The Full Strength of Cluster Gravitational Lensing: Mass Distribution of Galaxy Clusters from the CLASH Survey

Cluster Lensing And Supernova survey with Hubble

Keiichi Umetsu (ASIAA, Taiwan)
Cluster Gravitational Lensing

Key Objectives

**Intra-halo structure (1h)**
- Halo density profile, $\rho(r)$
- Halo mass, $M_\Delta$
- Concentration, $c = r_{200}/r_{-2}$
- Shape parameter, $\alpha_E$

\[
\frac{d \ln \rho(r)}{d \ln r} = -2 \left( \frac{r}{r_{-2}} \right)^{\alpha_E}
\]

**Surrounding LSS (2h)**
- Halo bias $b_h(M,z)$
- Assembly bias
- Clustering strength $\sigma_8$
Cluster Gravitational Lensing

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$$\frac{d \ln \rho(r)}{d \ln r} = -2 \left( \frac{r}{r_2} \right)^{\alpha_E}$$

Surrounding LSS (2h)
- Halo bias $b_h(M,z)$
- Assembly bias
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Weak Lensing Shear and Magnification

• **Shear**
  - Shape distortion: $\delta e_+ \sim \gamma_+ 

• **Magnification**
  - Flux amplification: $\mu F$
  - Area distortion: $\mu \Delta \Omega$

**Un-lensed sources**

**Lensed images**

**Sensitive to “modulated” matter density**

$$\Sigma_c \gamma_+ = \Delta \Sigma(R) \equiv \Sigma(<R) - \Sigma(R)$$

**Sensitive to “total” matter density**

$$\mu \approx 1 + 2\kappa; \quad \Sigma_c \kappa = \Sigma(R)$$
Shear doesn’t see mass sheet

Averaged lensing profiles in/around LCDM halos (Oguri & Hamana 11)

\[ \kappa = \Sigma(R) / \Sigma_c \]

\[ \gamma_+ = \Delta \Sigma(R) / \Sigma_c \]

• Tangential shear is a powerful probe of 1-halo term, or intra-halo structure.
• Shear alone cannot recover absolute mass, known as mass-sheet degeneracy:

\[ \gamma \text{ remains unchanged by } \kappa \rightarrow \kappa + \text{const.} \]
Combining Weak-Lensing Shear and Magnification

\[ p(\kappa \mid \text{WL}) \propto p(\text{WL} \mid \kappa) p(\kappa) = p(g_+ \mid \kappa) p(n_\mu \mid \kappa) p(\kappa) \]

*Subaru/Suprime-Cam data (e.g., Umetsu+11a,15a)*

- Mass-sheet degeneracy broken
- Total statistical precision improved by \( \sim 20-30\% \)
- Calibration uncertainties marginalized over:
  \[ c = \{ \langle W \rangle_s, f_{W,s}, \langle W \rangle_\mu, \overline{n}_\mu, s_{\text{eff}} \} \]
Multi-probe Lensing Approach
Combining azimuthally-averaged strong and weak lensing observables

\[ \{ M_{2D,i} \}_{i=1}^{N_{\text{SL}}} , \{ \langle g_{+},i \rangle \}_{i=1}^{N_{\text{WL}}} , \{ \langle n_{\mu,i} \rangle \}_{i=1}^{N_{\text{WL}}} . \]

\[ M_{2D}(R) = \int_{|R'|<R} \Sigma(R')d^2R' \]

\[ p(\kappa \mid WL, SL) \propto p(WL, SL \mid \kappa) p(\kappa) = p(g_+ \mid \kappa) p(n_{\mu} \mid \kappa) p(M_{2D} \mid \kappa) p(\kappa) \]
Multi-probe Lensing Approach

Combining azimuthally-averaged strong and weak lensing observables

\[
\begin{align*}
\{M_{2D,i}\}_{i=1}^{N_{SL}}, \{\langle g+,i \rangle_{i=1}^{N_{WL}}, \{\langle n_\mu,i \rangle_{i=1}^{N_{WL}}. \\
M_{2D}(R) &= \int_{|R'|<R} \Sigma(R')d^2R'
\end{align*}
\]

\[
p(\kappa | WL, SL) \propto p(WL, SL | \kappa) p(\kappa) = p(g_+ | \kappa) p(n_\mu | \kappa) p(M_{2D} | \kappa) p(\kappa)
\]

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Cluster Lensing And Supernova survey with Hubble

PI. Marc Postman (STScI)
http://www.stsci.edu/~postman/CLASH/Home.html
CLASH Objectives & Motivation

Before CLASH (2010), deep-multicolor Strong (HST) + Weak (Subaru) lensing data only available for a handful of “super lens” clusters

Total mass profile shape: consistent w self-similar NFW (cf. Newman+13; Okabe+13)
Degree of concentration: predicted superlens correction not enough if \( c_{\text{LCDM}} \simeq 3 \)?
Before CLASH (2010), deep-multicolor Strong (HST) + Weak (Subaru) lensing data only available for a handful of “super lens” clusters. 

Total mass profile shape: consistent w self-similar NFW (cf. Newman+13; Okabe+13)
Degree of concentration: predicted superlens correction is just enough if $c_{\text{LCDM}} \sim 4$
CLASH X-ray-selected Subsample

• High-mass clusters with smooth X-ray morphology
  – $T_x > 5\text{keV} \ (M_{200c} > 5\times10^{14}M_{\text{sun}}/h)$
  – Small BCG to X-ray-peak offset, $\sigma_{\text{off}} \sim 10\text{kpc}/h$
  – Smooth regular X-ray morphology

  ➔ Optimized for radial-profile analysis

• CLASH theoretical predictions (Meneghetti+14)
  – Composite relaxed (70%) and unrelaxed (30%) clusters
  – Mean $\langle c_{200c} \rangle = 3.9$, $c_{200c} = [3, 6]$
  – Small scatter in $c_{200c}$: $\sigma(\ln c_{200c}) = 0.16$
  – Largely free of orientation bias ($\sim2\%$ in $\langle M_{3D} \rangle$)
  – $>90\%$ of CLASH clusters to have strong-lensing features
CLASH: Joint Analysis of Strong-lensing, Weak-lensing Shear and Magnification Data for 20 CLASH Galaxy Clusters

Umetsu et al. 2015b, arXiv:1507.04385
(submitted to ApJ in mid July)
CLASH HST Lensing Dataset

CLASH *Subaru* Weak-lensing Dataset

No WL data for M1311, M2129

High-resolution space imaging with HST (ACS/WFC3) for strong lensing

Subaru/Suprime-Cam multi-color imaging for wide-field

34 arcmin
Joint Analysis of Strong-lensing, Weak-lensing Shear and Magnification Constraints

\[ \{M_{2D,i}\}_{i=1}^{N_{SL}}, \{\langle g_+,i \rangle\}_{i=1}^{N_{WL}}, \{\langle n_\mu,i \rangle\}_{i=1}^{N_{WL}}. \]

**HST** multiple-image constraints on \(M_{2D}(<R)\) (Zitrin et al. 15, ApJ, 801, 44)

\[ \Delta = 10''(R_{\text{Ein}}/22'')^{1/2}(N/17)^{-1/2} \text{ sampling, } R_{\text{max}} \sim 2<R_{\text{Ein}}> \sim 40'' \]

Strong-lensing mass integration radii: \(R=(10'', 20'', 30'', 40'')\)

\[ \left\langle \chi^2 / \text{dof} \right\rangle = 0.95 \text{ for 20 CLASH clusters} \]
CLASH Stacked Full-lensing Analysis of the X-ray-selected Subsample

Umetsu et al. 2015b, arXiv:1507.04385
Ensemble-averaged Surface Mass Density Profile

\[ \langle \Sigma(R) \rangle = \int \overline{\rho} \xi_{hm}(r \mid M) dx \]

- HST strong lensing
- Subaru weak lensing

33σ detection of the ensemble-averaged mass profile out to \( \sim 2r_{200m} \)
Characterizing the Averaged Mass Profile Shape

\[ \Sigma(R) = \int dl \Delta \rho(r), \]

**Models:**

1. No 2-halo term, no truncation \((f_t=1, \rho_{2h}=0)\)
2. With 2-halo term (Tinker+10)

\[ \Delta \rho(r) = f_t(r) \rho_h(r) + \rho_{2h}(r), \]

\[ f_t(r) = \left[ 1 + \left( \frac{r}{r_t} \right)^2 \right]^{-2}, \]
Comparison of Best-fit Models

Acceptable fits: $\rho$ values (PTE) > 0.05

<table>
<thead>
<tr>
<th>Model</th>
<th>$M_{200c}$ ($10^{14} M_\odot h^{-1}$)</th>
<th>$c_{200c}$</th>
<th>Shape/structural parameters</th>
<th>$b_h$</th>
<th>$\chi^2$/dof</th>
<th>PTEa</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFW</td>
<td>$14.4_{-1.0}^{+1.4}$</td>
<td>$3.76_{-0.27}^{+0.39}$</td>
<td>$\gamma_c = 1$</td>
<td></td>
<td>11.3/11</td>
<td>0.419</td>
<td>No truncation</td>
</tr>
<tr>
<td>gNFW</td>
<td>$14.1_{-1.1}^{+1.1}$</td>
<td>$4.04_{-0.32}^{+0.53}$</td>
<td>$\gamma_c = 0.85_{-0.31}^{+0.22}$</td>
<td></td>
<td>10.9/10</td>
<td>0.366</td>
<td>No truncation</td>
</tr>
<tr>
<td>Einasto</td>
<td>$14.7_{-1.1}^{+1.1}$</td>
<td>$3.53_{-0.36}^{+0.39}$</td>
<td>$\alpha_E = 0.232_{-0.038}^{+0.042}$</td>
<td></td>
<td>11.7/10</td>
<td>0.306</td>
<td>No truncation</td>
</tr>
<tr>
<td>DARKexp-$\gamma^b$</td>
<td>$14.5_{-1.1}^{+1.1}$</td>
<td>$3.53_{-0.42}^{+0.41}$</td>
<td>$\phi_0 = 3.90_{-0.45}^{+0.44}$</td>
<td></td>
<td>13.5/10</td>
<td>0.198</td>
<td>No truncation</td>
</tr>
<tr>
<td>Pseudo isotermal</td>
<td></td>
<td></td>
<td>$V_c = 1762_{-39}^{+40}$ km/s, $r_c = 69_{-7}^{+7}$ kpc</td>
<td></td>
<td>23.6/11</td>
<td>0.015</td>
<td>No truncation</td>
</tr>
<tr>
<td>Burkert</td>
<td>$11.6_{-0.8}^{+0.8}$</td>
<td></td>
<td>$r_{200c}/r_0 = 8.81_{-0.41}^{+0.42}$</td>
<td></td>
<td>29.9/11</td>
<td>0.002</td>
<td>No truncation</td>
</tr>
<tr>
<td>Power-law sphere</td>
<td>$12.5_{-0.8}^{+0.8}$</td>
<td></td>
<td>$\gamma_c = 1.78_{-0.02}^{+0.02}$</td>
<td></td>
<td>93.5/11</td>
<td>0.000</td>
<td>No truncation</td>
</tr>
</tbody>
</table>

Halo model$^c$:

<table>
<thead>
<tr>
<th>Model</th>
<th>$M_{200c}$ ($10^{14} M_\odot h^{-1}$)</th>
<th>$c_{200c}$</th>
<th>Shape/structural parameters</th>
<th>$b_h$</th>
<th>$\chi^2$/dof</th>
<th>PTEa</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFW+LSS (i)</td>
<td>$14.1_{-1.0}^{+1.4}$</td>
<td>$3.79_{-0.28}^{+0.30}$</td>
<td>$\gamma_c = 1$</td>
<td></td>
<td>9.3</td>
<td>0.450</td>
<td>$\Lambda$CDM $b_h(M)$ scaling</td>
</tr>
<tr>
<td>NFW+LSS (ii)</td>
<td>$14.4_{-1.3}^{+1.4}$</td>
<td>$3.74_{-0.30}^{+0.33}$</td>
<td>$\gamma_c = 1$</td>
<td>$7.4_{-4.7}^{+4.6}$</td>
<td>10.8/10</td>
<td>0.377</td>
<td>$b_h$ as a free parameter</td>
</tr>
<tr>
<td>Einasto+LSS (i)</td>
<td>$14.3_{-1.1}^{+1.1}$</td>
<td>$3.69_{-0.42}^{+0.36}$</td>
<td>$\alpha_E = 0.248_{-0.047}^{+0.051}$</td>
<td></td>
<td>9.3</td>
<td>0.385</td>
<td>$\Lambda$CDM $b_h(M)$ scaling</td>
</tr>
<tr>
<td>Einasto+LSS (ii)</td>
<td>$14.5_{-1.6}^{+1.9}$</td>
<td>$3.65_{-0.51}^{+0.47}$</td>
<td>$\alpha_E = 0.245_{-0.053}^{+0.047}$</td>
<td></td>
<td>9.3</td>
<td>0.301</td>
<td>$b_h$ as a free parameter</td>
</tr>
<tr>
<td>DARKexp+LSS (i)</td>
<td>$14.2_{-1.1}^{+1.2}$</td>
<td>$3.64_{-0.46}^{+0.44}$</td>
<td>$\phi_0 = 3.89_{-0.54}^{+0.51}$</td>
<td></td>
<td>9.3</td>
<td>0.308</td>
<td>$\Lambda$CDM $b_h(M)$ scaling</td>
</tr>
<tr>
<td>DARKexp+LSS (ii)</td>
<td>$14.0_{-1.6}^{+1.8}$</td>
<td>$3.69_{-0.57}^{+0.53}$</td>
<td>$\phi_0 = 3.85_{-0.61}^{+0.57}$</td>
<td>$10.1_{-5.1}^{+4.9}$</td>
<td>11.6/9</td>
<td>0.235</td>
<td>$b_h$ as a free parameter</td>
</tr>
</tbody>
</table>

$a$ Probability to exceed the observed $\chi^2$ value.

$b$ We use Dehnen–Tremaine $\gamma$-models with the central cusp slope $\gamma_c = 3 \log_{10} \phi_0 - 0.65 (1.7 \leq \phi_0 \leq 6)$ as an analytic fitting function for the DARKexp density profile.

$c$ For halo model predictions, we decompose the total mass overdensity $\Delta \rho(r) = \rho(r) - \rho_{\text{m}}$ as $\Delta \rho = f_{t} \rho_{h} + \rho_{2h}$, where $\rho_{h}(r)$ is the halo density profile, $\rho_{2h}(r) = \rho_{\text{m}} b_{h} \xi_m(r)$ is the two-halo term, and $f_{t}(r) = (1 + r^2/r_t^2)^{-2}$ describes the steepening of the density profile in the transition regime around the truncation radius $r_t$, which is assumed to be $r_t = 3 r_{200c}$.

- Consistent with cuspy density profiles (NFW, Einasto, DARKexp)
- Cuspy models that include $\Lambda$CDM 2-halo term ($b_h \approx 9.3$) give improved fits
- The best model reproduces the observed Einstein radius, $R_{\text{Ein}} \approx 20''$ at $z_s=2$
Concentration—Mass Relation of the CLASH X-ray-selected Subsample

Umetsu et al. 2015b, arXiv:1507.04385
Concentration—Mass Scaling Relation

Consider a power-law scaling relation of the form:

\[ c_{200c} = 10^\alpha \left( \frac{M_{200c}}{M_{\text{piv}}} \right)^\beta \left( \frac{1+z}{1+z_{\text{piv}}} \right)^\gamma, \]

with pivot mass and redshift \( M_{\text{piv}} = 10^{15} M_\text{sun} / h, z_{\text{piv}} = 0.34 \)

Define new independent \((X)\) and dependent \((Y)\) variables:

\[ Y \equiv \log_{10} \left[ \left( \frac{1+z}{1+z_{\text{piv}}} \right)^{-\gamma} c_{200c} \right], \]
\[ X \equiv \log_{10} \left( \frac{M_{200c}}{M_{\text{piv}}} \right). \]

Redshift slope \( \gamma \) is fixed to the theoretical prediction for the CLASH sample, \( \gamma = -0.668 \) (Meneghetti+14)
Bayesian Regression Analysis

We take into account:

- Covariance between observed $M$ and $c$
- Intrinsic scatter in $c$
- Non-uniformity in mass probability distribution $P(\log M)$

**Conditional probability** $P(y|x)$ with $(x,y) = \text{observed (X,Y)}$

\[
\ln P(y|x) = -\frac{1}{2} \sum_n \left[ \ln (2\pi \sigma_n^2) + \left( \frac{y_n - \langle y_n|x_n \rangle}{\sigma_n} \right)^2 \right],
\]

where $\langle y_n|x_n \rangle$ and $\sigma_n^2 \equiv \text{Var}(y_n|x_n)$ are the conditional mean and variance of $y_n$ given $x_n$, respectively:

\[
\langle y_n|x_n \rangle = \alpha + \beta \mu + \frac{\beta \tau^2 + C_{xy,n}}{\tau^2 + C_{xx,n}} (x_n - \mu),
\]

\[
\sigma_n^2 = \beta^2 \tau^2 + \sigma_{Y|X}^2 + C_{yy,n} - \frac{(\beta \tau^2 + C_{xy,n})^2}{\tau^2 + C_{xx,n}},
\]

where $\sigma_{Y|X}$ is the intrinsic scatter in the $Y-X$ relation;
Marginalized Posterior Distributions

\[ c_{200c} = 10^\alpha \left( \frac{M_{200c}}{M_{\text{piv}}} \right)^\beta \left( \frac{1 + z}{1 + z_{\text{piv}}} \right)^\gamma \]

- \( \alpha \): intercept
- \( \beta \): slope
- \( \sigma_{Y|X} \): scatter
- \( \mu \): Gaussian mean of \( P(\ln M) \)
- \( \tau \): Gaussian width of \( P(\ln M) \)

High \( \beta \) tail associated with small \( \tau \): i.e., localized \( P(\ln M) \)
CLASH: Lensing Observations vs. Predictions

Normalization, slope, & scatter are all consistent with LCDM when the CLASH selection function based on X-ray morphological regularity and the projection effects are taken into account.

\[ c_{200c} \bigg|_{z=0.34} = 3.95 \pm 0.35 \]

at \[ M_{200c} = 10^{15} M_{\odot} / h, \]

\[ \sigma(\ln c_{200c}) = 0.13 \pm 0.06 \]
Comparison with LCDM Models

Table 5
Comparison of measured and predicted concentrations for the CLASH X-ray-selected subsample

<table>
<thead>
<tr>
<th>Author</th>
<th>Sample</th>
<th>3D/2D</th>
<th>Function$^a$</th>
<th>$c^{(\text{obs})}/c^{(\text{pred})}$</th>
<th>$\chi^2$</th>
<th>PTE$^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Average$^c$</td>
<td>$\sigma^d$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theory:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Duffy et al. (2008)</td>
<td>full</td>
<td>3D</td>
<td>$c-M$</td>
<td>1.331 ± 0.108</td>
<td>0.334</td>
<td>22.6</td>
</tr>
<tr>
<td>Duffy et al. (2008)</td>
<td>relaxed</td>
<td>3D</td>
<td>$c-M$</td>
<td>1.165 ± 0.094</td>
<td>0.290</td>
<td>13.6</td>
</tr>
<tr>
<td>Prada et al. (2012)</td>
<td>full</td>
<td>3D</td>
<td>$c-\nu$</td>
<td>0.733 ± 0.065</td>
<td>0.244</td>
<td>24.6</td>
</tr>
<tr>
<td>Bhattacharya et al. (2013)</td>
<td>full</td>
<td>3D</td>
<td>$c-\nu$</td>
<td>1.169 ± 0.095</td>
<td>0.292</td>
<td>14.1</td>
</tr>
<tr>
<td>Bhattacharya et al. (2013)</td>
<td>relaxed</td>
<td>3D</td>
<td>$c-\nu$</td>
<td>1.131 ± 0.092</td>
<td>0.277</td>
<td>12.4</td>
</tr>
<tr>
<td>Dutton &amp; Macciò (2014)</td>
<td>full</td>
<td>3D</td>
<td>$c-M$</td>
<td>1.061 ± 0.086</td>
<td>0.262</td>
<td>10.4</td>
</tr>
<tr>
<td>Meneghetti et al. (2014)</td>
<td>full</td>
<td>3D</td>
<td>$c-M$</td>
<td>1.061 ± 0.089</td>
<td>0.279</td>
<td>10.2</td>
</tr>
<tr>
<td>Meneghetti et al. (2014)</td>
<td>relaxed</td>
<td>3D</td>
<td>$c-M$</td>
<td>0.990 ± 0.083</td>
<td>0.249</td>
<td>9.2</td>
</tr>
<tr>
<td>Diemer &amp; Kravtsov (2015)</td>
<td>full (median)</td>
<td>3D</td>
<td>$c-\nu$</td>
<td>1.021 ± 0.083</td>
<td>0.330</td>
<td>14.4</td>
</tr>
<tr>
<td>Diemer &amp; Kravtsov (2015)</td>
<td>full (mean)</td>
<td>3D</td>
<td>$c-\nu$</td>
<td>1.060 ± 0.086</td>
<td>0.326</td>
<td>13.8</td>
</tr>
<tr>
<td>Meneghetti et al. (2014)</td>
<td>full</td>
<td>2D</td>
<td>$c-M$</td>
<td>1.087 ± 0.092</td>
<td>0.336</td>
<td>13.5</td>
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<tr>
<td>Meneghetti et al. (2014)</td>
<td>relaxed</td>
<td>2D</td>
<td>$c-M$</td>
<td>1.040 ± 0.086</td>
<td>0.283</td>
<td>10.8</td>
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<tr>
<td>Meneghetti et al. (2014)</td>
<td>CLASH</td>
<td>2D</td>
<td>$c-M$</td>
<td>0.988 ± 0.078</td>
<td>0.227</td>
<td>9.6</td>
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<tr>
<td>Observations:</td>
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<tr>
<td>Merten et al. (2015)</td>
<td>CLASH</td>
<td>2D</td>
<td>$c-M$</td>
<td>1.133 ± 0.087</td>
<td>0.209</td>
<td>9.2</td>
</tr>
</tbody>
</table>

$^a$ $c-M$: power-law $c(M, z)$ relation; $c-\nu$: halo concentration given as a function of peak height $\nu(M, z)$.

$^b$ Probability to exceed the measured $\chi^2$ value assuming the standard $\chi^2$ probability distribution function.

$^c$ Weighted geometric average of observed-to-predicted concentration ratios.

$^d$ Standard deviation of the distribution of observed-to-predicted concentration ratios.

- Consistent with models that are calibrated for more recent cosmologies (WMAP7 and later)
- Better agreement is achieved when selection effects (overall degree of relaxation) are taken into account
**X-ray Regular vs. Superlens Clusters**

Umetsu+11b: 4 *superlens* clusters with $R_{\text{Ein}} > 30''$ at $z_s = 2$ (A1689, A1703, Cl0024, A370)

Higher normalization LCDM cosmology (WMAP7 and later) + “predicted” +60% superlens correction (e.g., Oguri+Blandford09) can explain superlens mass profiles!
Einasto Shape Parameter vs. Halo Mass

\( \alpha_E \): degree of curvature of the Einasto density profile

\[
\frac{d \ln \rho(r)}{d \ln r} = -2 \left( \frac{r}{r_-} \right)^{\alpha_E}
\]

\( \alpha_E \approx 0.155 + 0.0095 \nu^2 \) (Gao +08)

\[
\nu = \frac{\delta_c}{\sigma(M)}
\]

**Preliminary results**
Einasto Shape Parameter vs. Halo Mass

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$$\frac{d \ln \rho(r)}{d \ln r} = -2 \left( \frac{r}{r_2} \right)^{\alpha_E}$$

$\alpha_E \approx 0.155 + 0.0095 \nu^2$ (Gao + 08)

$\nu = \frac{\delta_c}{\sigma(M)}$

Preliminary results
Einasto Shape Parameter vs. Halo Peak Height

\( \alpha_E \): degree of curvature of the Einasto density profile

\[
\frac{d \ln \rho(r)}{d \ln r} = -2 \left( \frac{r}{r_2} \right)^{\alpha_E}
\]

\( \alpha_E \approx 0.155 + 0.0095\nu^2 \) (Gao +08)

\[
\nu = \frac{\delta_c}{\sigma(M)}
\]

---

**Preliminary results**
Ensemble Calibration of Cluster Masses

Umetsu et al. 2015b, arXiv:1507.04385
Planck13 CMB vs. Cluster Cosmology

- Planck: $3\sigma$ tension between SZ cluster counts and CMB cosmology
- Assumes $M_{\text{Planck}} / M_{\text{true}} = (1-b) = 0.8$
- Calibrated with XMM hydrostatic masses (Arnaud et al. 2010) + simulations

suggested explanations:

- **mass bias underestimated** (and no accounting for uncertainties)
- $2.9\sigma$ detection of neutrino masses: $\Sigma m_\nu = (0.58 \pm 0.20) \text{ eV}$
  (Planck+WMAPpol+ACT+BAO: $\Sigma m_\nu < 0.23 \text{ eV, 95% CL}$)

*Slide taken from Anja von der Linden’s presentation*
Comparison with *Planck* Masses: *It’s not so simple!!!*

Mass-dependent bias (20-45%) observed for *Planck* mass estimates

![Graph showing comparison between CLASH-WL and Planck masses](image)

- $b \sim 0.2$
- $b = \text{const.} = 0.2$
- Fiducial value assumed by the Planck team
- $b \sim 0.45$

Sereno, Ettori, & Moscardini 15, CoMaLit II (arXiv:1407.7869)
CLASH Internal Consistency

$M(<r)$ de-projected assuming spherical NFW density profiles

$$\left\langle \frac{M_{3D}(WL + SL)}{M_{3D}(WL)} \right\rangle$$

Individual clusters
1:1 relation
Geometric mean
1σ uncertainty

Systematic uncertainty in the overall mass calibration is empirically derived to be < 5%, which is insignificant compared to the statistical uncertainty of ~6% with N=20 clusters

Umetsu+15b, arXiv:1507.04385
Mass Comparisons with Other WL Surveys

**WtG** [Subaru] (Applegate+14)

LoCuSS [Subaru] (Okabe & Smith 15)

CCCP [CFHT] (Hoekstra+15)

17 clusters

5 clusters

6 clusters

\[
\begin{align*}
\langle M_{\text{WtG}} / M_{\text{CLASH}} \rangle &= 1.03 \pm 0.09 \quad (\Delta = 200) \\
&= 1.07 \pm 1.12 \quad (\Delta = 500) \\
&= 1.07 \pm 1.12 \quad (\Delta = 1000) \\
\langle M_{\text{LoCuSS}} / M_{\text{CLASH}} \rangle &= 1.00 \pm 0.15 \quad (\Delta = \Delta_{\text{vir}}) \\
&= 0.98 \pm 0.13 \quad (\Delta = 200) \\
&= 0.93 \pm 0.10 \quad (\Delta = 500) \\
&= 0.84 \pm 0.22 \quad (\Delta = 2500) \\
\langle M_{\text{CCCP}} / M_{\text{CLASH}} \rangle &= 0.84 \pm 0.10 \quad (\Delta = 500) \\
&= 0.91 \pm 0.24 \quad (\Delta = 2500)
\end{align*}
\]

Umetsu+15b, arXiv:1507.04385
Summary

- **Ensemble-averaged mass profile shape**
  - Data favor cuspy density profiles predicted for collisionless-DM-dominated halos in gravitational equilibrium (NFW, Einasto, DARKexp)
  - The highest-ranked model is the 2-parameter NFW+LSS model including the 2-halo term using the LCDM $b$-$M$ relation ($b_h \sim 9.3$)
  - $c_{200c} = 3.8 \pm 0.3$ at $M_{200c}=10^{15}M_{\odot}/h$, $z=0.34$

- **Concentration vs. mass relation**
  - Fully consistent with LCDM when the CLASH selection function based on X-ray morphological regularity and the projection effects are taken into account

- **Mass calibration**
  - Internal consistency better than 5% +/- 6% by comparison with the WL-only analysis of Umetsu et al. (2014)
Reionization Lensing Cluster Survey (RELICS)

Newly approved 190-orbit *HST* survey (7 ACS/WFC3 filters) of 41 high-mass clusters primarily selected from the *Planck* survey (P.I. Dan Coe; Oct 2015 – Apr 2017)

http://hstrelics.weebly.com
Supplemental Slides
Ensemble-averaged Error Budget

Diagonal elements \( (C_{ii}) \) averaged over all CLASH clusters

Residual mass-sheet uncertainty (Umetsu+14)

\[
(C_{\text{sys}})_{ii} \sim \text{const.} \sim (0.02)^2
\]

Intrinsic profile variations due to triaxiality, substructure, and c-M scatter (Gruen+15)

\[
(C_{\text{int}})_{ii} \approx (0.2)^2 \kappa_i^2
\]
In hierarchical structure formation, $<c>$ is predicted to correlate with $M$.

DM halos that are more massive collapse later on average, when the mean background density of the universe is correspondingly lower (e.g., Bullock+01).
Concentration is sensitive to cosmology

$c \sim 2.9$ vs. $3.6$ @ $10^{15}M_{\text{sun}}/h$

Dutton & Maccio 2014
Intrinsic Scatter in $c(M)$: Mass Assembly Histories (MAH)

- Scatter is due to another DoF ($\alpha$), related to MAH (Ludlow+13)
- Larger values of $\alpha$ correspond to halos that have been assembled more rapidly than the NFW curve
- Halos with average $c_{200}$ have the NFW-equivalent $\alpha \sim 0.18$
Key Predictions of nonlinear structure formation models

(3) Halo bias: surrounding large-scale structure

$$\delta(x) := \frac{\rho - \bar{\rho}}{\bar{\rho}} = \int \frac{d^3k}{(2\pi)^3} \tilde{\delta}(k) e^{ik \cdot x}$$

$$\left\langle \tilde{\delta}(k) \tilde{\delta}(k') \right\rangle = (2\pi)^3 \delta_D^3(k + k') P(k)$$
Halo Bias Factor: $b_h$

Clustering of matter around halos with $M$: 

$$
\xi_{m} (r | M) \equiv \langle \delta_h (x | M) \delta_m (x + r) \rangle 
$$

$$
= \frac{\langle \rho_h (r | M) \rangle}{\bar{\rho}} + b_h (M) \xi_{mm} (r)
$$

2h term

**Correlated matter distribution (2h term)**

**Matter correlation function:** 

$$
\xi_{mm} (r) \equiv \langle \delta_m (x) \delta_m (x + r) \rangle = \int \frac{d^3k}{(2\pi)^3} P(k) e^{i k \cdot r}
$$

$$
\propto \sigma_8^2
$$

**Linear halo bias:** 

$$
b_h (\nu) \approx 1 + \frac{\nu^2 - 1}{\delta_c}
$$

$$
\nu \equiv \frac{\delta_c}{\sigma (M, z)} \sim 3 - 4 \text{ for clusters}
$$

Tinker+10 LCDM simulations
Non-local substructure effect

A substructure at $R \sim r_{\text{vir}}$ of the main halo, modulating

$$\Delta \Sigma(R) = \Sigma(< R) - \Sigma(R)$$

Known 5%-10% negative bias in mass estimates from tangential-shear fitting, inherent to rich substructure in outskirts (Rasia+12)
Magnification bias effects

Flux-limited source counts:

\[ n_{\text{obs}} (> f) = \mu^{-1} n(> \mu^{-1} f) \]

Broadhurst, Taylor & Peacock 1995

Geometric area distortion

\[ \frac{n}{\mu} \]

Flux amplification
Averaged Halo Density Profile $\Sigma(R)$

Stacking lensing signals of individual clusters by

$$\langle \Sigma \rangle = \left( \sum_n W_n \right)^{-1} \left( \sum_n W_n \Sigma_n \right),$$

Summing over clusters ($n=1, 2, ..$)

with individual “sensitivity” matrix

$$(W_n)_{ij} \equiv \Sigma_{(c,\infty)n}^{-2} \left(C_n^{-1}\right)_{ij},$$

defined with total covariance matrix

$$C = C^{\text{stat}} + C^{\text{sys}} + C^{\text{lss}} + C^{\text{int}},$$

With “trace-approximation”, averaging (stacking) is interpreted as

$$\langle M_\Delta \rangle = \frac{\sum_n \text{tr}(W_n) M_{\Delta,n}}{\sum_n \text{tr}(W_n)}$$