Basics of Radiation

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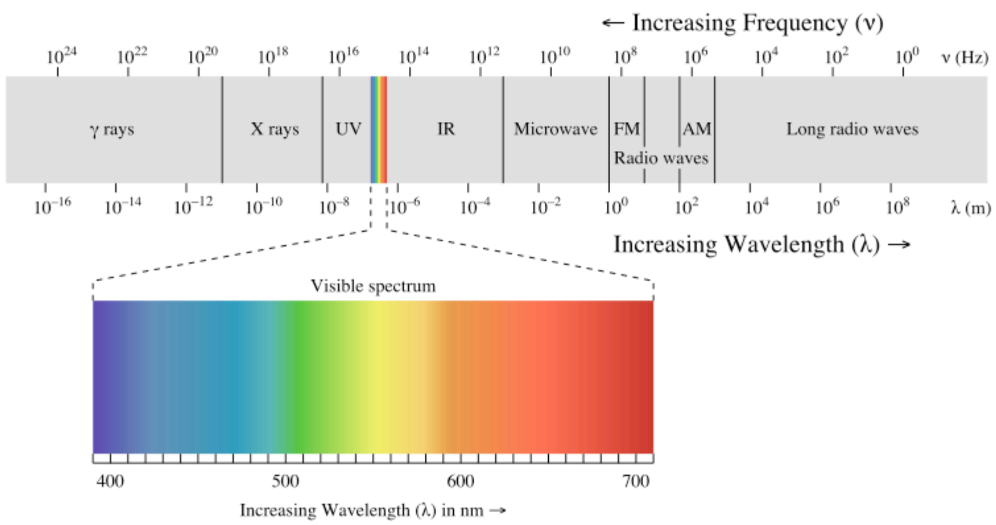
1. Electromagnetic Radiation



All wavelengths are important in astronomy!
Wavelength = Energy of photon ⇔ Physical Process

Emission Mechanism

Acceleration of electric charge

$$\mathbf{E}_{\text{rad}}(\mathbf{r},t) = \frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R}\times\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}\right], \qquad \boldsymbol{\beta}\equiv\frac{\mathbf{u}}{c}, \qquad \kappa\equiv 1-\mathbf{n}\cdot\boldsymbol{\beta}.$$

$$\mathbf{B}_{\text{rad}}(\mathbf{r},t) = [\mathbf{n}\times\mathbf{E}_{\text{rad}}].$$

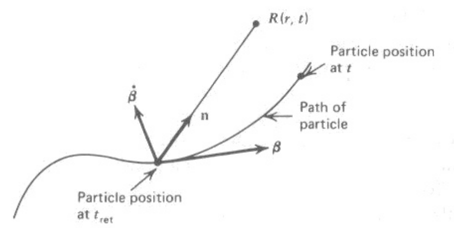


Figure 3.1 Geometry for calculation of the radiation field at R from the position of the radiating particle at the retarded time.

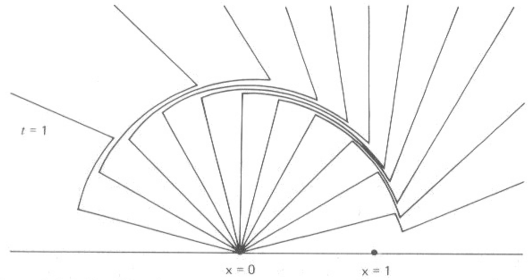
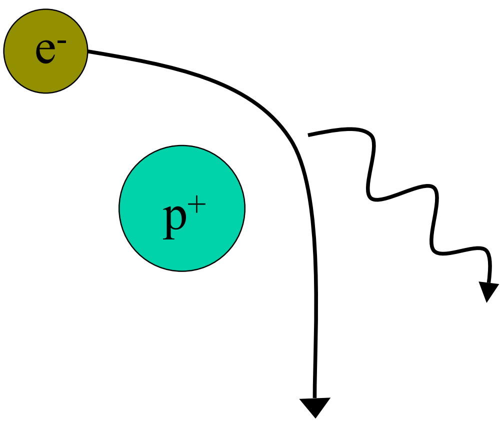


Figure 3.2 Graphical demonstration of the 1/R acceleration field. Charged particle moving at uniform velocity in positive x direction is stopped at x = 0 and t = 0.

Rybicki & Lightman (1979, Radiative Processes in Astrophysics)

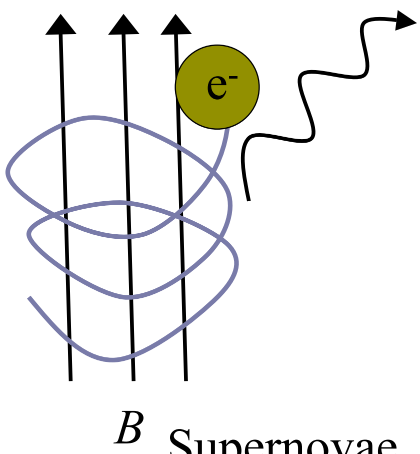
Examples

Bremsstrahlung (free-free)



Ionized regions:
e.g., H II region
Intracluster medium

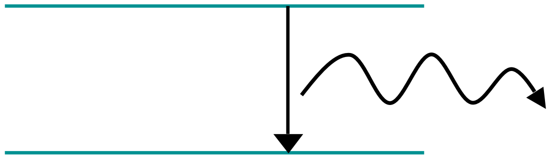
Synchrotron (magnetic brems.)



Supernovae
AGN jet

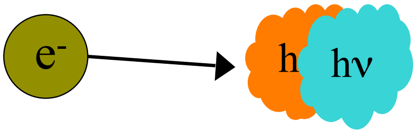
Radiation as Photons

Radiative transition



electric ~ UV, optical
molecular vibrational ~ near-IR
molecular rotational ~ radio

(Inverse) Compton Radiation



Intracluster medium (Sunyaev-Zeldovich effect)
AGN jet, accretion disk

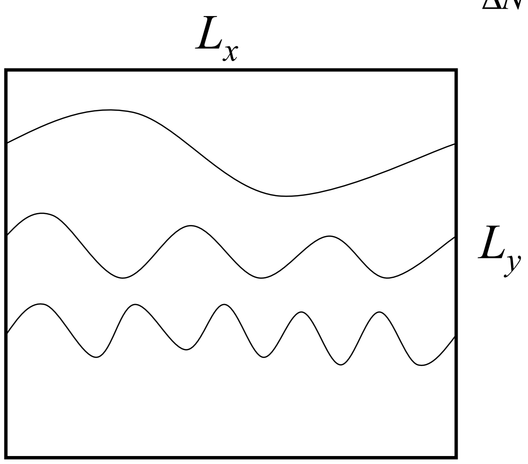
Two Things to Understand Today!

(1) What radiation spectrum appear if the matter and the radiation are in thermal equilibrium? What radiation spectrum appear if the matter is in thermal equilibrium? (Blackbody, TE, LTE)

(2) What is the equation to describe the radiation transport? (Radiation transfer equation)

2. Blackbody Radiation

The Planck function



$$k_x L_x = 2\pi n_x$$
$$k_y L_y = 2\pi n_y$$
$$k_z L_z = 2\pi n_z$$

$$\Delta N = \Delta n_x \Delta n_y \Delta n_z = \frac{L_x L_y L_z}{(2\pi)^3}\Delta k_x \Delta k_y \Delta k_z = \frac{V}{(2\pi)^3} d^3\mathbf{k}$$

$$k = \frac{2\pi\nu}{c}$$

$$d^3\mathbf{k} = k^2 dk d\Omega = \frac{(2\pi)^3 \nu^2 d\nu d\Omega}{c^3}$$

↓

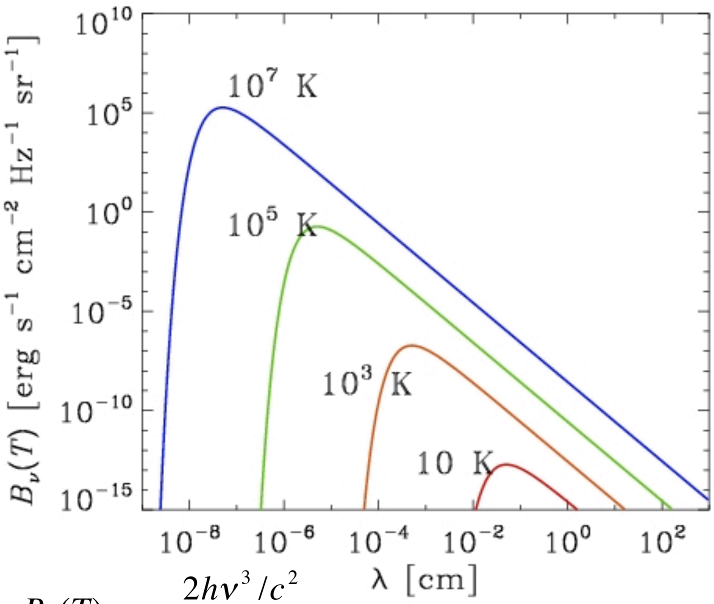
$$2\Delta N = \frac{2\nu^2 V d\nu d\Omega}{c^3}$$

$$2\bar{E}\Delta N = \boxed{\frac{2\nu^2}{c^3}\frac{h\nu}{\exp(h\nu/kT)-1}} V d\nu d\Omega$$

$u_\nu(T)$ [erg/cm^3/Hz/sr]

$$B_\nu(T) = c u_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT)-1}$$ [erg/s/cm^2/Hz/sr]

The Properties of $B_\nu(T)$



$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT)-1}$$

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{\exp(hc/k\lambda T)-1}$$

(1) Monotonic increase with T.

(2) Peak ($\partial B_\lambda/\partial\lambda|_{\lambda=\lambda max} = 0$): $\lambda_{max} T = 0.29$ cm deg (Wien displacement law).

(3) $h\nu << kT \Rightarrow B_\nu(T) \sim (2\nu^2/c^2)kT$ (Rayleigh-Jeans law)

(4) $\int_0^\infty \pi B_\nu(T) d\nu = \sigma T^4$

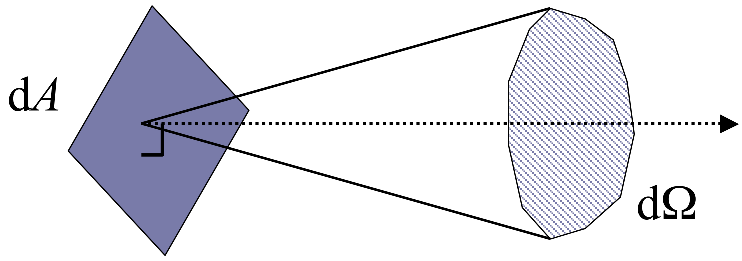
$\sigma = 2\pi^5 k^4/(15c^2h^3)$
Stefan-Boltzmann constant

3. Radiation Transfer Equation

Definition of Intensity

$dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$

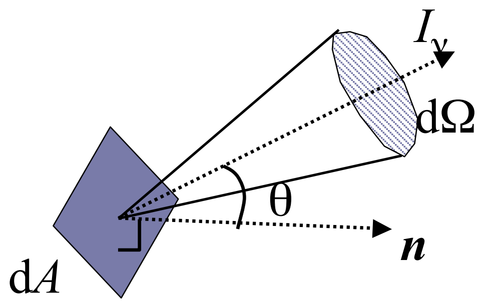
I_ν [erg/cm^2/s/sr/Hz]



Definition of Flux

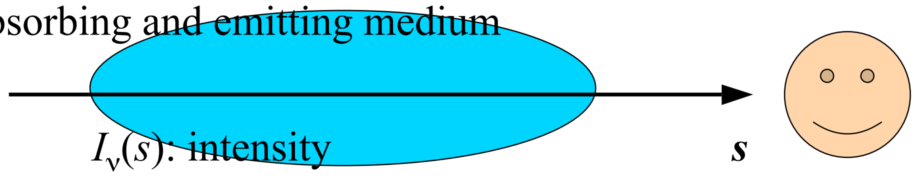
$F_\nu = \int I_\nu \cos\theta \, d\Omega$

F_ν [erg/cm^2/s/Hz]



What is Radiation Transfer Equation?

dI_ν/ds = (Emission) – (Absorption)

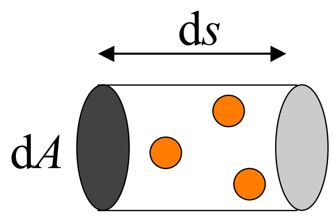


$I_\nu(s)$: intensity
(energy per unit time per unit area per unit solid angle)

Emission

$dE = j_\nu \, dV \, d\Omega \, dt \, d\nu$ ($dV = dA \, ds$)
(spontaneous) emission coefficient
$dI_\nu = j_\nu ds$: radiative transfer eq. for emission only

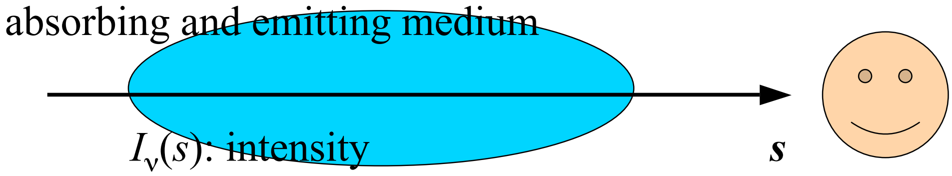
Absorption



Total absorbing area: $n\sigma_\nu dA \, ds$
n: number density of the absorber
σ_ν: absorption cross section of the absorber
$\Rightarrow -dI_\nu \, dA \, d\Omega \, dt \, d\nu = I_\nu (n\sigma_\nu dA \, ds) d\Omega \, dt \, d\nu$
$\Rightarrow dI_\nu = -n\sigma_\nu I_\nu \, ds$
$n\sigma_\nu = \rho\kappa_\nu$
ρ: mass density
κ_ν: mass absorption coefficient

Radiative transfer equation for absorption only
$dI_\nu = -\rho\kappa_\nu I_\nu \, ds \Rightarrow I_\nu(s) = I_\nu(0)\exp(-n\sigma_\nu s)$
Mean free path: $l = 1/(n\sigma_\nu) \leftarrow n\sigma_\nu l = 1$
On average, a photon is absorbed if it travels for a length l.

Radiation Transfer Equation



(energy per unit time per unit area per unit solid angle)

$$dI_\nu/ds = -\rho\kappa_\nu I_\nu + j_\nu$$

Optical Depth

Definition: $d\tau_\nu = \rho\kappa_\nu ds$ ($= n\sigma_\nu ds$)

Radiation transfer equation:

$$dI_\nu/ds = -\rho\kappa_\nu I_\nu + j_\nu$$

$$dI_\nu/d\tau_\nu = -I_\nu + S_\nu$$

$S_\nu = j_\nu/(\rho\kappa_\nu)$: source function

Formal solution:

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} S_\nu(\tau'_\nu)d\tau'_\nu$$

If the source function is constant,

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

$s = l$ (mean free path) $\Leftrightarrow \tau_\nu = 1$

Optically Thin/Thick

Formal solution without background light

$$I_\nu(\tau_\nu) = S_\nu(1 - e^{-\tau_\nu})$$

Optically thin: $\tau_\nu << 1 \Rightarrow I_\nu = S_\nu \tau_\nu = j_\nu s$

Optically thick: $\tau_\nu >> 1 \Rightarrow I_\nu = S_\nu$

Scattering + Absorption

Simplest case: Coherent, isotropic scattering

contribution from scattering to j_ν:

$j_\nu = \eta_\nu J_\nu$ (η_ν: scattering coefficient) $\quad J_\nu = \frac{1}{4\pi}\int I_\nu d\Omega$

Radiation transfer equation with scattering and absorption:

$dI_\nu/ds = -\alpha_\nu(I_\nu - B_\nu) - \eta_\nu(I_\nu - J_\nu)$

$= -(\alpha_\nu + \eta_\nu)(I_\nu - S_\nu) \qquad \alpha_\nu = \rho\kappa_\nu$

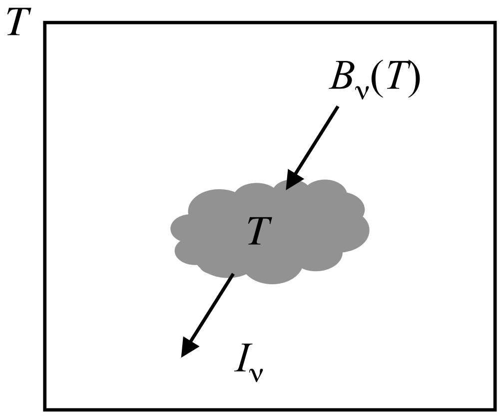
$S_\nu = (1 - \varepsilon_\nu) J_\nu + \varepsilon_\nu B_\nu$

$1 - \varepsilon_\nu = \eta_\nu/(\alpha_\nu + \eta_\nu)$: single-scatteing albedo

J_ν contains the integration of an unknown function I_ν.

$\Rightarrow$ integro-differential equation

4. Thermal Radiation



$dI_\nu/d\tau_\nu = -I_\nu + S_\nu$

$I_\nu = B_\nu$ is satisfied only if $S_\nu = B_\nu$: The matter is in local thermodynamic equilibrium (LTE).

> Thermal Radiation: $S_\nu = B_\nu$
> ($j_\nu = \rho\kappa_\nu B_\nu$): Kirchhoff's law

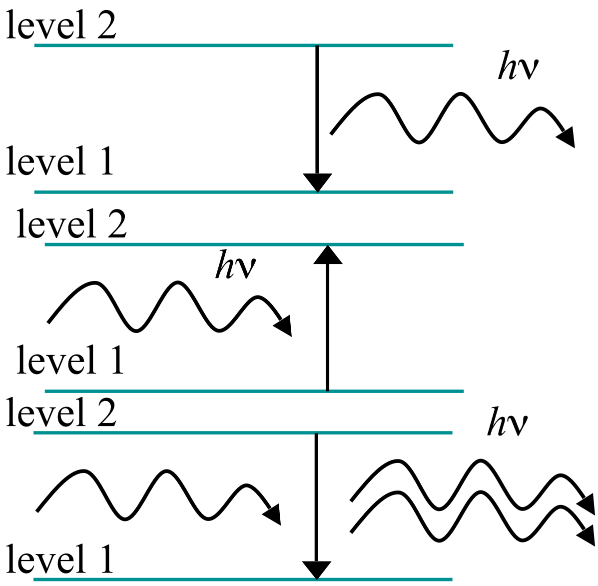
Formal solution for constant source function ⇒

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + B_\nu(1 - e^{-\tau_\nu})$$

$$I_\nu(\tau_\nu) = B_\nu \text{ for } \tau_\nu \to \infty$$

> Optically thick limit for the thermal radiation = Blackbody

5. The Einstein Coefficients



Spontaneous emission

A_{21} (transition probability per unit time)

Absorption

$B_{12}\mathscr{J}$ (transition probability per unit time)

Stimulated emission

$B_{21}\mathscr{J}$ (transition probability per unit time)

$$\mathscr{J} = \int_0^\infty J_\nu \phi(\nu) d\nu$$

$\phi(\nu)$: line profile function

$$\int_0^\infty \phi(\nu) d\nu = 1$$

Einstein Relations

Thermodynamic equilibrium

$$n_1 B_{12} \mathcal{J} = n_2 A_{21} + n_2 B_{21} \mathcal{J} \Rightarrow \quad \mathcal{J} = \frac{A_{21}/B_{21}}{(n_1/n_2)(B_{12}/B_{21}) - 1}$$

$$\frac{n_1}{n_2} = \frac{g_1}{g_2} \exp(h\nu/kT)$$

$$\mathcal{J} = \frac{A_{21}/B_{21}}{(g_1 B_{12}/g_2 B_{21})\exp(h\nu/kT) - 1}$$

should be the Planck function.

$$g_1 B_{12} = g_2 B_{21}$$
$$A_{21} = (2h\nu^3/c^2) B_{21}$$

Absorption and Emission Coefficients

$$j_\nu = \frac{h\nu}{4\pi} n_2 A_{21} \phi(\nu) \qquad \alpha_\nu = \frac{h\nu}{4\pi} \phi(\nu)(n_1 B_{12} - n_2 B_{21})$$

$$S_\nu = \frac{n_2 A_{21}}{n_1 B_{12} - n_2 B_{21}}$$

Using the Einstein relations,

$$\alpha_\nu = \frac{h\nu}{4\pi} n_1 B_{12} (1 - g_1 n_2 / g_2 n_1) \phi(\nu)$$

$$S_\nu = \frac{2h\nu^3}{c^2} \left(\frac{g_2 n_1}{g_1 n_2} - 1 \right)^{-1}$$

The above equation holds also for nonthermal emission

Derive α_ν and S_ν in LTE.