489. **Rabi: Scientist and Citizen,** John S. Rigden (Basic Books, New York, 1987). (I)

490. **Moments in the Life of a Scientist,** Bruno Rossi (Cambridge U. P., New York, 1990). "Autobiographical notes," including 65 photographs, describing Rossi's career as an experimental physicist and a physics educator. Covers his important work in cosmic-ray physics, his wartime years at Los Alamos, and his later interests in space research. (I)

491. **Rutherford: Simple Genius,** David Wilson (MIT Press, Cambridge, MA, 1983). (A)

492. **"Shôichi Sakata—His Physics and Methodology,"** Shunkichi Hirokawa and Shûzô Ogawa, in Ref. 66, pp. 67–81. Focuses on Sakata's work before and during World War II. (A)

493. **Abdus Salam, A Nobel Laureate from a Muslim Country: A Biographical Sketch,** Abdul Ghani (Ma'aref, Karachi, Pakistan, 1982). (A)

494. **Ideals and Realities: Selected Essays of Abdus Salam,** 3rd ed., edited by C. H. Lai and Azim Kidwai (World Scientific, Singapore, 1989). A selection of essays and talks on the International Centre for Theoretical Physics (Trieste), science in developing countries, Islam, and various aspects of physics. Includes Salam's Nobel lecture of 1979 (Ref. 378) and biographical sketches of Salam by N. Calder, J. Ziman, R. Walgate, and others. Also contains a section with biographical data. (I)

495. **"Fifty Years Up and Down a Strenuous and Scenic Trail,"** Emilio Segrè, Ann. Rev. Nucl. Part. Sci. **31**, 1–18 (1981). An autobiographical account of the author's participation in the development of nuclear and particle physics, 1928–72. (E)

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497. **Tabibito** ("The Traveler"), Hideki Yukawa, translated by L. M. Brown and R. Yoshida (World Scientific, Singapore, 1982). An autobiography of Yukawa's early years. (I)

See also Refs. 52, 237, 274, 278, and 446 and the festschrifts and collected works listed in Secs. IV B and IV E.

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Simple demonstration of storing macroscopic particles in a "Paul trap"

H. Winter and H. W. Ortjohann
Institut für Kernphysik der Universität Münster, Wilhelm-Klemm-Strasse 9, D-4400 Münster, Germany

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A simple experimental setup can be used to demonstrate the storage of macroscopic dust particles in a "Paul trap." The trap is operated at atmospheric pressure, which results in an efficient "cooling" of the stored particles. Under these conditions, a single anthracene dust particle has been confined for over 2 months. The device is well suited to show the formation of ordered structures when a number of "cooled" species are trapped. Due to its overall simplicity, the setup can be used in lectures, student laboratories, etc.

I. INTRODUCTION

Different concepts of trapping atomic particles nowadays allow one to perform experiments in which individual species are quasipermanently kept at rest in free space. These developments make feasible a number of conceptually new experiments with accuracies of data that are orders of magnitude smaller than those obtained with conventional spectroscopic methods. A representative example is the outstanding work by H. G. Dehmelt and his coworkers (University of Washington, Seattle) with respect to trapping single electrons. The resulting structure of electronic terms of such an electron confined in a "Penning trap" (static magnetic field and static electric quadrupole field) is ideally suited for high-precision measurements of the electron g factor. These experiments currently provide the most stringent tests on the concepts of quantum electrodynamics. In a recent paper appearing in this Journal, Dehmelt briefly reviews features and perspectives of spectroscopy with trapped particles.[1]

The purpose of this paper is to describe a simple setup to demonstrate features of a "Paul trap," i.e., trapping of charged particles in ac–electric quadrupole fields. This trap was developed by W. Paul (Universität Bonn)[2] and has proven to be a powerful method in storage and spectroscopy of ions. Fascinating new types of experiments with this type of trap have been performed in recent years: storage and detection of a single ion,[3] laser sideband "cooling" of the motion,[4] and formation of ordered structures[5,6] of trapped atomic ions, studies with respect to "quantum jumps,"[7,8] microwave[9] and laser spectroscopy[10] with resonances of very high-quality factors. Direct applications of those experiments are expected to provide improved frequency/time standards.[11]

This very exciting physics, which was given prominent public attention by the decision of the Nobel prize committee in 1989, also can be expected to enter physics education at universities. Successful curricula in physics should comprise—whenever possible—experimental demonstrations, and we will give examples with respect to the "Paul trap" below.

The first experiments of storing macroscopic charged particles were performed by Straubel[12] who trapped charged oil drops in a simple ac quadrupole trap from a ring and a large electrode that operated in air. Later, Wuerker *et al.*[13] nicely demonstrated in many experiments with aluminum dust particles the specific features of the physics of a "Paul trap." However, the overall complexity and the cost of such an apparatus does not allow reproduction of that setup for lecture demonstrations or for use by inexperienced students as a laboratory exercise. The same authors subsequently published experiments with a trap, where the application of three-phase voltages connected to opposite faces of a hollow cube produced electric fields of cubic symmetry.[14] Friker greatly simplified this approach with respect to "injection" of charged particles, vacuum, etc., so that such an experiment can be performed by students in a laboratory.[15] We have adapted Friker's concepts to the geometry of a "Paul trap" and have succeeded in establishing a simple and low-cost setup, which makes possible fascinating demonstrations on the trapping of particles.

II. PAUL TRAP

A brief introduction to the physics of particle confinement in a "Paul trap"[13] is helpful to a discussion of our setup and experiment. A "Paul trap" (see Fig. 1) consists of three electrodes: a ring electrode and two end caps. The present setup uses the ring electrode kept at ground potential, with the end caps biased by an ac voltage of amplitude V_{ac} and angular frequency Ω. A dc voltage V_{dc} also can be applied in series across the end caps.

The electrode structure has axial symmetry with respect to the z axis, and Fig. 2 shows a section in the z-r plane $(r^2 = x^2 + y^2)$. With end caps and a ring electrode that have shapes given by $z^2 = z_0^2 + r^2/2$ and $z^2 = (r^2 - r_0^2)/2$, respectively, and adjusting $r_0^2 = 2z_0^2$, we obtain the potential distribution

$$V(z,r) = (V_{dc} - V_{ac} \cos \Omega t) \times (1/4z_0^2)[2z^2 + (r_0^2 - r^2)], \tag{1}$$

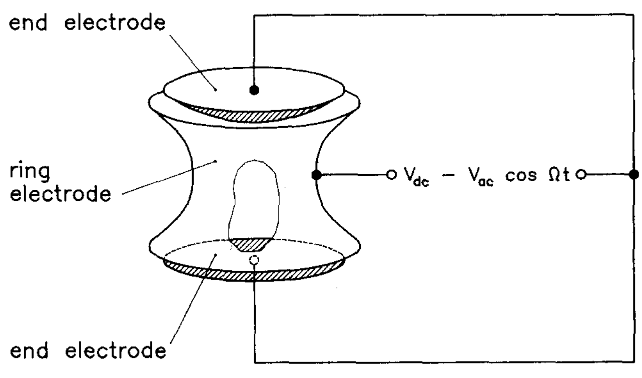
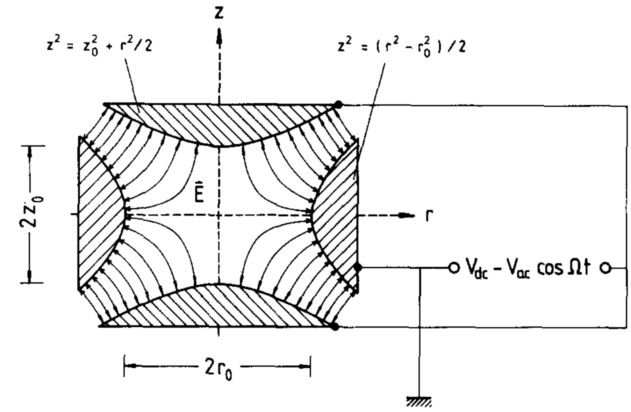


Fig. 1. Paul trap.



$r_0^2 = 2z_0^2$:

Fig. 2. Electric field in Paul trap (section in z-r plane).

within the enclosed volume. Note that this result gives electric field components that vary linearly with their coordinates:

$$E_z = -\frac{\partial V}{\partial z} = -\frac{(V_{dc} - V_{ac} \cos \Omega t)}{z_0^2} z \tag{2a}$$

and

$$E_r = -\frac{\partial V}{\partial r} = \frac{(V_{dc} - V_{ac} \cos \Omega t)}{2z_0^2} r = \frac{(V_{dc} - V_{ac} \cos \Omega t)}{r_0^2} r. \tag{2b}$$

Thus the equations of motion of a particle with mass M and charge Q in this field become

$$\frac{d^2 z}{dt^2} - \frac{Q}{M} \frac{(V_{dc} - V_{ac} \cos \Omega t)}{z_0^2} z = 0 \tag{3a}$$

and

$$\frac{d^2 r}{dt^2} + \frac{Q}{M} \frac{(V_{dc} - V_{ac} \cos \Omega t)}{2z_0^2} r = 0. \tag{3b}$$

For an ideal quadrupole potential [as in Eq. (1)], the motions in z and r are independent, and both equations differ only in the sign and size of their main factors. With

$$x = \Omega t/2$$

$$a_z = -2a_r = 4 \frac{Q}{M} \frac{V_{dc}}{z_0^2} \frac{1}{\Omega^2},$$

$$q_z = -2q_r = 2 \frac{Q}{M} \frac{V_{ac}}{z_0^2} \frac{1}{\Omega^2}. \tag{4}$$

Equations (3a) and (3b) take the normal form of the Mathieu differential equation,[16]

$$\frac{d^2 u}{dx^2} + (a - 2q \cos 2x)u = 0. \tag{5}$$

This type of equation generally is obtained for a periodic variation of parameters in an oscillating system. These phenomena of "parametric excitation"[17] can be demonstrated nicely by a pendulum with an oscillating length or similar setup. For most applications, a specification of the stability of a solution of Eq. (5) is sufficient rather than a detailed functional dependence. The two parameters in Eq. (5) de-

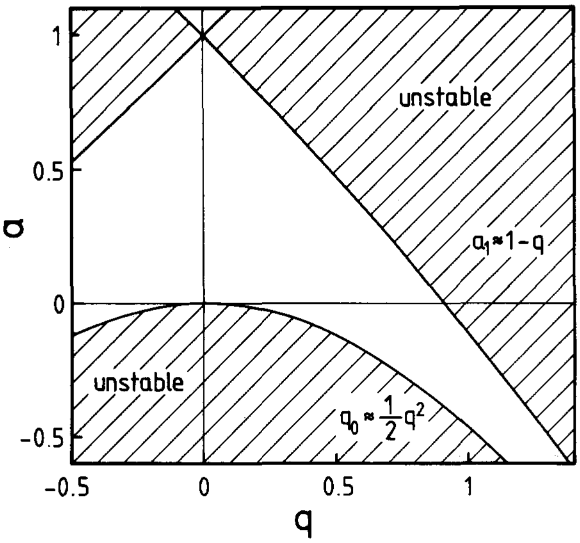


Fig. 3. Section of stability plot. The shaded area represents configurations of parameters a,q [defined by Eq. (5)] where the motion of an undamped particle is unstable.

termine the stability of the solutions, so that an a-q stability plot can be used to determine whether a system is stable (e.g., trapping of particle) or unstable (loss of particle).

Figure 3 displays a small section of a stability plot in which combinations of the parameters a and q leading to unstable solutions (no trapping) are marked by shaded areas. In order to keep the setup as simple as possible, the present experiments choose $V_{dc} = 0$, i.e., $a = 0$. In this case, bound trajectories of particles are obtained for $q < 0.908$ (see Fig. 3). Equation (4) then gives for the specific charge

$$\frac{Q}{M} < \frac{0.908}{2} \frac{z_0^2}{V_{ac}} \Omega^2, \tag{6a}$$

where the upper limit is inversely proportional to the ac voltage amplitude. It is important for our study that Q/M for macroscopic particles is orders of magnitude smaller than for atomic species. Therefore, the driving frequency Ω may be chosen to be rather low. Instead of using oscillators providing high frequencies (MHz, GHz) we simply apply voltages from a transformer connected to the line ($\Omega = 2\pi$ 50 Hz in European countries). With $V_{ac} = 1500$ V and $2z_0 = 11$ mm our setup traps particles with

$$Q/M < 9 \times 10^{-4} \text{ C/kg}. \tag{6b}$$

It should be noted that a strong damping of particle motion in our experiments increases the limits of stability as will be outlined below. The motion of a particle in the trap can be separated into an oscillation, $\delta(t)$, with driving frequency Ω and an averaged secular motion $\bar{z}(t)$.[13,18] The position along z is given by

$$z(t) = \delta(t) + \bar{z}(t).$$

Inserting Eq. (6c) in Eq. (5) ($a = 0$) and assuming that δ varies much faster than $\bar{z}$ we obtain

$$\frac{d^2 z}{dx^2} \approx \frac{d^2 \delta}{dx^2} = 2q_z \bar{z}(t) \cos(2x), \tag{7}$$

so that

$$\delta(t) = -(q_z/2)\bar{z}(t) \cos(\Omega t). \tag{8}$$

Averaging over a period of the micromotion in Eq. (7) yields the differential equation

$$\frac{d^2 \bar{z}}{dx^2} = -\frac{q_z^2}{2} \bar{z}, \tag{9}$$

with the solution

$$\bar{z}(t) = \bar{z}_0 \cos(\bar{\omega}_z t) \tag{10}$$

and

$$\bar{\omega}_z = (q_z/2^{1/2})(\Omega/2). \tag{11}$$

The secular motion proceeds in a quasipotential

$$Q\bar{U} = \tfrac{1}{2} M \bar{\omega}_z^2 z^2, \tag{12}$$

with a depth at the center of the trap

$$\bar{U}_{max} = \frac{1}{2} \frac{\bar{\omega}_z^2 z_0^2}{Q/M} = \frac{1}{8} q_z V_{ac} \tag{13}$$

$$\bar{U}_{max} \leqslant 0.114 V_{ac}. \tag{14}$$

For efficient trapping of charged particles U_{max} should be kept as large as possible, especially for operation of a trap under atmospheric pressure. This is achieved simply by large amplitudes V_{ac}. However, since Eq. (6a) gives Q/M inversely proportional to V_{ac}, an increase of V_{ac} simultaneously reduces the upper limit of specific charges of trapped particles. Thus the essential problem in the operation of this setup has been to establish a proper choice of V_{ac} (see also the experiment).

The action of an external force, F_z, along z is described by adding a term $4F_z/m\Omega^2$ to the right side of Eq. (9). With this term a static force (gravitation, static electric field) induces a shift in position from the center of the trap approximated by

$$z_F = \frac{4F_z}{m\Omega^2(q^2/2)} = \frac{F_z}{m\bar{\omega}_z^2}. \tag{15}$$

Since the experiments are performed under atmospheric pressure, the motion of the particles in the trap is strongly damped by the surrounding air. Assuming a laminar air stream, we add in Eq. (5) a term that accounts for friction and that depends linearly on the velocity, $b\, du/dx$, with

$$b = 9\eta/\rho R^2 \Omega, \tag{16}$$

where R is the radius of a spheric particle with density ρ and η is the viscosity of the medium (air). For anthracene dust particles with $2R \approx 30$ μm and $\rho = 1.28$ g/cm^3, we obtain, with η (air) $= 1.8 \times 10^{-4}$ P, $b = 1.8$.

In order to investigate some properties of such a strongly damped system, we solved the Mathieu differential equation with a damping term by a numerical Runge–Kutta method.[19] It is obvious that damping increases the range of stability of the system described by the plot in Fig. 3. In Fig. 4 we display results from some numerical studies that show that the maximum of the parameter q for a stable motion, q_{max}, increases with the damping constant b. For typical settings in the experiments, the limit of stability increases from $q_{max} = 0.908$ ($b = 0$, undamped motion) to $q_{max} = 2.13$ for $b = 1.8$, i.e., damping enlarges here the limits of stability by a factor of 2 [see Eqs. (6a) and (6b)]. From Eq. (16) we find $b \sim R^{-2}$ so that the range of stability for particles with the same specific charge but smaller radii is further increased.

The effect of damping on the trajectory of a particle within the trap volume is shown for the z coordinate in Fig. 5. In numerical calculations, we have added an external

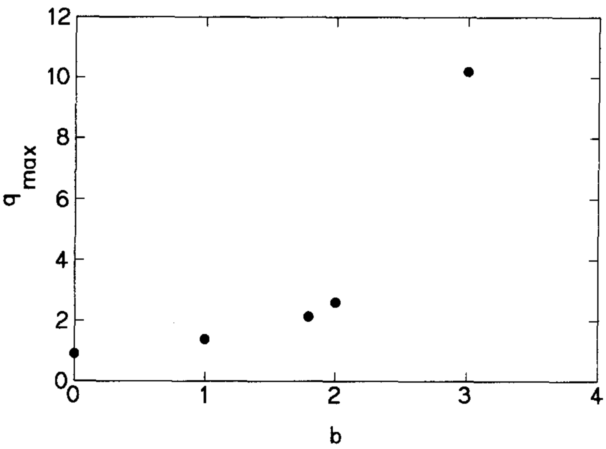


Fig. 4. Maximum of parameter q, q_{max}, for obtaining bound motion in the trap in dependence on damping constant b defined by Eq. (16).

constant force along z to account for the effect due to gravitation on the motion of the particle. With $q = 0.8$ and $g = 9.81$ m/s^2, we obtain a motion along z given by curve a (note the beating between micro- and secular motion). With finite damping ($b = 1.8$, curve b) the secular motion is damped out quickly, and the trajectory is characterized by a micromotion with clearly reduced amplitude. Interpretation of the energy stored in the secular motion of a bound particle as a temperature implies a "cooling" of the particle. According to Eq. (8) we have for the amplitude of the micromotion $q_z\bar{z}/2$, so that a compensation of the shift along z due to gravity is expected to reduce this amplitude further. This feature is demonstrated with curve c, where the gravitational force is overcompensated by a static electric field to 10% of the original value. This reduction in oscillation amplitude and some consequences of cooling can be observed in experiments with our setup and are discussed in Sec. III.

The energy dissipated during a period of the secular motion ΔE, compared to the energy stored in this motion, E_z, is given by the ratio

$$\frac{\Delta E}{E_z} = \frac{9\pi\eta}{\rho R^2\bar{\omega}_z} = \frac{\pi\Omega}{\bar{\omega}_z}\, b, \qquad (17)$$

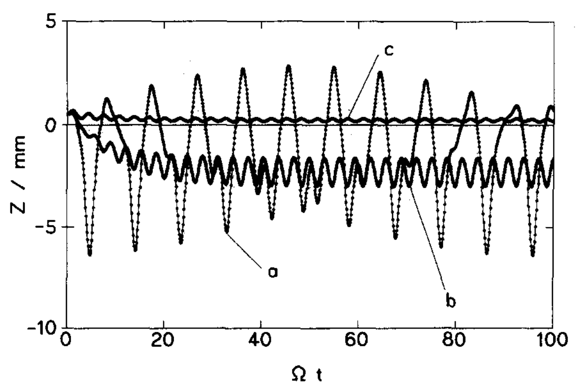


Fig. 5. Dependence of the z position of a particle on Ωt in a "Paul trap." The experimental parameters correspond to $q = 0.8$, and the particle position is shifted by the gravitational force. Curve a: No damping of the particle motion ($b = 0$); curve b (open circles): Damping of the particle motion ($b = 1.8$). Curve c: Same as b, but the gravitational force has been overcompensated to 10% of its initial value by an electric field along z.

which amounts to about 15 for our conditions. Thus the secular motion of a single particle is strongly "cooled" to possibly the limit of Brownian motion.

III. EXPERIMENTAL SETUP AND RESULTS

The main goal of this investigation was to establish a setup for demonstrating the operation of a "Paul trap" under very simple conditions. Several direct consequences of this approach are discussed in the following list.

(1) The trap may be operated under atmospheric pressure, thereby eliminating the need for vacuum vessels and pumps. In addition, operation under atmospheric pressure makes possible the study of "cooling" phenomena in the trap. Furthermore, results from our experiments clearly demonstrate that such conditions do not inhibit long particle storage times at all. A single particle has been stored in the trap for 2 months!

(2) The shape of the electrodes of the trap are kept as simple as possible. Instead of using the "complicated" shape as shown in Fig. 1, which results in a high-quality quadrupole potential, we replaced the ring electrode by a simple wire ring (e.g., NW16CF ring of a UHV setup) and the end caps by small spheres (see below and Fig. 6). This geometry still gives a useful trap potential, especially in the center of the trap, and the relatively small size of the electrodes allows good observation of processes within the trap volume.

(3) The voltages applied to the electrodes of the trap are available in any laboratory. We chose $V_{dc} = 0$ and generated the ac voltages with a high-voltage transformer that was directly connected to the line. Our experiments used $V_{ac} \approx 1500$ V at the line frequency $\Omega = 2\pi \times 50$ Hz.

(4) We used anthracene dust particles, which show efficient fluorescence, when irradiated by uv light.[15] In principle, however, almost any kind of dust is applicable. The dust particles were charged up and "injected" into the trap with a simple plastic syringe (available at any pharmacy). We used a syringe of 10-mm o.d. and about 70-mm length with a 1-mm opening for the injection needle (needle removed). The syringe was filled with a small amount of dry anthracene powder. The filling corresponded to about the quantity of powder on the tip of a small screwdriver. Charging of the particles proceeded via frictional electricity in the syringe. The piston of the syringe was pulled back from its end position by some millimeters, and the opening

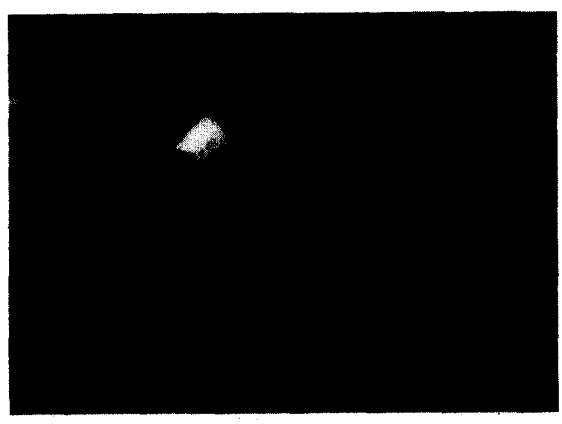


Fig. 6. Most simple version of the experimental setup. The diameter of the coin is 23 mm.

of the syringe was directed, at a distance of about 10–20 mm from the ring, toward the center of the trap. Distance and direction at the "injection" could be chosen within wide limits. Then, by a quick push to the piston, the particles were accelerated toward the inner volume of the trap.

(5) The particles were detected by irradiation with light from a filament lamp or a uv light source (fluorescence). The trapped particles could be seen clearly with bare eyes. For measurements of amplitudes, geometrical shifts, etc. a simple microscope was helpful. For demonstrations in lectures, etc. a videocamera operating in "macromode" of the lens would provide a nice visualization of processes in the trap.

A photograph of our simplest setup is shown in Fig. 6. The ring electrode is a 3-mm wire ring with an inner diameter $2r_0 \approx 16$ mm. The end cap electrodes, 8-mm-diam copper spheres that are separated by $2z_0 \approx 11$ mm, fulfill the condition $r_0^2 \approx 2z_0^2$. A sketch of the setup and the electrical connections is given in Fig. 7 (upper part). The ring electrode is kept at electrical ground and the two end electrodes are connected to a high-voltage transformer operating with a fixed voltage from the line (or with adjustable voltage from a variable transformer).

An ac voltage of about 1500 V yields an efficient operation with respect to injection and trapping of anthracene dust particles (Bayer 5H657, $C_{14}H_{10}$). This is consistent with the discussion in Sec. II [Eqs. (6b) and (14)]. It is advantageous to provide "deep" trapping potentials. This leads to more favorable particle capture during the injection cycle and reduces the perturbation due to the stream of the surrounding air on trapped particles. However, operation at higher ac voltages also reduces the limit of specific charge of the trapped species. For successful operation of the trap, this limit has to be adapted to the specific charges of particles as produced by our technique of injection. This condition is fulfilled by a proper choice of V_{ac}. It should be noted that early tests with our setup failed, because the voltages were too high with respect to the specific charges of the injected particles.

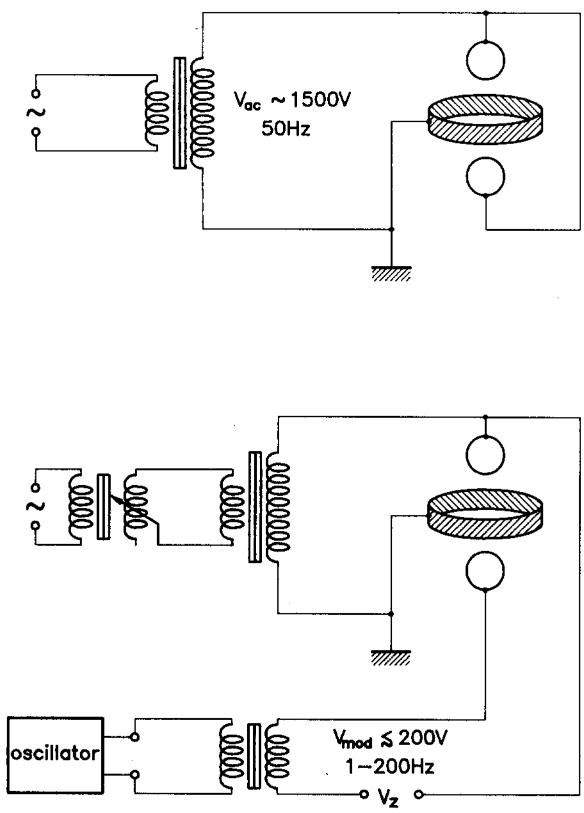


Fig. 7. Sketch of the experimental setups. The use of additional protective high-Ohmic resistors between transformer and trap electrodes is recommended.

Fig. 8. A single anthracene particle in the trap. The trap is surrounded by a transparent foil which is hit by the light beam at both sides (bright spots). The picture of the particle is slightly blurred by various effects.

Figure 8 is a picture showing a single anthracene particle confined in our trap. Any slight stream of air in the vicinity of the trap—caused by the experimentalist or whatever—can be a serious disturbance. However, these problems can easily be overcome with a shield made out of, e.g., part of an overhead projection foil. The two bright spots outside the trap are light scattered by the protecting foil. The light beam which illuminates the trapped particles is generated by a filament lamp and focused by a lens ($f = 100$ mm) to the center of the trap without hitting the electrodes. Under these conditions the particles in the trap can be observed directly. Since the photograph required an exposure time of about 1 s, the image of the particle is slightly blurred.

The gravitational force shifts the z position of the particle by $z_F = -g/\bar{\omega}_z^2$, as about 2 mm in our setup. Therefore, the particle is shifted below the center of trap where the increasing electric field leads to an increase of the amplitude in its periodic motion. This results also from our theoretical analysis as given above (see Fig. 5). The shift due to gravity can be simply compensated by an electric field along z which is obtained by biasing the end electrodes with a voltage V_z (see lower part of Fig. 7). We then find for the z position

$$z_F = -\frac{g}{\bar{\omega}_z^2} + \frac{Q}{M}\frac{V_z}{2z_0}\frac{1}{\bar{\omega}_z^2}. \tag{18}$$

Variation of V_z shifts particles along z in the trap, and it can be demonstrated nicely that the amplitude of oscillation reaches a minimum at the center of the trap and strongly increases outside from that region. Most of our experiments have indicated negative particle charges, because particles have been lifted with the application of positive voltages to the upper end electrode.

Varying V_z and recording the shift Δz_F yields a simple method to extract the specific charges of trapped particles. We obtain from Eq. (18)

$$\frac{Q}{M}=\frac{1}{4}\frac{z_0^3\Omega^2}{V_{ac}^2}\frac{\Delta V_z}{\Delta z_F}. \quad (19)$$

Δz_F is obtained by observation normal to the z axis with a small microscope. We find typical data $\Delta V_z/\Delta z_F \approx 100$ V/mm so that $Q/M \approx 8\times10^{-4}$ C/kg. This is about the theoretical limit of stability for undamped particle motion.

When gravity is not compensated with voltage V_z, a gradual increase of specific charge Q/M is observed with time. This increase, which also has been observed by Friker in a hexapole trap,[15] is attributed to collisions of the oscillating particles with the molecules of the air. The stored particle is lost after a time interval from hours up to a day, because gradually Q/M increases to the limits of stability.

However, this storage-time limitation can be overcome by keeping the particle in the center of the trap with $V_z \approx 150$ V. Then any amplitude of oscillation is reduced to $\sim\mu$m, and Q/M is observed to be rather stable with time. In Fig. 9 we have plotted the specific charge of a single anthracene particle that was stored in our trap for 2 months! After that period the particle was still bound, but the experiment was ceased, because we had to remove the setup from the laboratory.

For an estimate of the mass and charge of a trapped anthracene particle we assume a spherical shape and obtain with the density $\rho = 1.28$ g/cm^3 and a diameter $2R \approx 30$ μm $M \approx 1.9\times10^{-11}$ kg so that $Q \approx 1.5\times10^{-14}$ C. Then confined particles carry about 10^5 units of elementary charge.

In the setup shown in the lower part of Fig. 7 we can apply an additional ac voltage from an oscillator between the end caps. With amplitudes of some 100 V and at frequencies that scale with the driving frequency of the trap (50 Hz), beating phenomena in the motion of trapped particles can be seen. This is a simple demonstration for visualizing the (fast) micromotion.

Particles trapped in the pseudopotential can be excited with respect to the secular motion by the external ac voltage at the end caps. They can be forced to oscillate along z over a larger domain of the trap volume by increasing the amplitude of the ac signal (take care that the particles do not hit one of the end electrodes). A sudden turning off of the ac amplitude results in an instantaneous stopping of the particles. This clearly shows the strong damping of the motion for operating at atmospheric pressure. As a consequence, the resonance of the secular motion at $\bar{\omega}_z \approx 2\pi\times 18$ Hz is so strongly broadened that it is only poorly resolved.

Fig. 10. Two anthracene particles in the trap.

On the other hand, such a strong damping depletes most of the kinetic energy of the particles (see Sec. II), i.e., the particles are "cooled" very efficiently. Then, for confinement of more than one particle, the energies of interparticle Coulomb interactions are much larger than the "thermal" energies of individual particles. We then expect that the Coulomb coupling parameter $\Gamma = Q^2/\bar{r}kT \gg 1$, where $\bar{r}^3 = 4\pi N/3V$ (N = number of particles, and V = volume of system). Slattery *et al.* have found that for $\Gamma > 170$ ordered structures are obtained.[20] From the cooling limit of Brownian motion we deduce for the experiment Γ of about 10^5, i.e., well above the threshold for crystallization. However, this number is expected to be reduced clearly by "parasitic heating," a transfer of kinetic energy from the driven micromotion to the secular motion[21,22] via Coulomb interaction and collisions with the air.

Those structures were first observed for macroscopic particles.[12,13] In recent years such a crystallization also has been demonstrated with a small number of laser-cooled ions.[5,6] In Fig. 10 we display a stable arrangement of two particles in our setup. Generation of a structure with more particles is a question of the experimenter's skill. Up to several hundred particles have been stored in our setup. Figure 11 shows as an example the structure obtained for the confinement of about 30 particles.

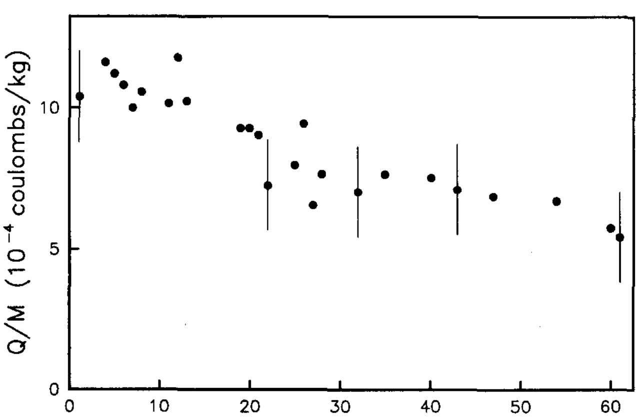


Fig. 9. Specific charge of a single anthracene particle that has been confined for 2 months.

Fig. 11. Stable structure which is observed for the storage of a larger number of particles.

IV. CONCLUSION

In conclusion, we have reported on a setup for demonstrating the storage of macroscopic particles in a "Paul trap." The simplicity of the experiment allows direct applications in lectures, laboratories, etc. which demonstrate features of the confinement of particles. Finally, we note that we also have succeeded in establishing a "Paul-storage-ring" by similar experimental procedures. In this ring, the lining up of particles can be visualized nicely, a property that one hopes will be achieved in future storage rings for fast and cooled ions.

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Motion of a leaky tank car

K. T. McDonald
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544

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A solution is presented to the following problem: Describe the motion of a tank car initially at rest once an off-center drain opens. The tank car rolls without friction on a horizontal surface, and the water flows out of the drain vertically in the rest frame of the car.

I. INTRODUCTION

The motion of a leaky tank car is surprisingly complex. We approach a solution in four steps: a brief introductory discussion of the motion, a discussion of the forces that cause the motion, a general analysis, and last, two detailed examples. (This problem has appeared in recent years on qualifying exams in Russia.)

There are no external horizontal forces on the system of tank car + water (including the water that has drained out), so the momentum is conserved and the center of mass (c.m.) of the whole system must remain fixed. As the water drains out of the off-center hole the tank car initially moves opposite to the direction of the drain to keep the c.m. fixed. But if this motion persisted until all the water drained out, then both the car and the fallen water would have momen-