perfectly logarithmic pattern?

2.7 Based on your answer in Prob 2.5, draw the two-arm spiral pattern between the inner Lindblad resonance and the inner Q-barrier.