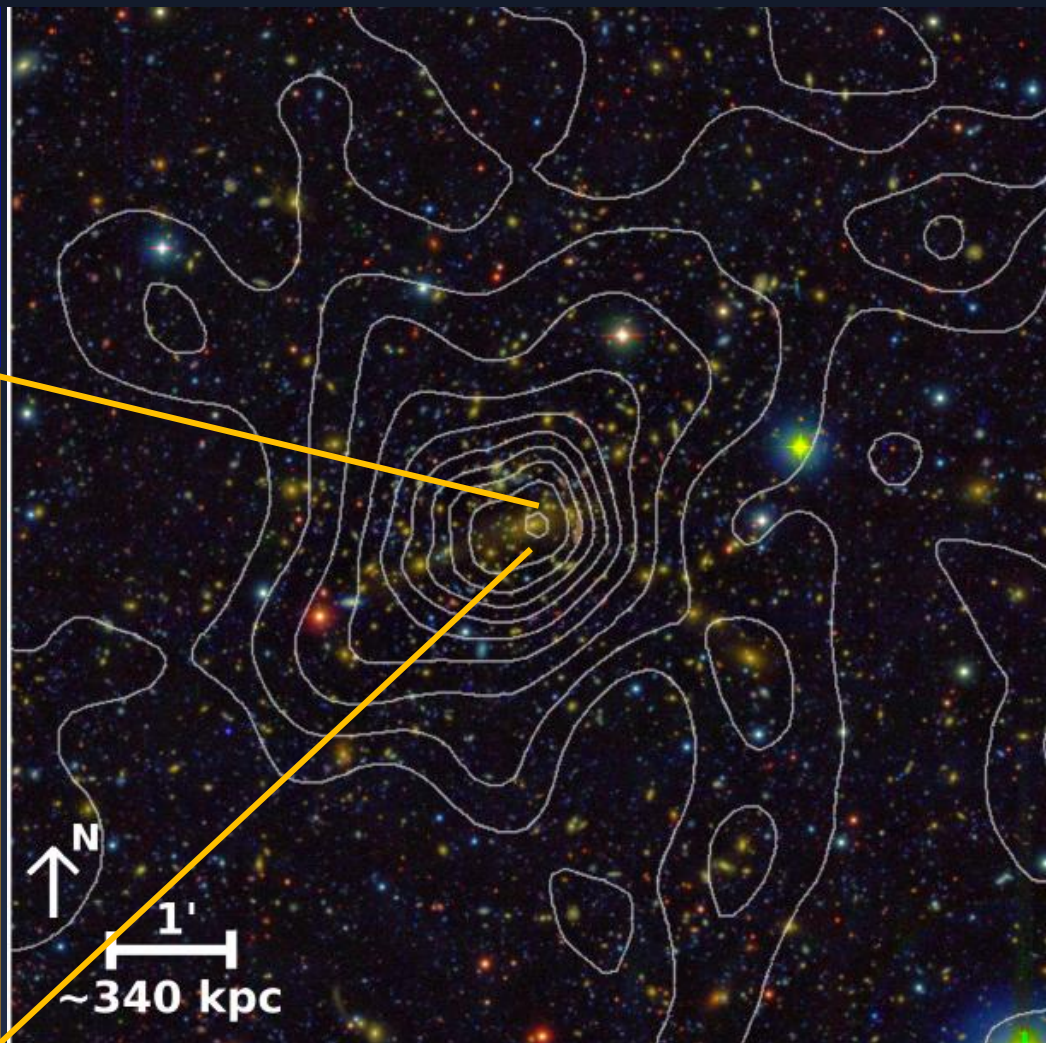
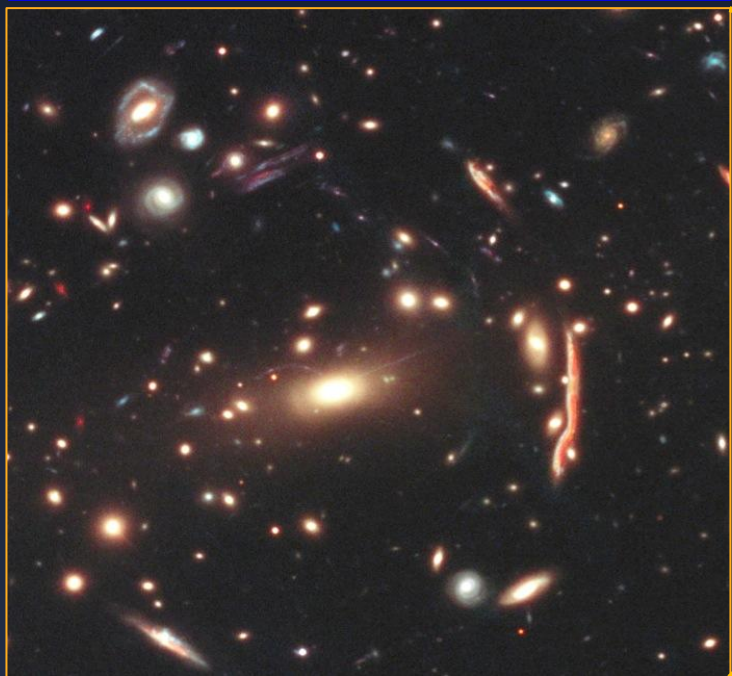


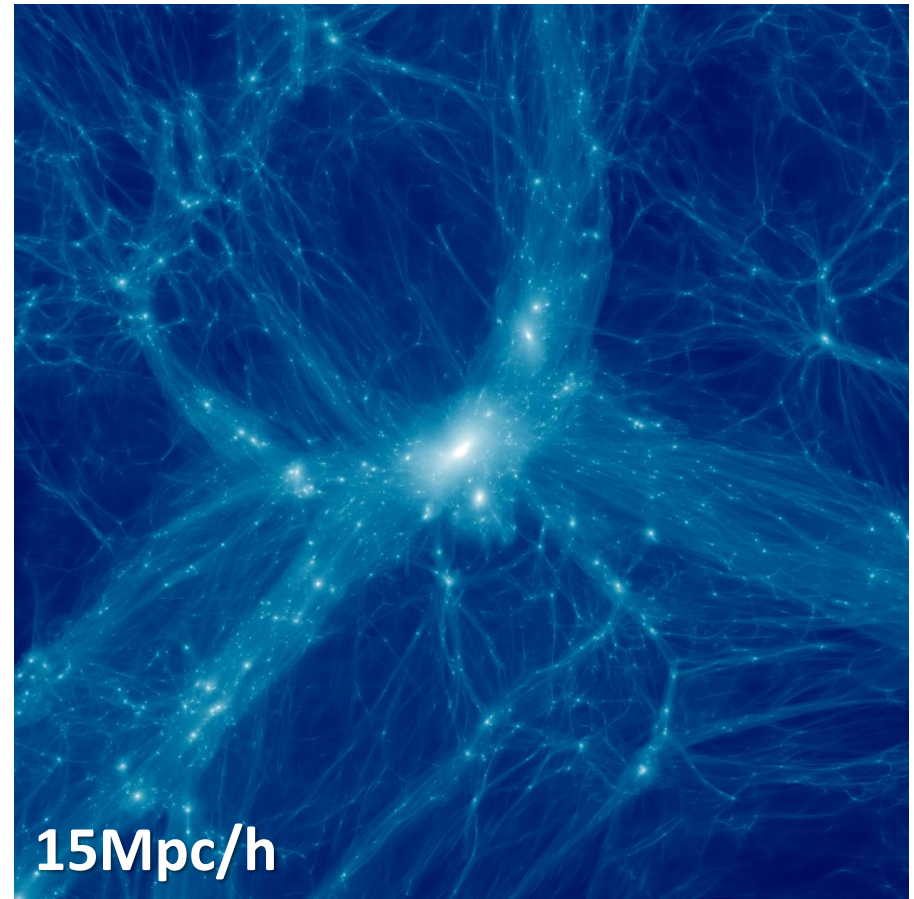
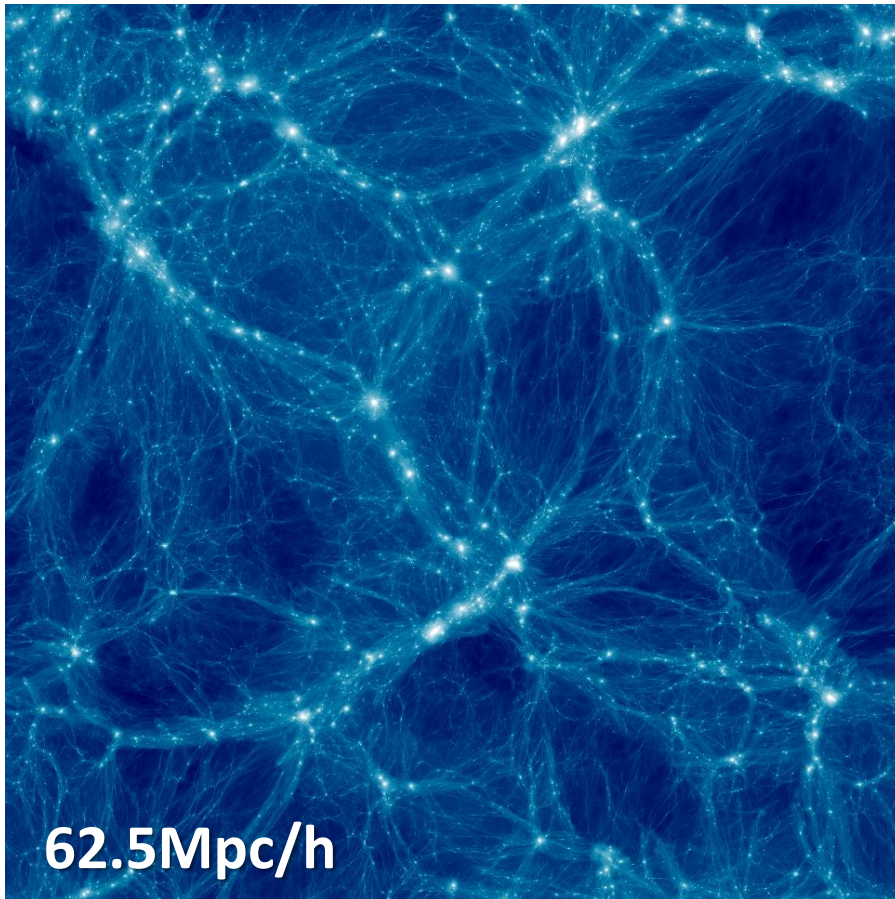
# Recent Progress in Cluster Weak Lensing

Cluster Lensing And Supernova survey with Hubble



# Galaxy Clusters as Cosmological Probe

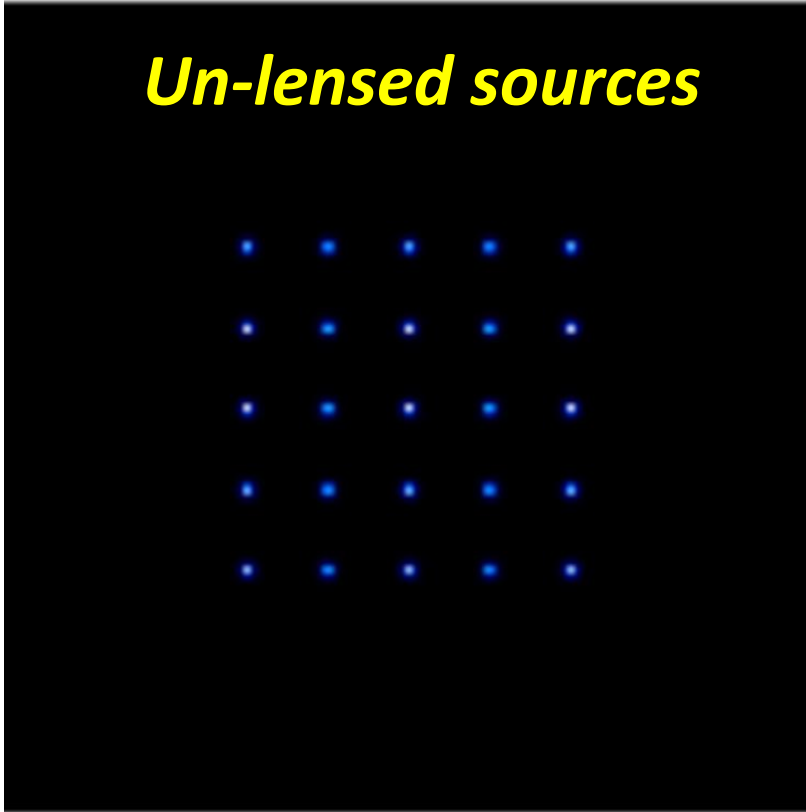
Cluster counts  $n(M, z)$  are exponentially sensitive to *cosmology*,  
but also to *mass calibration*!!!



# Weak Gravitational Lensing

Idlis & Gridneva 1960

*Un-lensed sources*



*Lensed images*



- **Shear** (Kaiser 92, 93, 95)

✓ Shape distortion:  $\delta e \sim \gamma$

*Sensitive to “modulated” matter density*

$$\Sigma_c \gamma_+ = \Delta \Sigma(R) \equiv \Sigma(< R) - \Sigma(R)$$

- **Magnification** (Broadhurst 95)

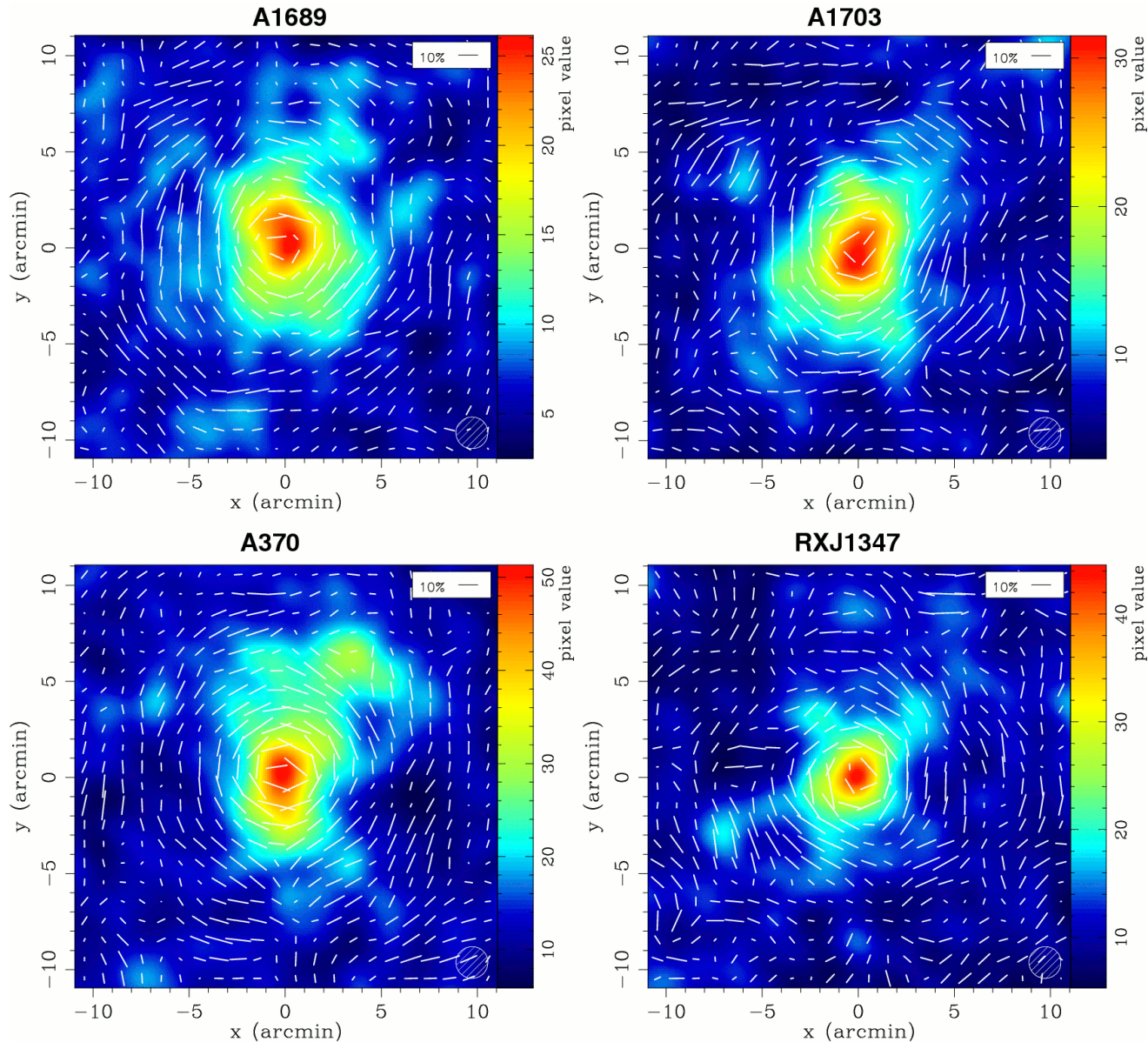
✓ Flux amplification:  $\mu F$

*Sensitive to “total” matter density*

$$\mu \approx 1 + 2\kappa; \quad \Sigma_c \kappa = \Sigma(R)$$

✓ Area distortion:  $\mu \Delta \Omega$

# Shear fields around Tom's favorite clusters

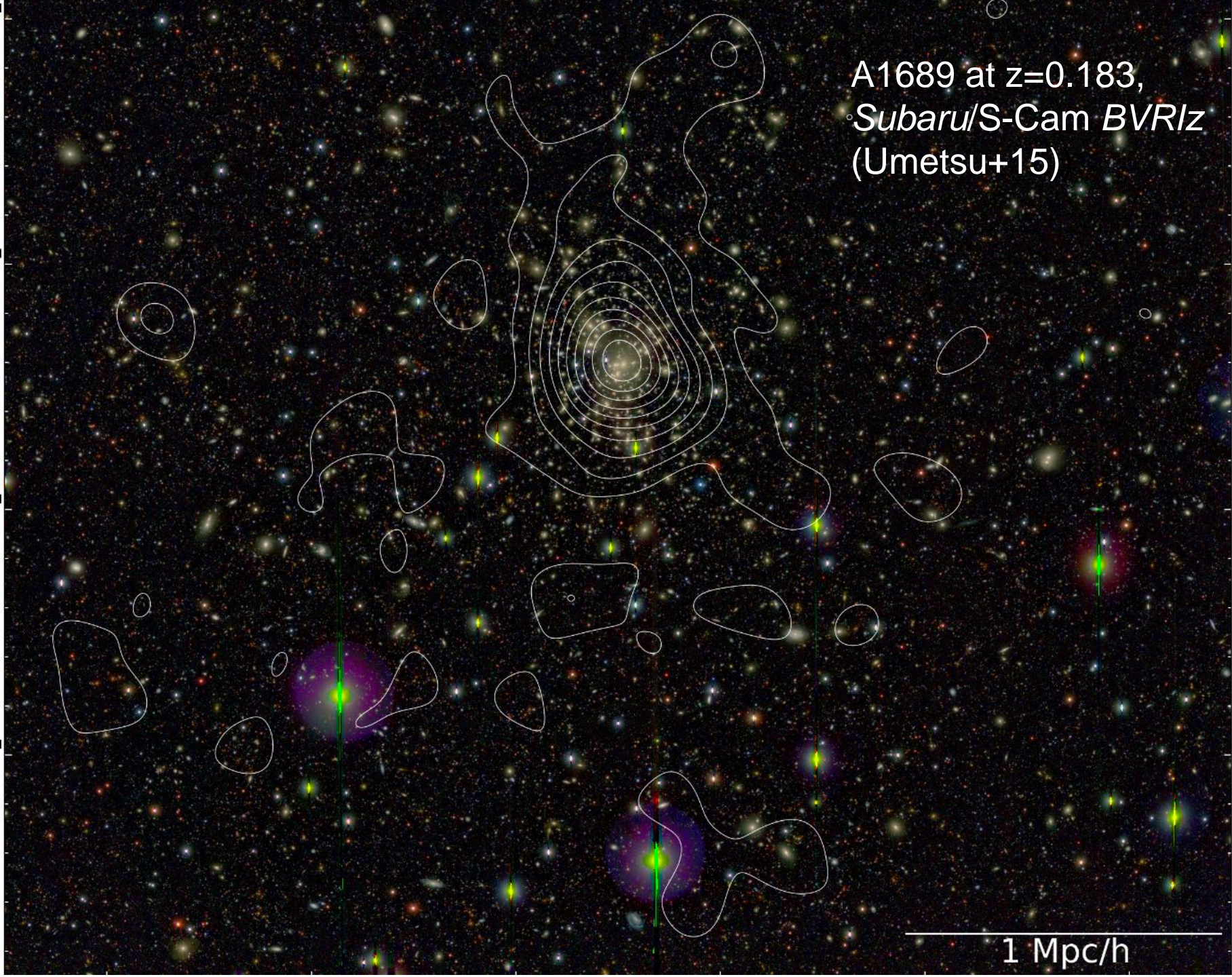


Map: **RS** galaxies  
Whiskers: **shear**

*Subaru*  
Suprime-Cam  
archival data

Broadhurst,  
Umetsu,  
Medezinski+08

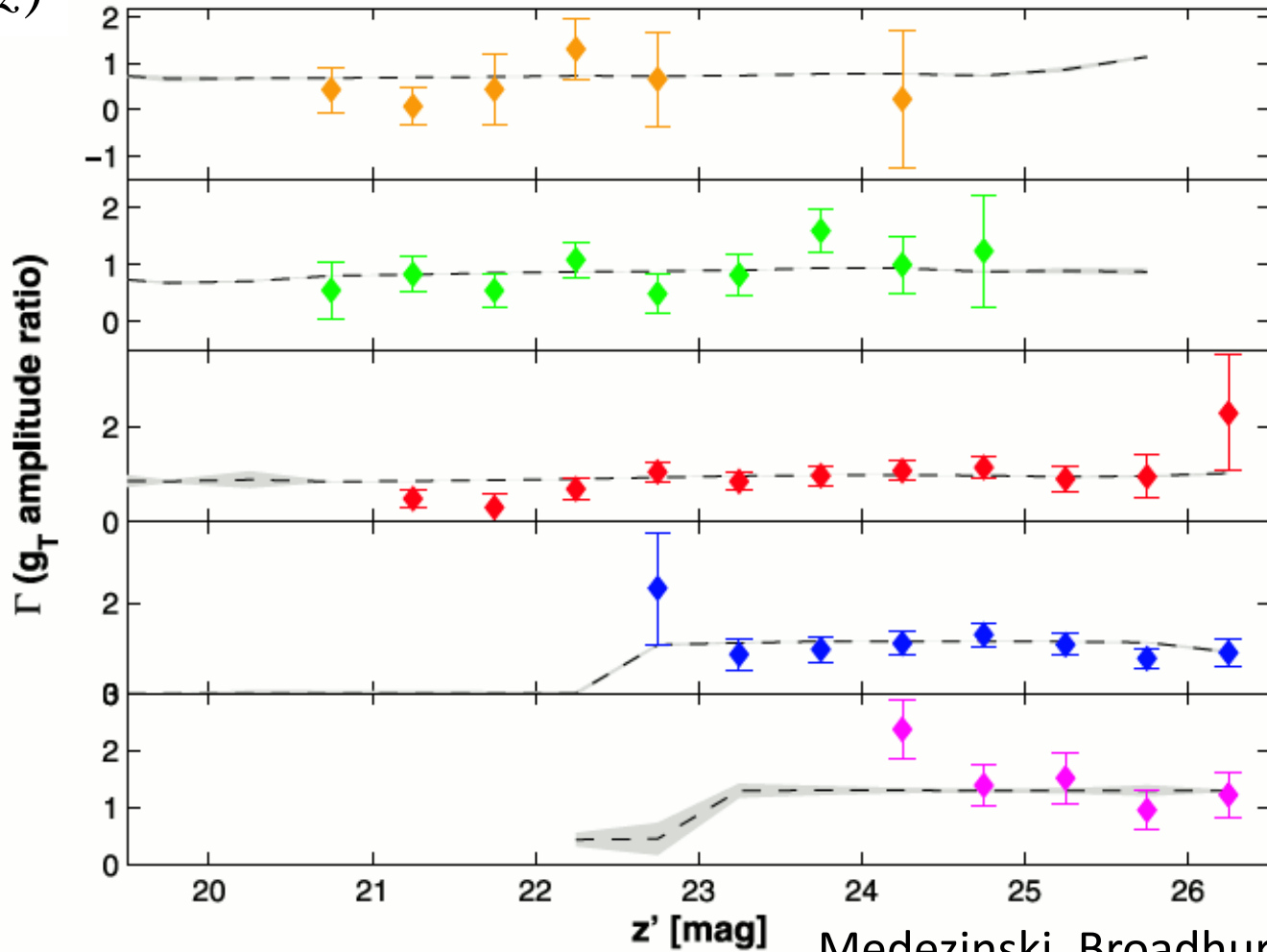
A1689 at  $z=0.183$ ,  
Subaru/S-Cam *BVRiz*  
(Umetsu+15)



# Shear strength as function of magnitude

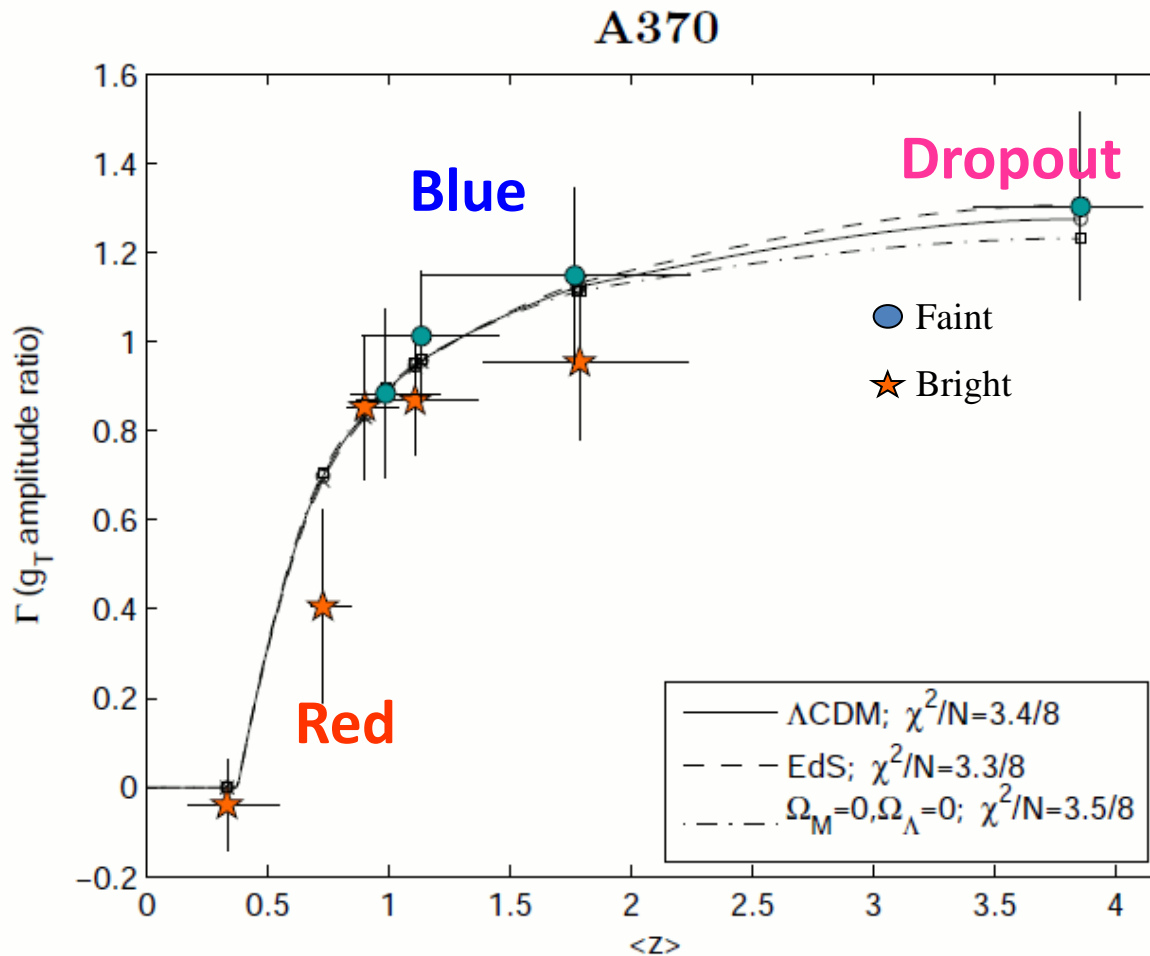
$$\gamma_{\infty} \frac{D_{LS}(z)}{D_S(z)}$$

KSB+ (Umetsu+10) pipeline

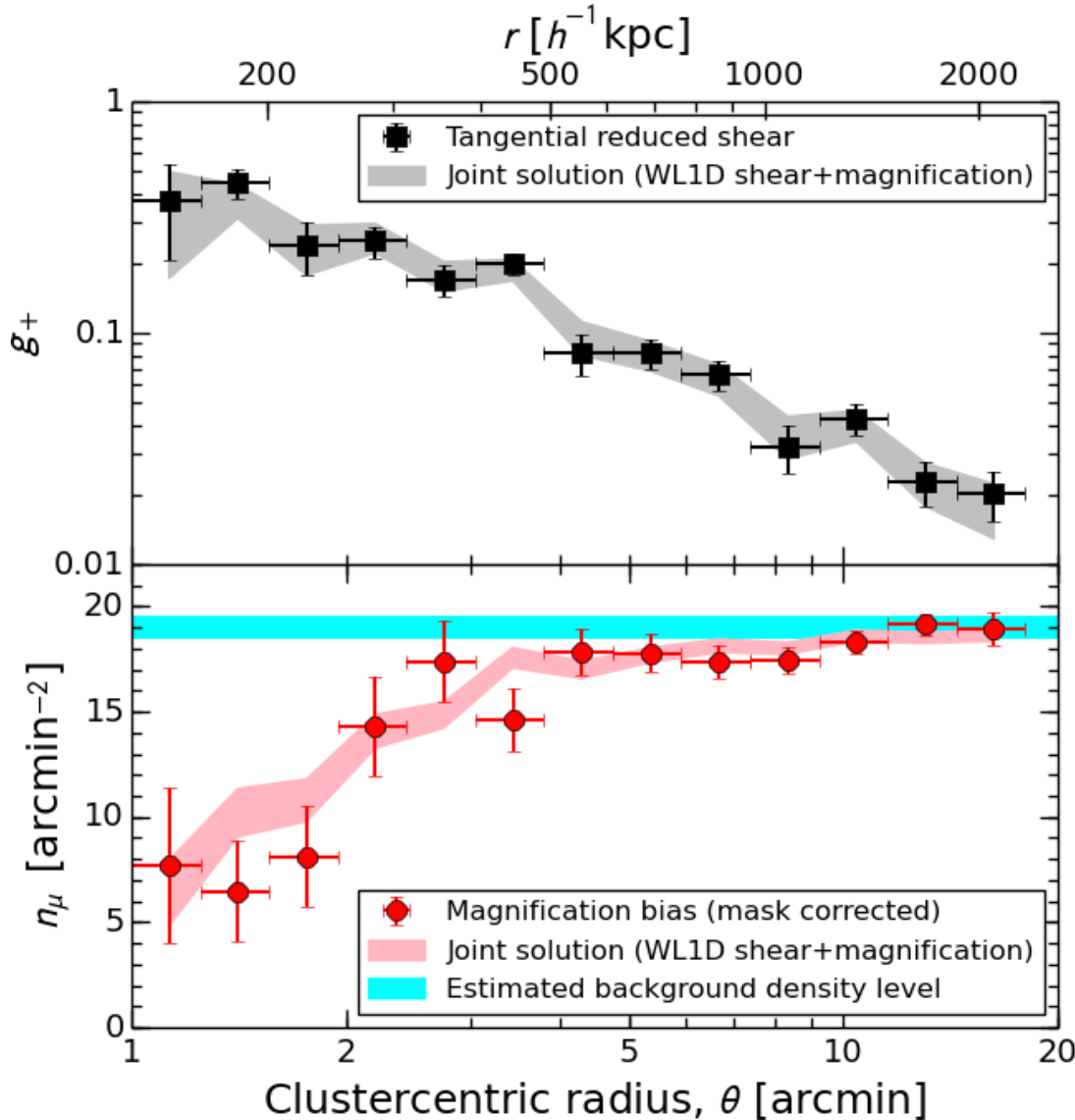


# Shear strength as function of $z$ (KSB+)

First detection of WL distance vs. redshift relation!!!



# Shear vs. Magnification



**Reduced tangential shear**

$$g_+ \approx \gamma_+ = \Delta\Sigma / \Sigma_c$$

**Number count depletion due to magnification bias**

(Broadhurst, Taylor, & Peacock 95)

$$n_\mu = \bar{n}_\mu \mu^{-1+2.5s_{\text{eff}}}$$

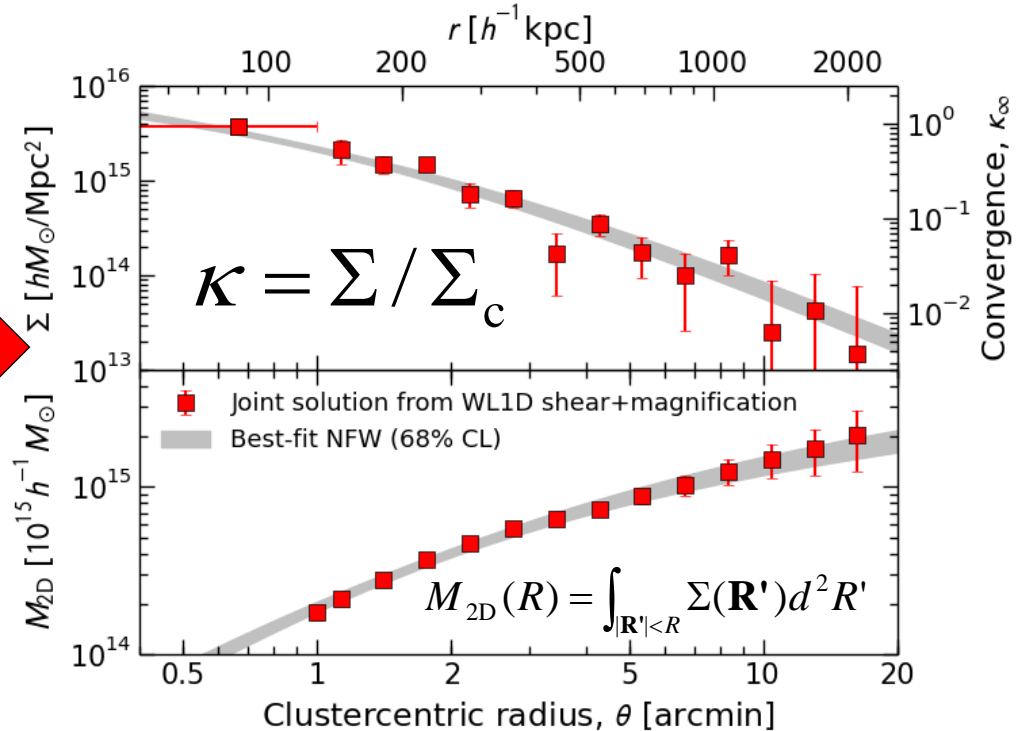
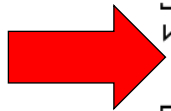
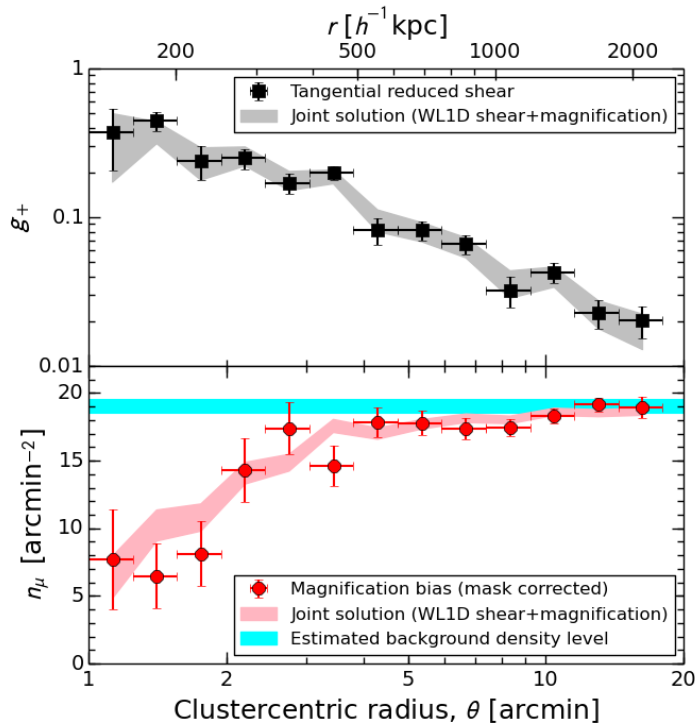
Subaru *BVR/iz* data: A1689  
(Umetsu et al. 2015)



# Combining Shear and Magnification

Methodology: Umetsu et al. 2011a, ApJ, 729, 127

$$P(\kappa | \text{WL}) \propto P(\text{WL} | \kappa)P(\kappa) = P(\mathbf{g}_+ | \kappa)P(\mathbf{n}_\mu | \kappa)P(\kappa)$$



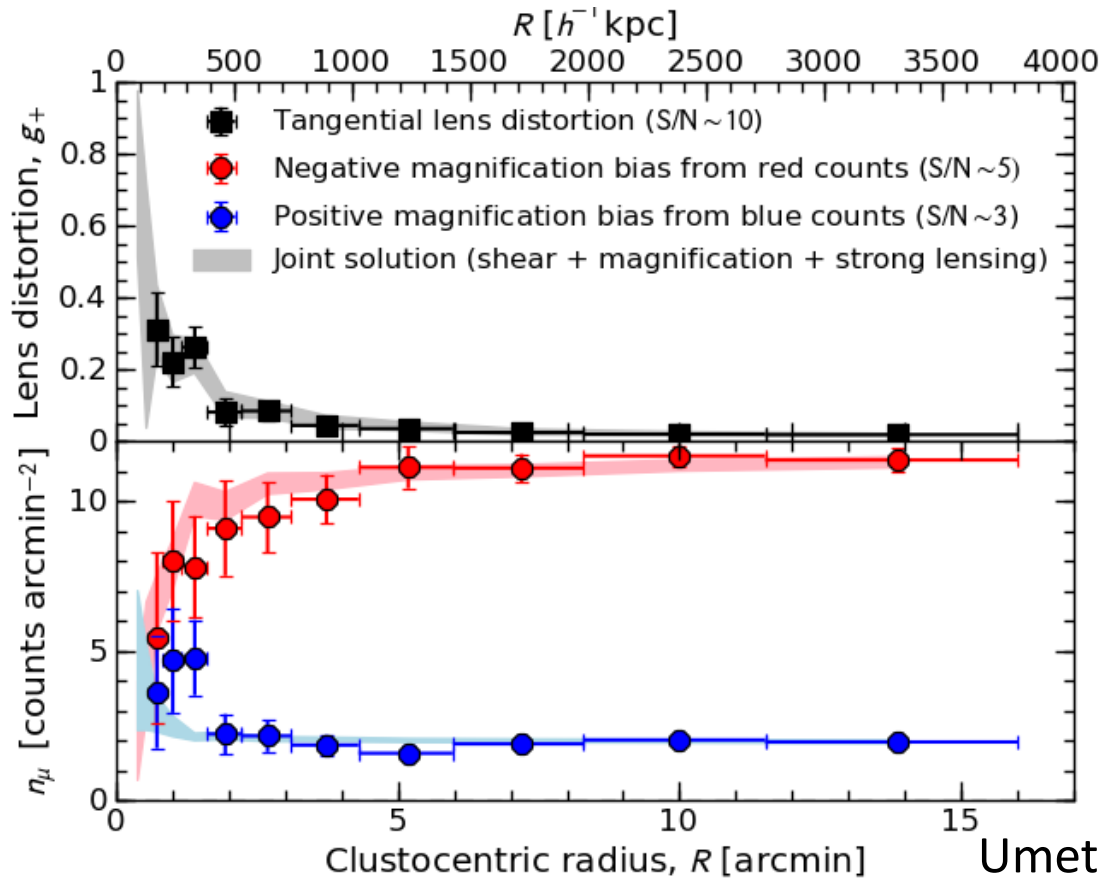
- Mass-sheet degeneracy broken
- Total statistical precision improved by  $\sim 20\text{-}30\%$
- Calibration uncertainties marginalized over:  $c = \{\langle W \rangle_s, f_{W,s}, \langle W \rangle_\mu, \bar{n}_\mu, s_{\text{eff}}\}$ .

# Multi-probe Lensing Approach

## Combining azimuthally-averaged strong and weak lensing observables

$$\{M_{2D,i}\}_{i=1}^{N_{SL}}, \{\langle g_{+,i} \rangle\}_{i=1}^{N_{WL}}, \{\langle n_{\mu,i} \rangle\}_{i=1}^{N_{WL}} \quad M_{2D}(< R) = \int_{|\mathbf{R}'| < R} \Sigma(\mathbf{R}') d^2 R'$$

$$P(\boldsymbol{\kappa} | \text{WL,SL}) \propto P(\text{WL,SL} | \boldsymbol{\kappa})P(\boldsymbol{\kappa}) = P(\mathbf{g}_+ | \boldsymbol{\kappa})P(\mathbf{n}_\mu | \boldsymbol{\kappa})P(\mathbf{M}_{2D} | \boldsymbol{\kappa})P(\boldsymbol{\kappa})$$



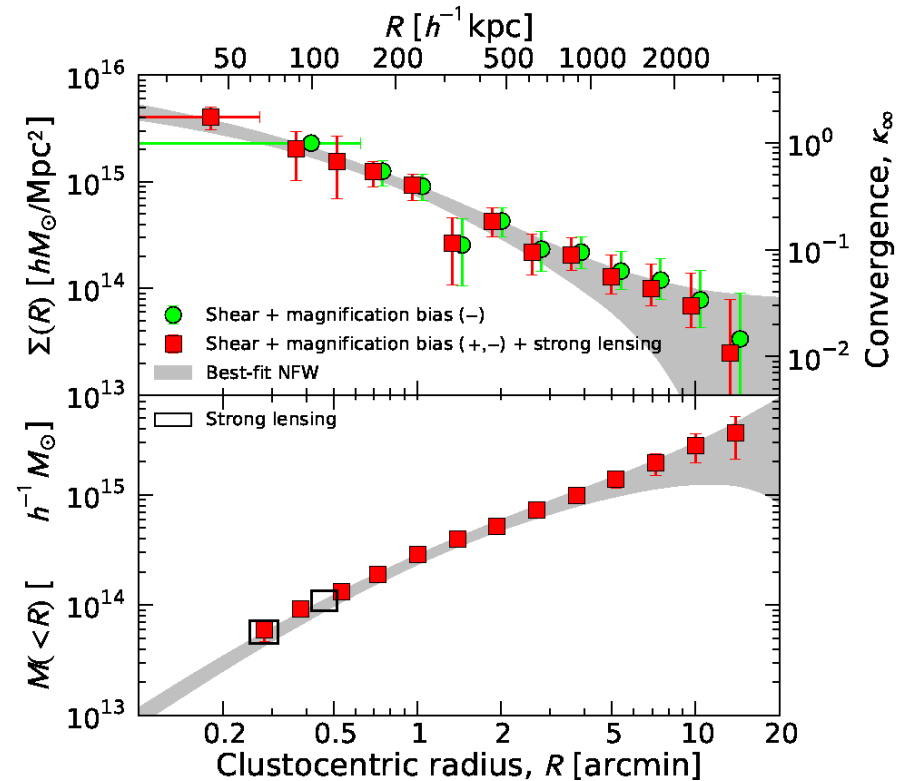
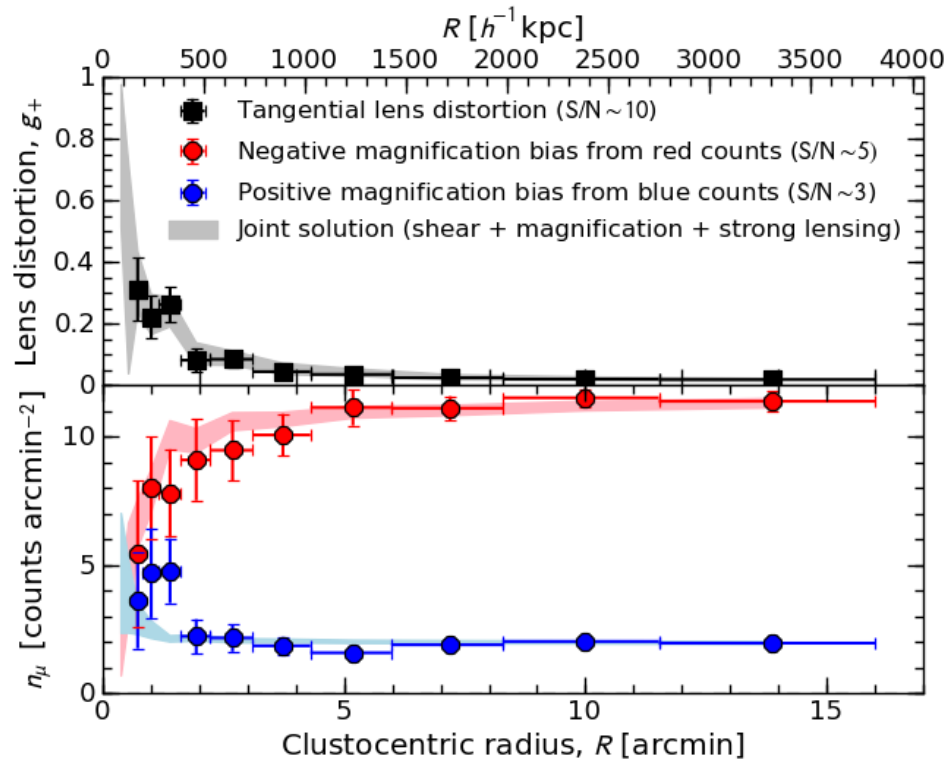
# Multi-probe Lensing Approach

Combining azimuthally-averaged strong and weak lensing observables

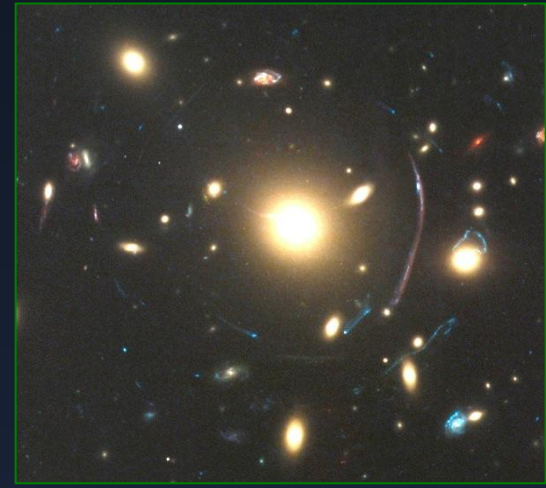
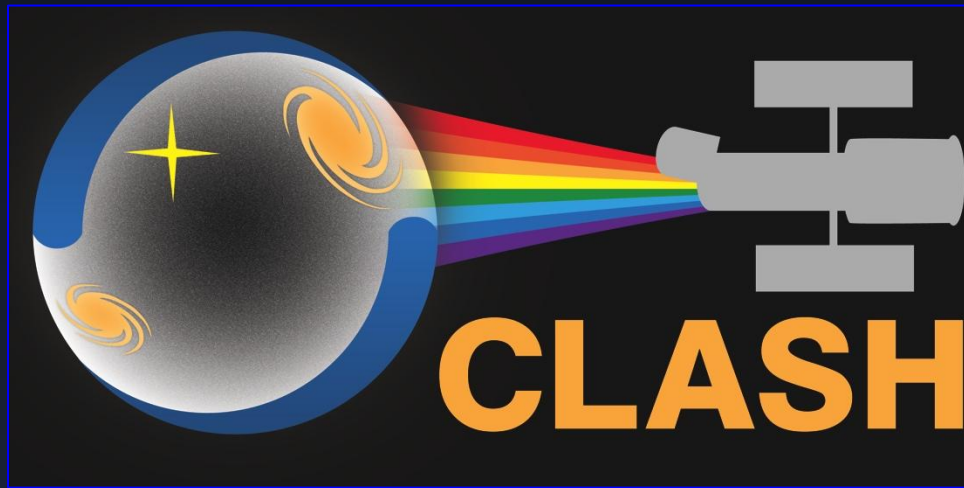
$$\{M_{2D,i}\}_{i=1}^{N_{SL}}, \{\langle g_{+,i} \rangle\}_{i=1}^{N_{WL}}, \{\langle n_{\mu,i} \rangle\}_{i=1}^{N_{WL}}.$$

$$M_{2D}(< R) = \int_{|\mathbf{R}'| < R} \Sigma(\mathbf{R}') d^2 R'$$

$$P(\boldsymbol{\kappa} | \text{WL,SL}) \propto P(\text{WL,SL} | \boldsymbol{\kappa})P(\boldsymbol{\kappa}) = P(\mathbf{g}_+ | \boldsymbol{\kappa})P(\mathbf{n}_\mu | \boldsymbol{\kappa})P(M_{2D} | \boldsymbol{\kappa})P(\boldsymbol{\kappa})$$



# Cluster Lensing And Supernova survey with Hubble

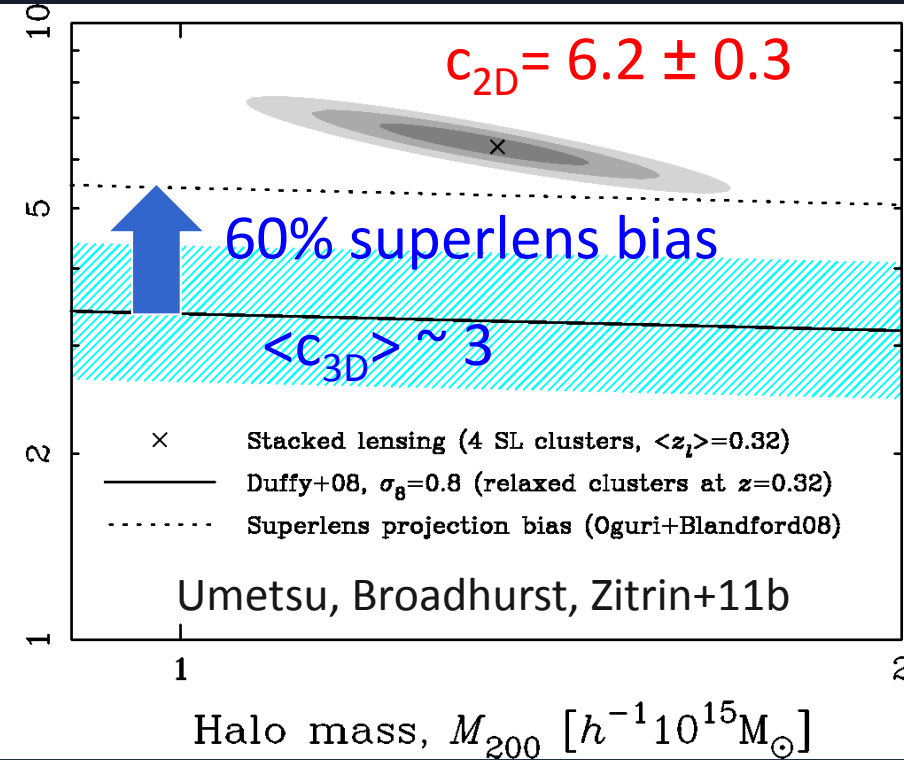
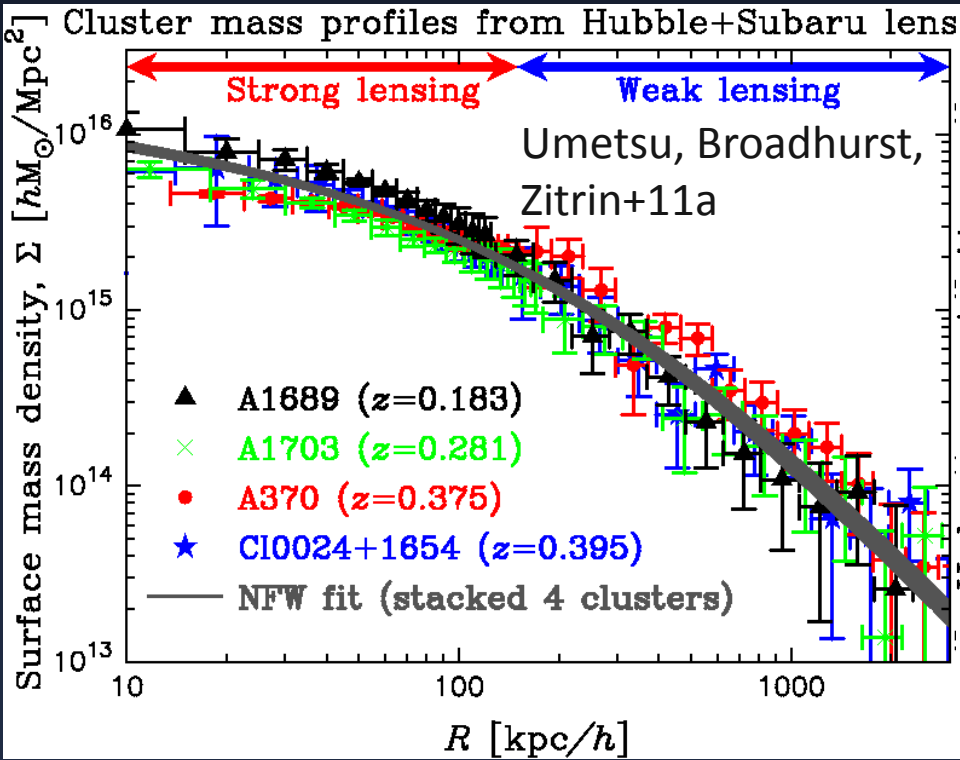


PI. Marc Postman (STScI)

<http://www.stsci.edu/~postman/CLASH/Home.html>

# CLASH Objectives & Motivation

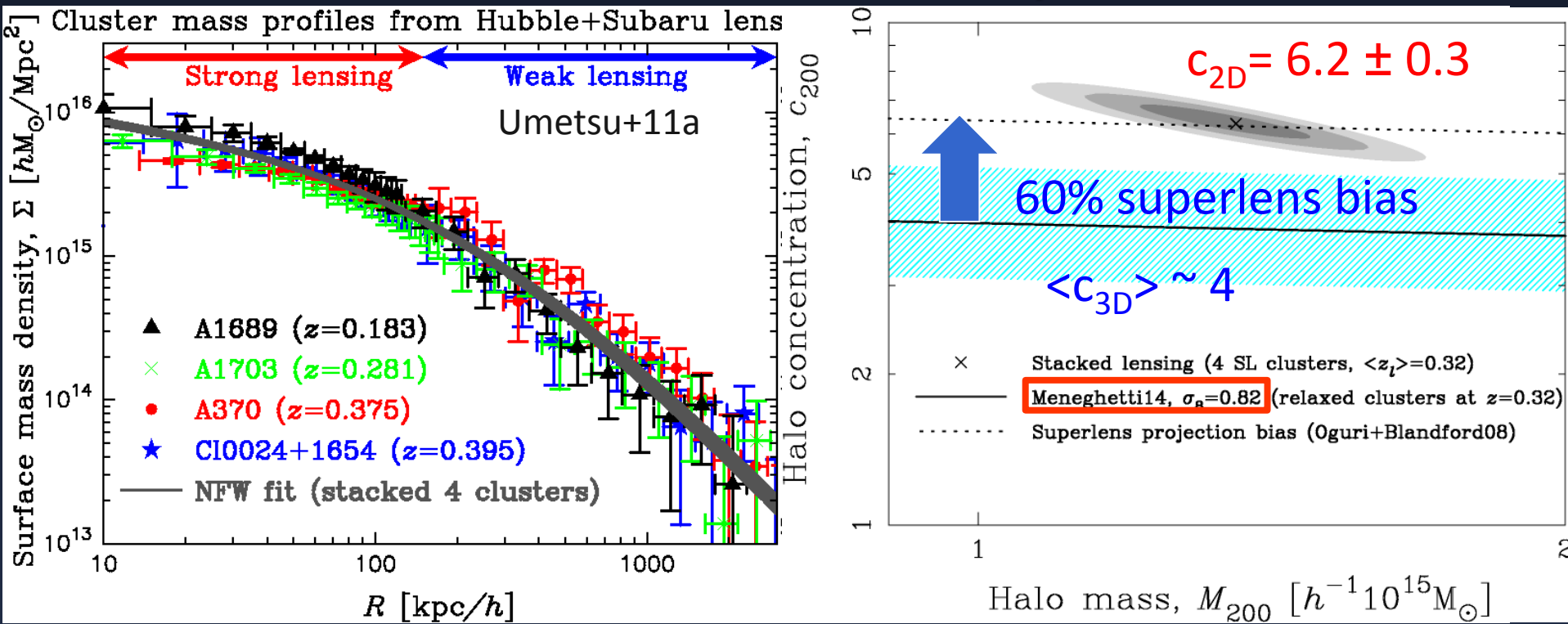
Before CLASH (2010), deep-multicolor Strong (*HST*) + Weak (*Subaru*) lensing data only available for a handful of “super lens” clusters



**Total mass profile shape:** consistent w self-similar NFW (cf. Newman+13; Okabe+13)  
**Degree of concentration:** predicted superlens correction not enough if  $\langle c_{\text{LCDM}} \rangle \sim 3$ ?

# CLASH Objectives & Motivation

Before CLASH (2010), deep-multicolor Strong (*HST*) + Weak (*Subaru*) lensing data only available for a handful of “superlens” clusters



**Total mass profile shape:** consistent w self-similar NFW (cf. Newman+13; Okabe+13)

**Degree of concentration:** predicted superlens correction is just enough if  $\langle c_{\text{LCDM}} \rangle \sim 4$



# CLASH X-ray-selected Subsample

- **High-mass clusters with smooth X-ray morphology**
  - $T_x > 5\text{keV}$  ( $M_{200c} > 5e14M_{\text{sun}}/h$ )
  - Small BCG/X-ray peak offset,  $\sigma_{\text{off}} \sim 10\text{kpc}/h$
  - Smooth regular X-ray morphology
  - **Optimized for radial-profile analysis**
- **CLASH theoretical predictions** (Meneghetti+14)
  - Composite relaxed (70%) and unrelaxed (30%) clusters
  - Mean  $\langle c_{200c} \rangle = 3.9$ ,  $c_{200c} = [3, 6]$
  - Small scatter in  $c_{200c}$ :  $\sigma(\ln c_{200c}) = 0.16$
  - Largely free of orientation bias ( $\sim 2\%$  in  $\langle M_{3D} \rangle$ )
  - $>90\%$  of CLASH clusters to have strong-lensing features



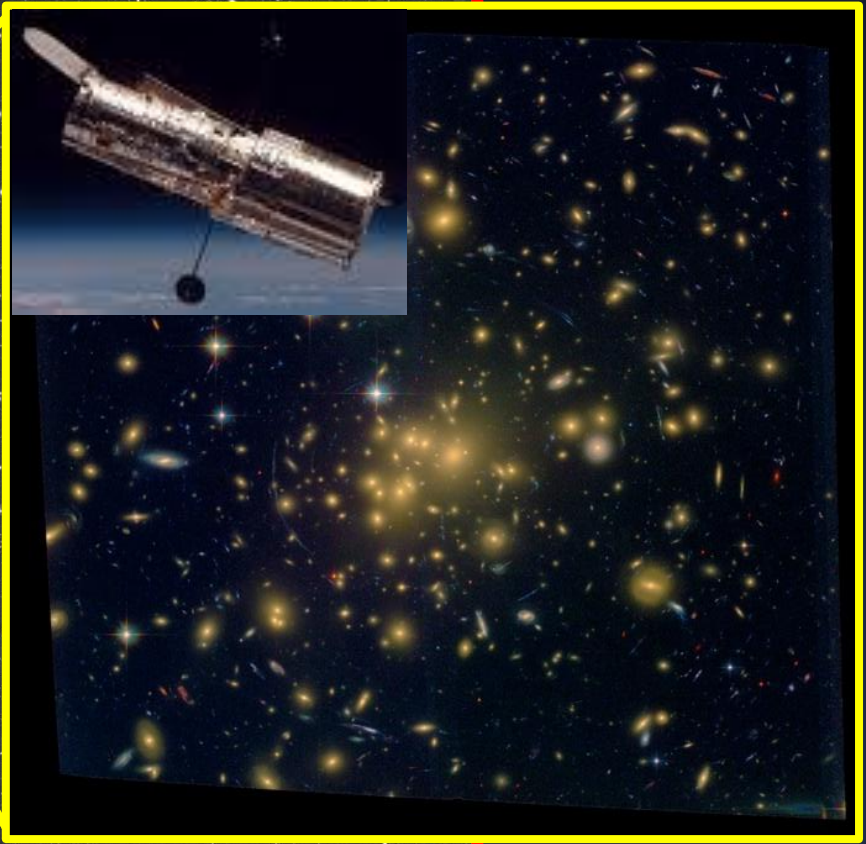
**CLASH: Joint Analysis of Strong-lensing,  
Weak-lensing Shear and Magnification  
Data  
for 20 CLASH Galaxy Clusters**

Umetsu et al. 2016, arXiv:1507.04385



*Subaru/Suprime-Cam* multi-color imaging for wide-field

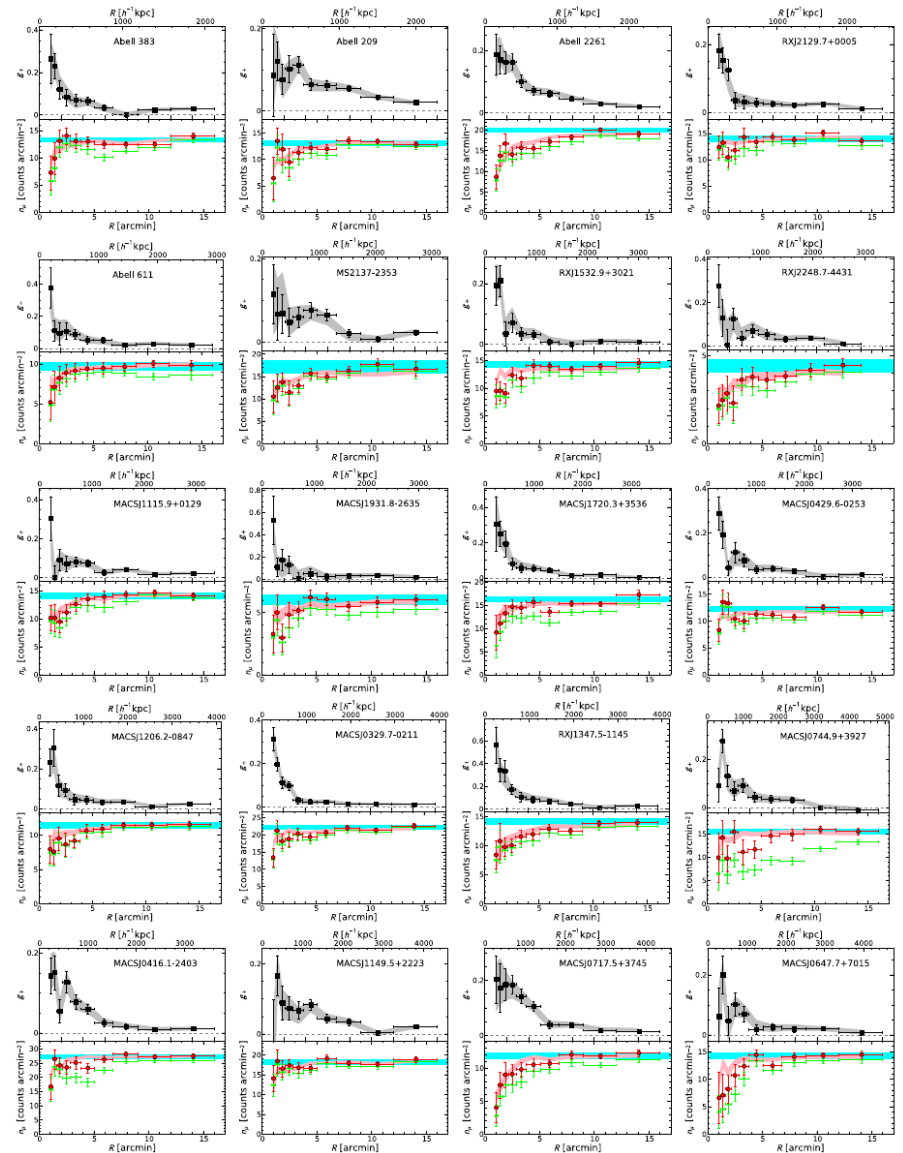
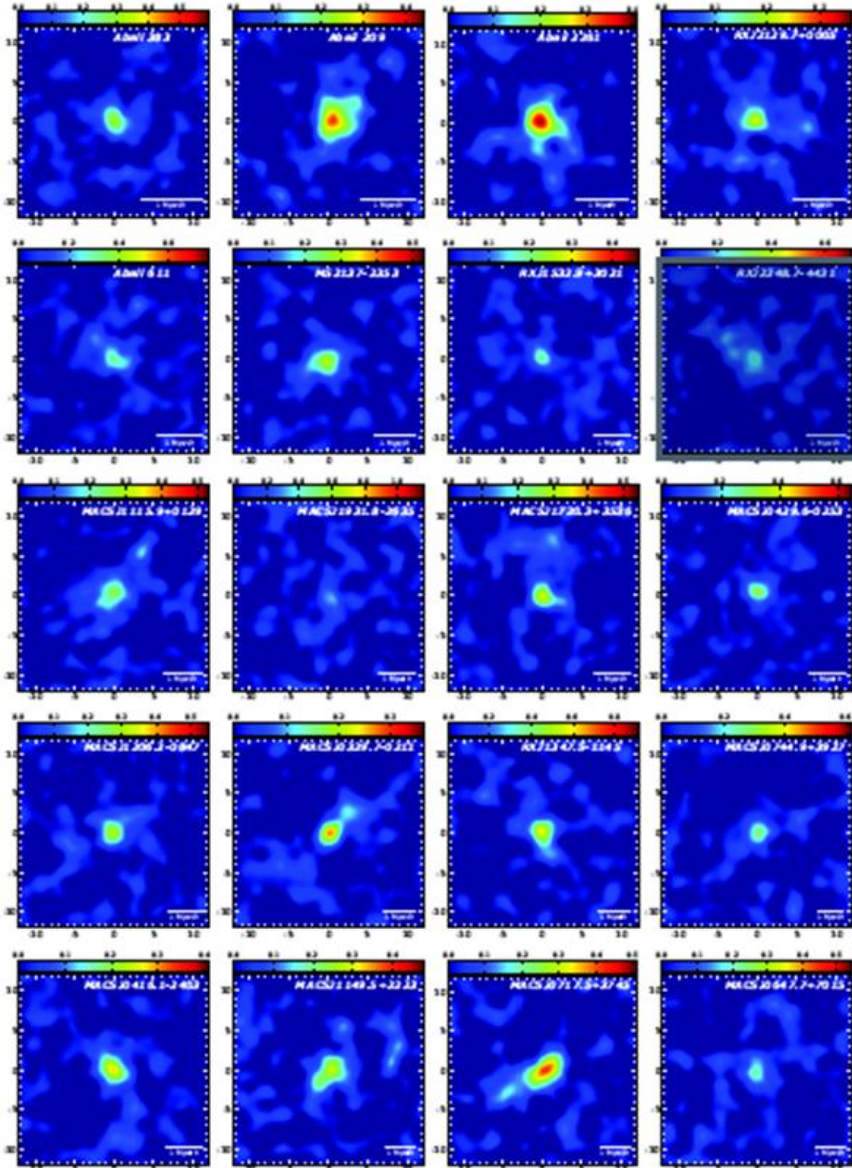
High-resolution space imaging with *HST* (ACS/WFC3) for strong lensing



34 arcmin



# CLASH *Subaru* Weak-lensing Dataset





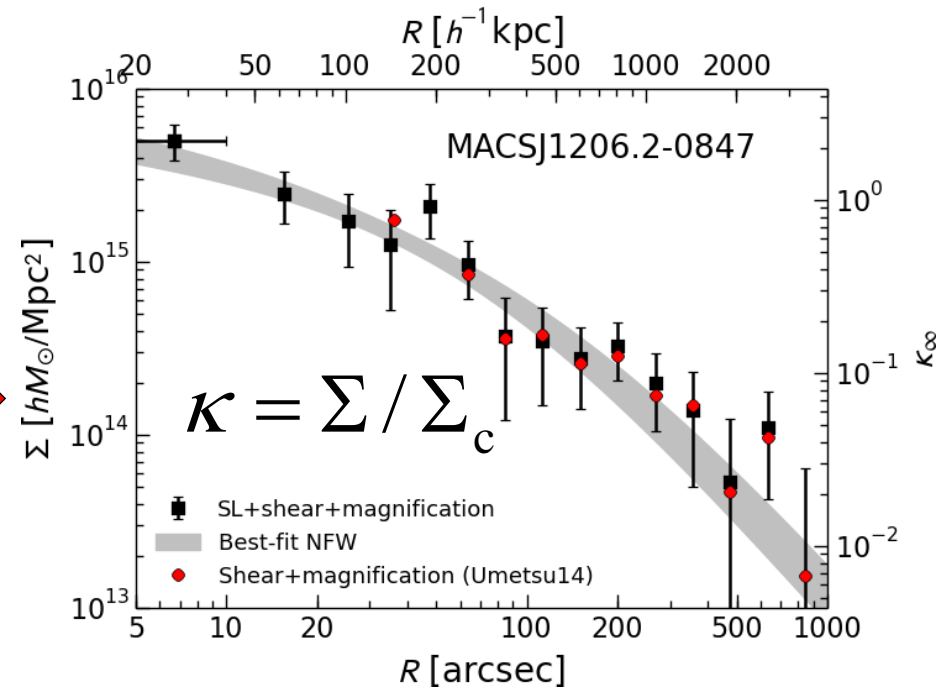
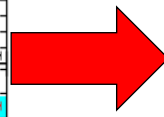
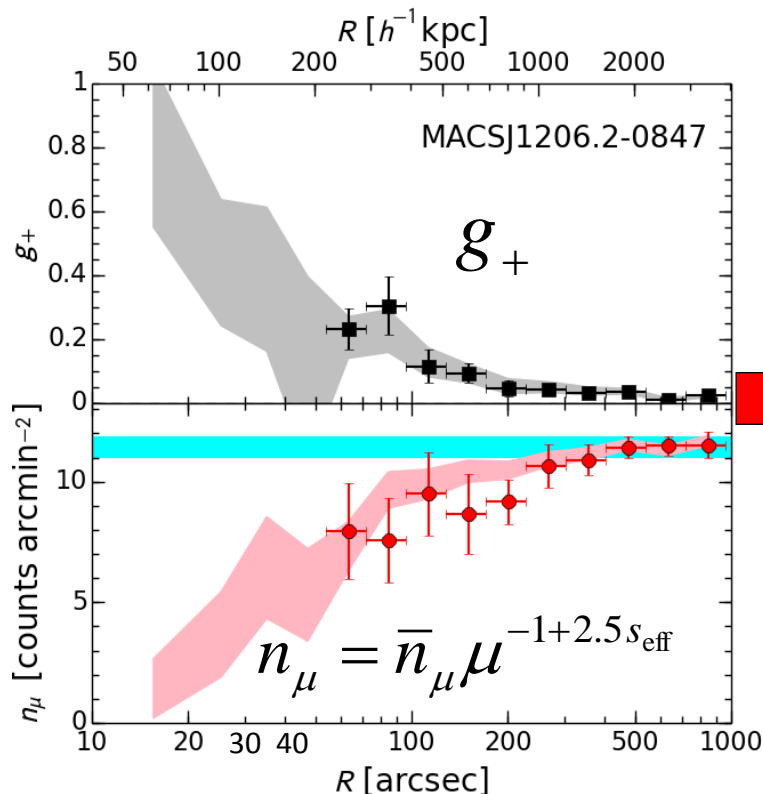
# Joint Analysis of SL & WL shear+magnification

$$\{M_{2D,i}\}_{i=1}^{N_{SL}}, \{\langle g_{+,i} \rangle\}_{i=1}^{N_{WL}}, \{\langle n_{\mu,i} \rangle\}_{i=1}^{N_{WL}}.$$

## Determination of $M_{2D}(<R)$ from detailed *HST* SL modeling (Zitrin+15)

- Effective resolution:  $\Delta R = 10'' (\langle R_{Ein} \rangle / 22'') (\langle N \rangle / 17)^{-1/2}$
- Maximum integration radius:  $R_{max} \sim 2 \langle R_{Ein} \rangle \sim 40''$

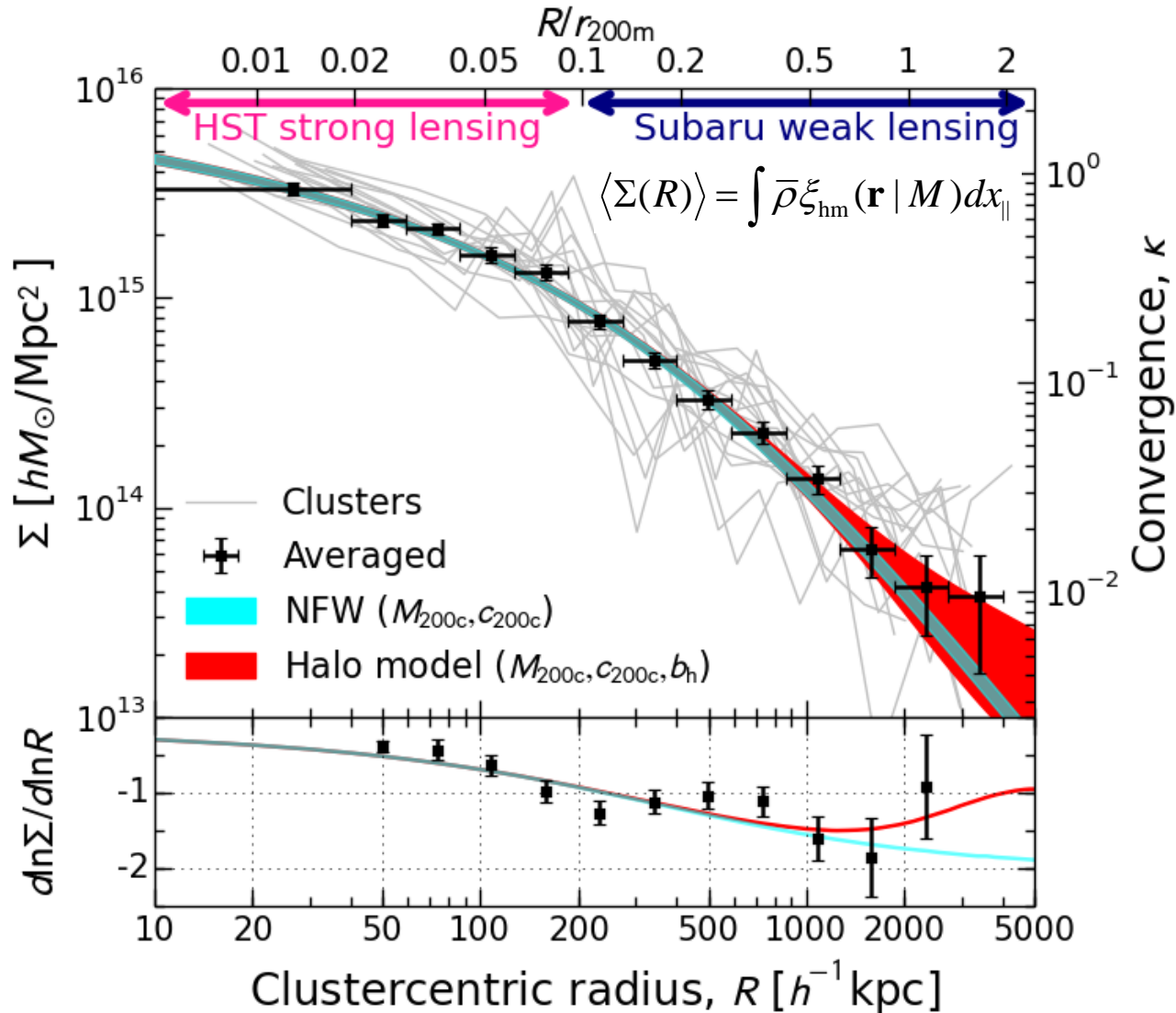
## HST-SL mass integration radii: $R=(10'', 20'', 30'', 40'')$



$\langle \chi^2 / \text{dof} \rangle = 0.95$  for 20 CLASH clusters



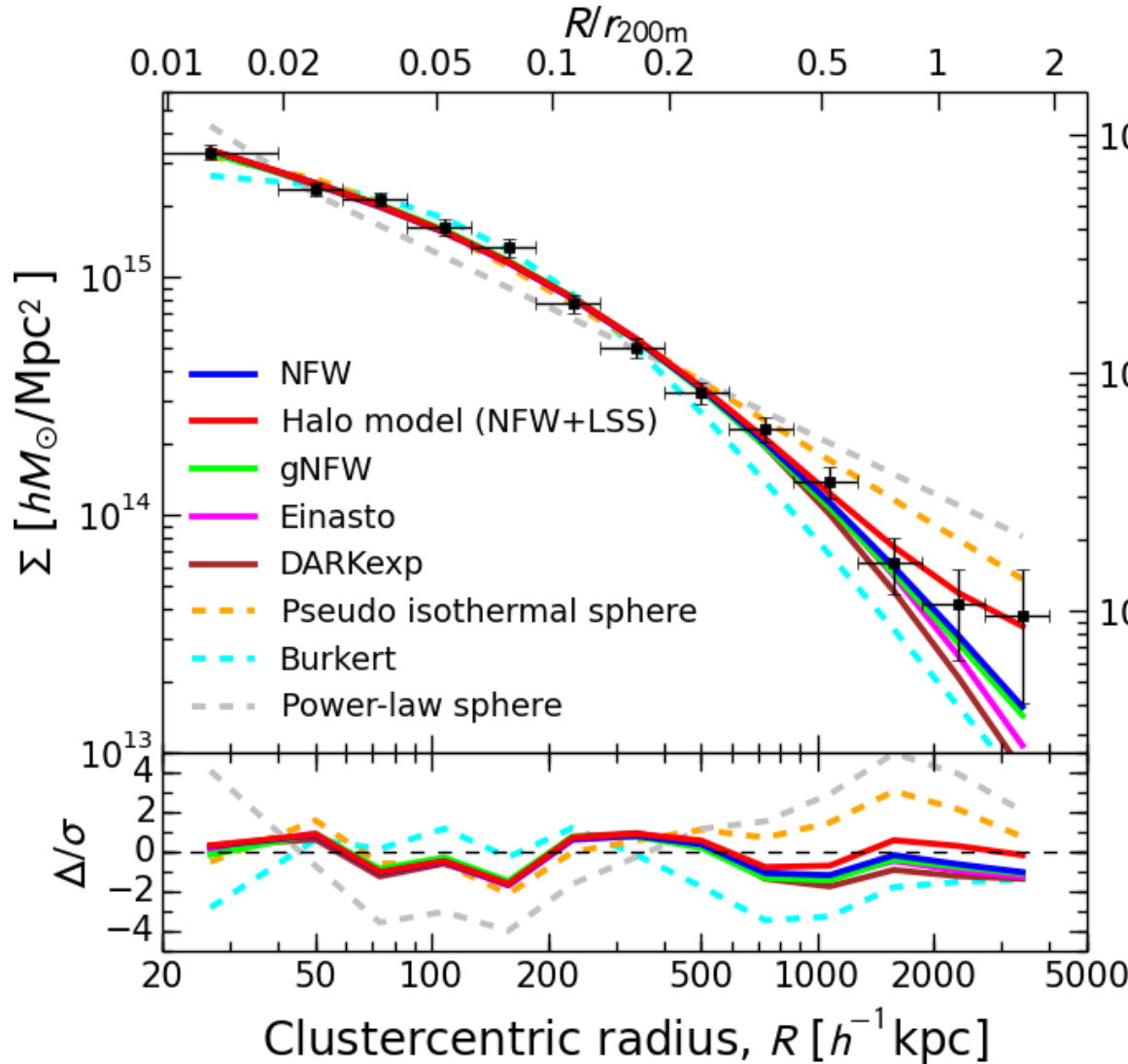
# CLASH Ensemble Mass Profile



33 $\sigma$  detection of the ensemble-averaged mass profile out to  $\sim 2r_{200m}$



# Characterizing the Ensemble Mass Profile



$$\Sigma(R) = \int dl \Delta\rho(r),$$

Convergence,  $\kappa$

## Models:

1. No 2-halo term ( $f_t=1, \rho_{2h}=0$ )
2. With 2-halo term (Tinker+10)

$$\Delta\rho(r) = f_t(r) \rho_h(r) + \rho_{2h}(r),$$

$$f_t(r) = \left[ 1 + \left( \frac{r}{r_t} \right)^2 \right]^{-2},$$



# Comparison of Best-fit Models

Acceptable fits:  $p$  values (PTE)  $> 0.05$

**Table 4**  
Best-fit models for the stacked mass profile of the CLASH X-ray-selected subsample

Model	$M_{200c}$ ( $10^{14} M_{\odot} h_{70}^{-1}$ )	$c_{200c}$	Shape/structural parameters	$b_h$	$\chi^2/\text{dof}$	PTE <sup>a</sup>	Notes
NFW	$14.4^{+1.1}_{-1.0}$	$3.76^{+0.29}_{-0.27}$	$\gamma_c = 1$	—	11.3/11	0.419	No truncation
gNFW	$14.1^{+1.1}_{-1.1}$	$4.04^{+0.53}_{-0.52}$	$\gamma_c = 0.85^{+0.22}_{-0.31}$	—	10.9/10	0.366	No truncation
Einasto	$14.7^{+1.1}_{-1.1}$	$3.53^{+0.36}_{-0.39}$	$\alpha_E = 0.232^{+0.042}_{-0.038}$	—	11.7/10	0.306	No truncation
DARKexp- $\gamma^b$	$14.5^{+1.2}_{-1.1}$	$3.53^{+0.42}_{-0.42}$	$\phi_0 = 3.90^{+0.41}_{-0.45}$	—	13.5/10	0.198	No truncation
Pseudo isothermal	—	—	$V_c = 1762^{+40}_{-39}$ km/s, $r_c = 69^{+7}_{-7}$ kpc	—	23.6/11	0.015	No truncation
Burkert	$11.6^{+0.8}_{-0.8}$	—	$r_{200c}/r_0 = 8.81^{+0.42}_{-0.41}$	—	29.9/11	0.002	No truncation
Power-law sphere	$12.5^{+0.8}_{-0.8}$	—	$\gamma_c = 1.78^{+0.02}_{-0.02}$	—	93.5/11	0.000	No truncation
<b>Halo model<sup>c</sup>:</b>							
NFW+LSS (i)	$14.1^{+1.0}_{-1.0}$	$3.79^{+0.30}_{-0.28}$	$\gamma_c = 1$	9.3	10.9/11	0.450	$\Lambda$ CDM $b_h(M)$ scaling
NFW+LSS (ii)	$14.4^{+1.4}_{-1.3}$	$3.74^{+0.33}_{-0.30}$	$\gamma_c = 1$	$7.4^{+4.6}_{-4.7}$	10.8/10	0.377	$b_h$ as a free parameter
Einasto+LSS (i)	$14.3^{+1.1}_{-1.1}$	$3.69^{+0.36}_{-0.42}$	$\alpha_E = 0.248^{+0.051}_{-0.047}$	9.3	10.7/10	0.385	$\Lambda$ CDM $b_h(M)$ scaling
Einasto+LSS (ii)	$14.5^{+1.9}_{-1.6}$	$3.65^{+0.47}_{-0.61}$	$\alpha_E = 0.245^{+0.061}_{-0.053}$	$8.7^{+5.3}_{-5.6}$	10.6/9	0.301	$b_h$ as a free parameter
DARKexp+LSS (i)	$14.2^{+1.2}_{-1.1}$	$3.64^{+0.44}_{-0.46}$	$\phi_0 = 3.89^{+0.51}_{-0.54}$	9.3	11.7/10	0.308	$\Lambda$ CDM $b_h(M)$ scaling
DARKexp+LSS (ii)	$14.0^{+1.8}_{-1.6}$	$3.69^{+0.53}_{-0.57}$	$\phi_0 = 3.85^{+0.57}_{-0.61}$	$10.1^{+4.9}_{-5.1}$	11.6/9	0.235	$b_h$ as a free parameter

<sup>a</sup> Probability to exceed the observed  $\chi^2$  value.

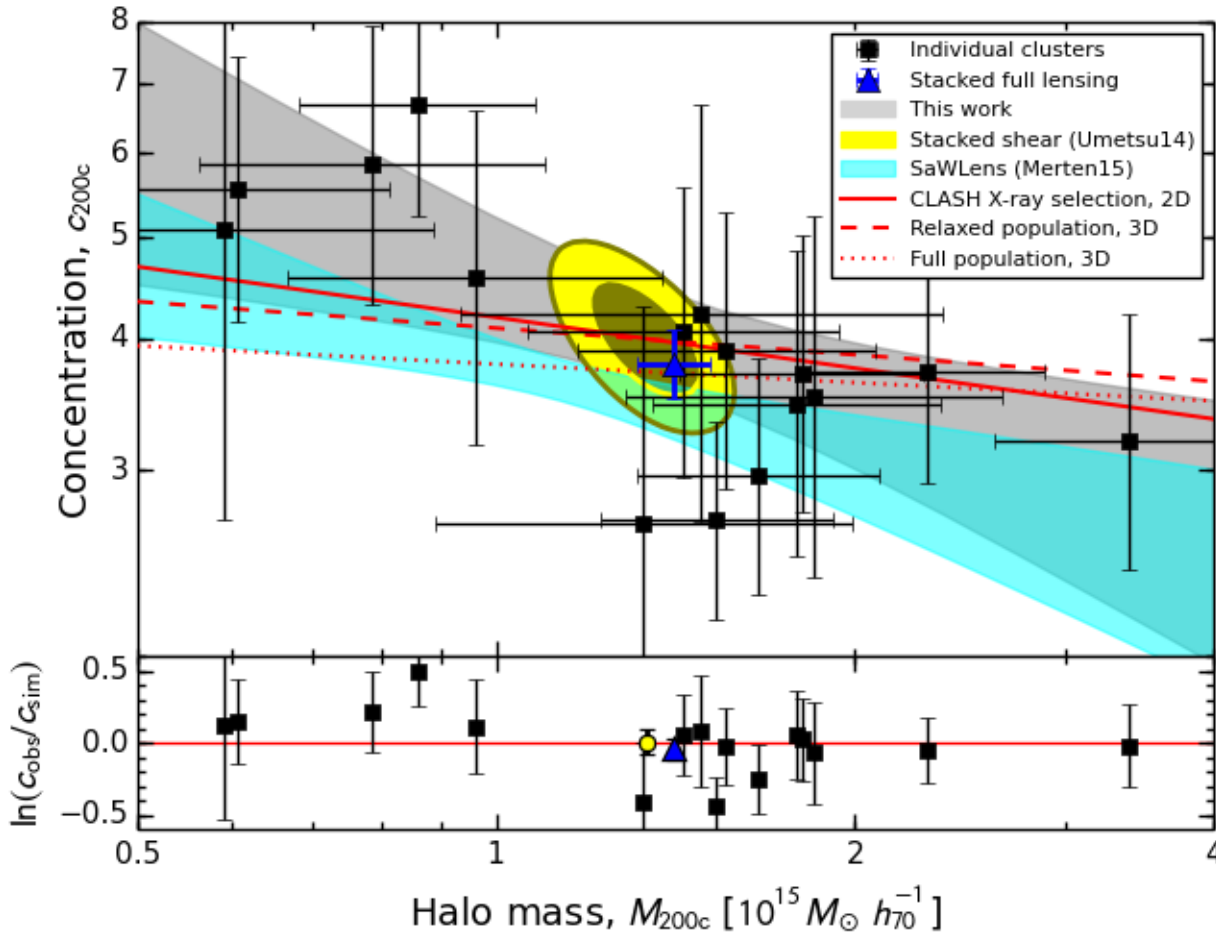
<sup>b</sup> We use Dehnen–Tremaine  $\gamma$ -models with the central cusp slope  $\gamma_c = 3 \log_{10} \phi_0 - 0.65$  ( $1.7 \leq \phi_0 \leq 6$ ) as an analytic fitting function for the DARKexp density profile.

<sup>c</sup> For halo model predictions, we decompose the total mass overdensity  $\Delta\rho(r) = \rho(r) - \bar{\rho}_m$  as  $\Delta\rho = f_t \rho_h + \rho_{2h}$  where  $\rho_h(r)$  is the halo density profile,  $\rho_{2h}(r) = \bar{\rho}_m b_h \xi_m^L(r)$  is the two-halo term, and  $f_t(r) = (1 + r^2/r_t^2)^{-2}$  describes the steepening of the density profile in the transition regime around the truncation radius  $r_t$ , which is assumed to be  $r_t = 3r_{200c}$ .

- Consistent with cuspy density profiles (NFW, Einasto, DARKexp)
- Cuspy models that include  $\Lambda$ CDM 2-halo term ( $b_h \sim 9.3$ ) give improved fits



# CLASH Concentration vs. Mass Relation



**Predicted (M14):**

$$\langle c_{200c} \rangle = 3.9,$$

$$3 \leq c_{200c} \leq 6,$$

$$\sigma(\ln c_{200c}) = 0.16$$

**Observed:**

$$c_{200c} |_{z=0.34} = 3.95 \pm 0.35$$

$$\text{at } M_{200c} = 10^{15} M_{\text{sun}} / h,$$

$$\sigma(\ln c_{200c}) = 0.13 \pm 0.06$$

Normalization, slope, & scatter are all consistent with LCDM when the CLASH selection function based on X-ray morphological regularity and the projection effects are taken into account



# Comparison with LCDM $c(M)$ models

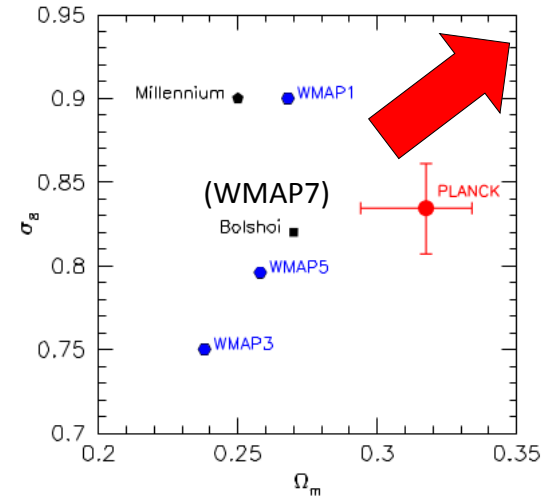
**Table 5**

Comparison of measured and predicted concentrations for the CLASH X-ray-selected subsample

Author	Sample	3D/2D	Function <sup>a</sup>	$c^{(obs)}/c^{(pred)}$ Average <sup>c</sup>	$\sigma^d$	$\chi^2$	PTE <sup>b</sup>
<b>Theory:</b>							
Duffy et al. (2008)	full	3D	$c-M$	$1.331 \pm 0.108$	0.334	22.6	0.046
Duffy et al. (2008)	relaxed	3D	$c-M$	$1.165 \pm 0.094$	0.290	13.6	0.399
Prada et al. (2012)	full	3D	$c-\nu$	$0.733 \pm 0.065$	0.244	24.6	0.026
Bhattacharya et al. (2013)	full	3D	$c-\nu$	$1.169 \pm 0.095$	0.292	14.1	0.369
Bhattacharya et al. (2013)	relaxed	3D	$c-\nu$	$1.131 \pm 0.092$	0.277	12.4	0.494
Dutton & Macciò (2014)	full	3D	$c-M$	$1.061 \pm 0.086$	0.262	10.4	0.659
Meneghetti et al. (2014)	full	3D	$c-M$	$1.061 \pm 0.089$	0.279	10.2	0.675
Meneghetti et al. (2014)	relaxed	3D	$c-M$	$0.990 \pm 0.083$	0.249	9.2	0.760
Diemer & Kravtsov (2015)	full (median)	3D	$c-\nu$	$1.021 \pm 0.083$	0.330	14.4	0.349
Diemer & Kravtsov (2015)	full (mean)	3D	$c-\nu$	$1.060 \pm 0.086$	0.326	13.8	0.391
Meneghetti et al. (2014)	full	2D	$c-M$	$1.087 \pm 0.092$	0.336	13.5	0.413
Meneghetti et al. (2014)	relaxed	2D	$c-M$	$1.040 \pm 0.086$	0.283	10.8	0.628
Meneghetti et al. (2014)	CLASH	2D	$c-M$	$0.988 \pm 0.078$	0.227	9.6	0.730
<b>Observations:</b>							
Merten et al. (2015)	CLASH	2D	$c-M$	$1.133 \pm 0.087$	0.209	9.2	0.754

WMAP5

High normalization



<sup>a</sup>  $c-M$ : power-law  $c(M, z)$  relation;  $c-\nu$ : halo concentration given as a function of peak height  $\nu(M, z)$ .

<sup>b</sup> Probability to exceed the measured  $\chi^2$  value assuming the standard  $\chi^2$  probability distribution function.

<sup>c</sup> Weighted geometric average of observed-to-predicted concentration ratios.

<sup>d</sup> Standard deviation of the distribution of observed-to-predicted concentration ratios.

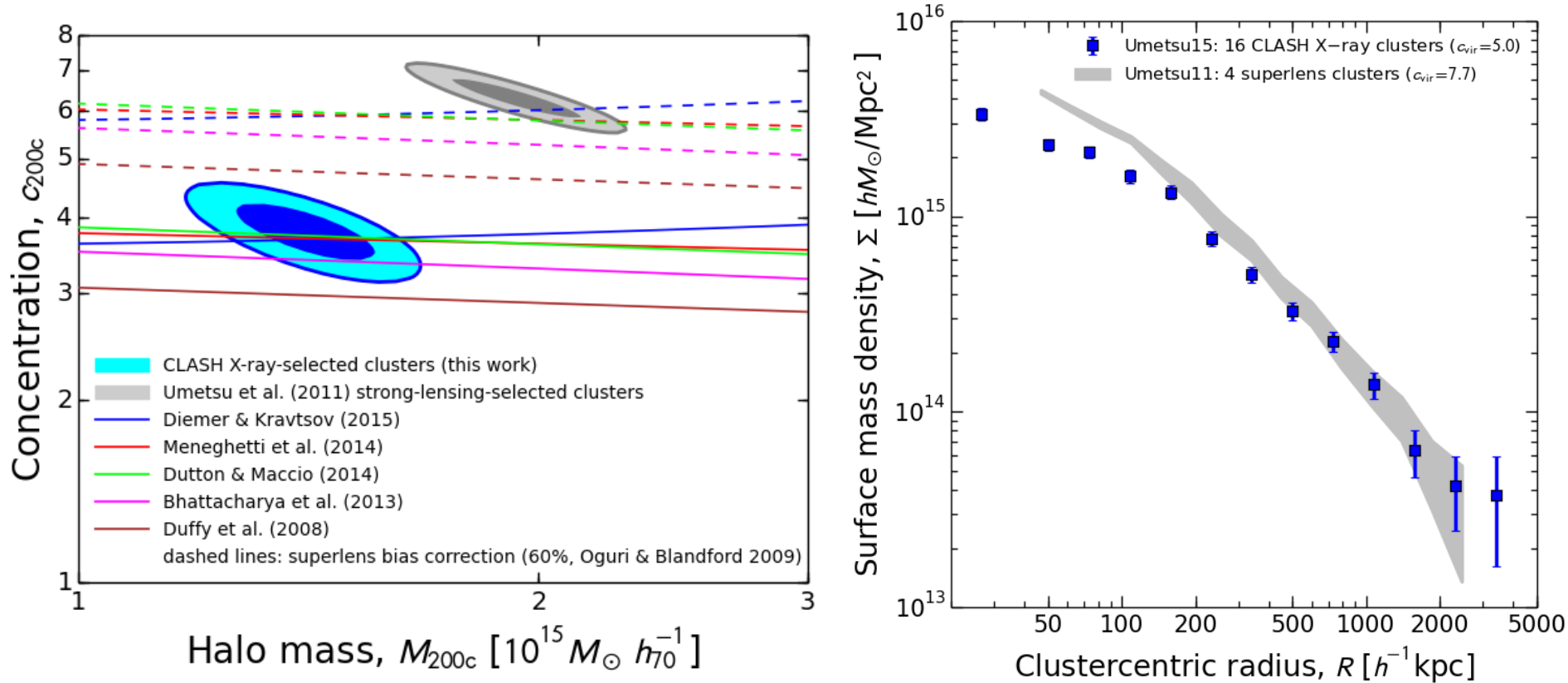
- Consistent with models that are calibrated for more recent cosmologies (WMAP7 and later)
- Better agreement is achieved when selection effects (overall degree of relaxation) are taken into account





# X-ray regular vs. Superlens Clusters

Umetsu+11b: 4 *superlenses* with  $R_{\text{Ein}} > 30''$  at  $z_s=2$  (A1689, A1703, Cl0024, A370)



Higher normalization LCDM cosmology (WMAP7 and later) + “predicted” +60% superlens correction (e.g., Oguri+Blandford09) can explain superlens mass profiles!

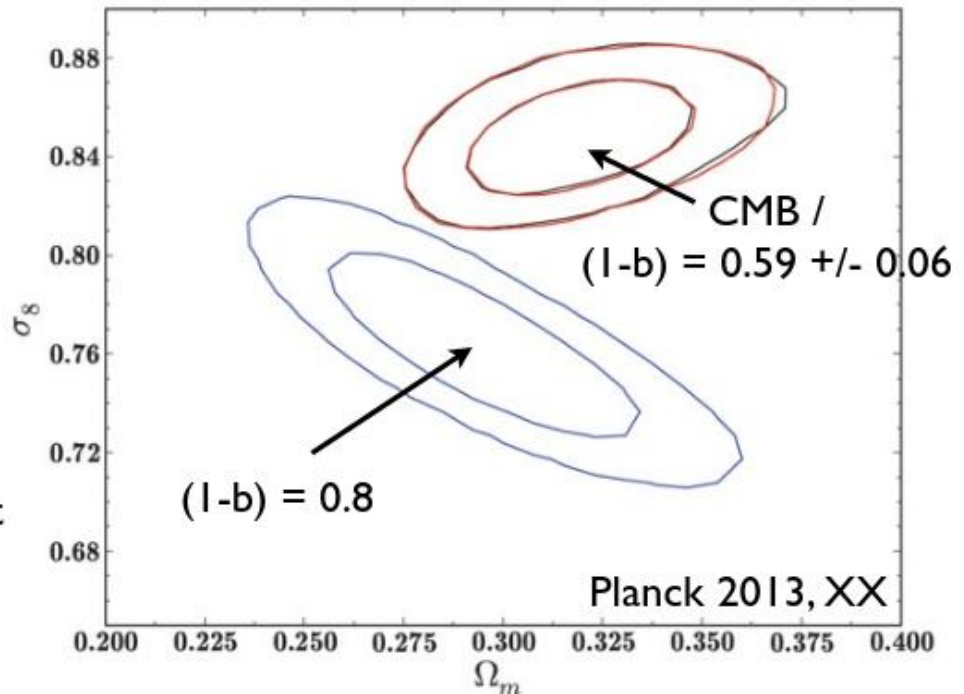


# Ensemble Calibration of Cluster Masses

# Planck13 CMB vs. Cluster Cosmology

$b=0.2?? - 0.4??$

- Planck:  $3\sigma$  tension between SZ cluster counts and CMB cosmology
- assumes  $M_{\text{Planck}} / M_{\text{true}} = (1-b) = 0.8$
- calibrated with XMM hydrostatic masses (Arnaud et al. 2010) + simulations



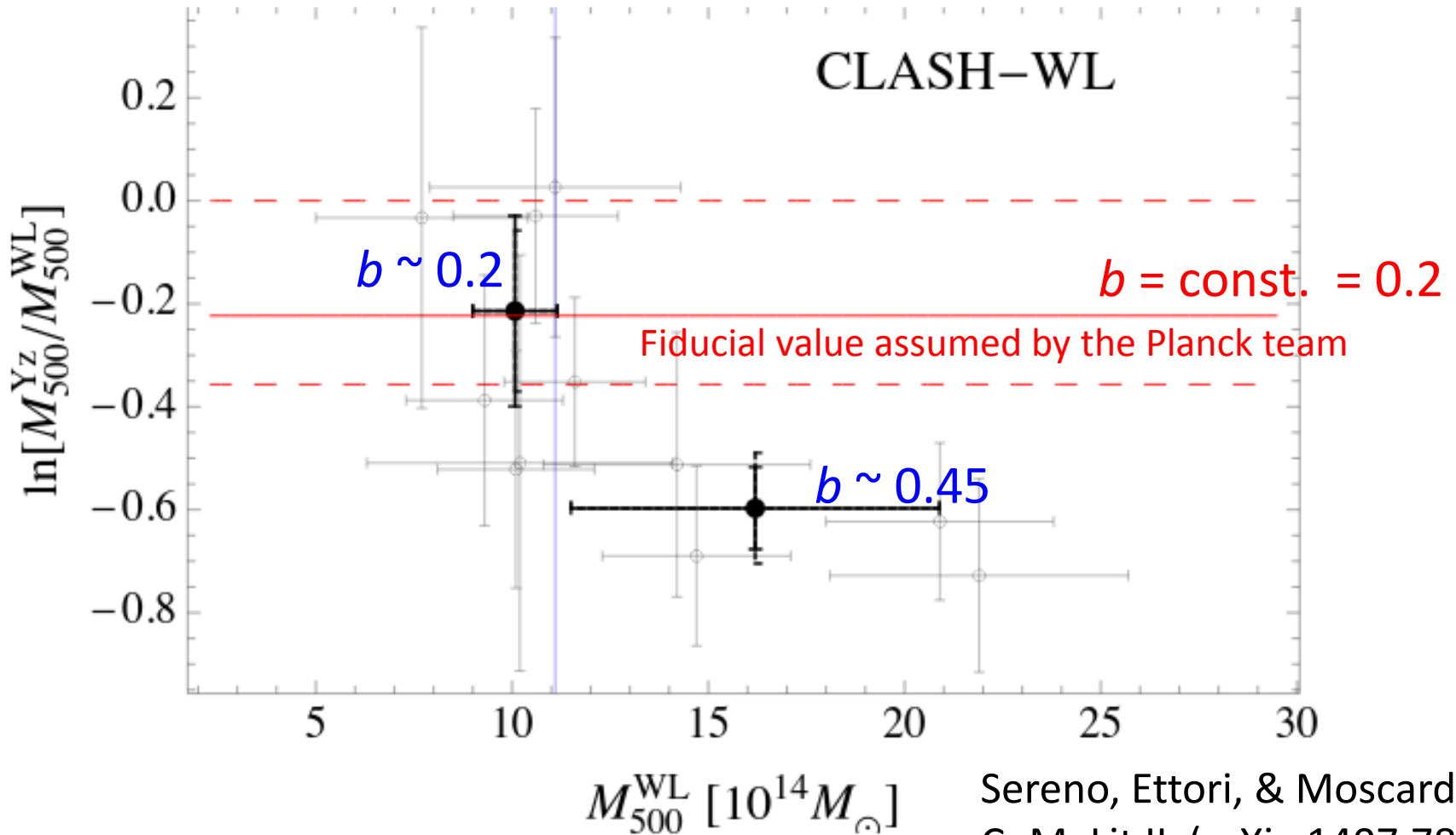
suggested explanations:

- **mass bias underestimated** (and no accounting for uncertainties)
- $2.9\sigma$  detection of neutrino masses:  $\Sigma m_\nu = (0.58 \pm 0.20) \text{ eV}$   
(Planck+WMAPpol+ACT+BAO:  $\Sigma m_\nu < 0.23 \text{ eV}$ , 95% CL)

*Slide taken from Anja von der Linden's presentation*

# Comparison with *Planck* Masses – Not so Simple

Mass-dependent bias (20-45%) observed for *Planck* mass estimates

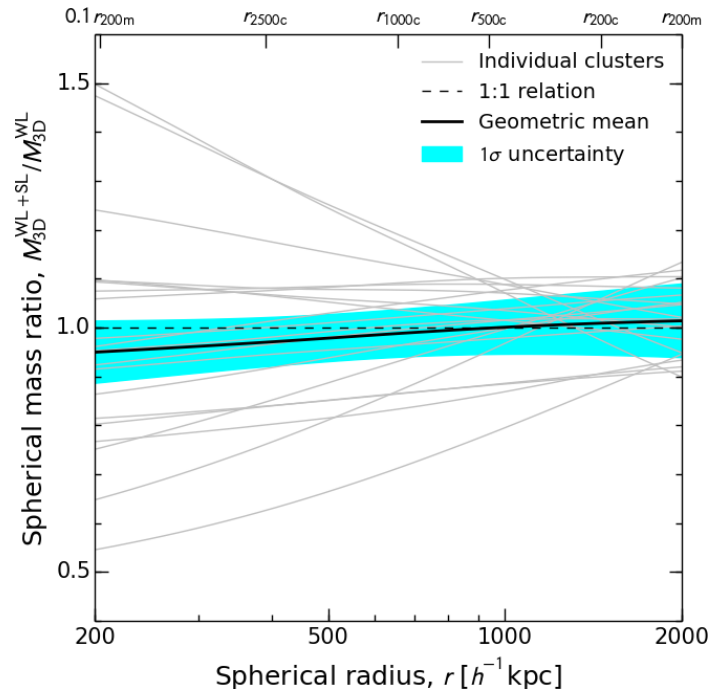


Sereno, Ettori, & Moscardini 15,  
CoMaLit II (arXiv:1407.7869)



# CLASH Internal Consistency

$M_{3D}(<r)$  de-projected assuming spherical NFW density profiles



**WL (U14) and WL+SL (U16)  
are consistent within 5% at  
 $r = [200, 2000]$  kpc/h**

Umetsu+16,  
arXiv:1507.04385

## CLASH ensemble mass calibration uncertainty

- Statistical uncertainty with  $N=20$  clusters:  $28\%/\sqrt{20} = 6.3\%$
- Systematic uncertainty: 5.6% (5% shear calibration, 2% dilution)
- Mass modeling bias (deviations from NFW, orientation bias): 3%
- Total calibration uncertainty: 9%

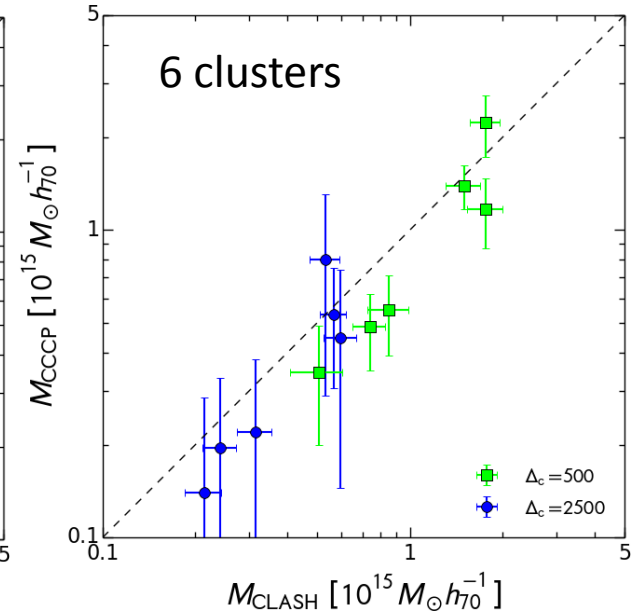
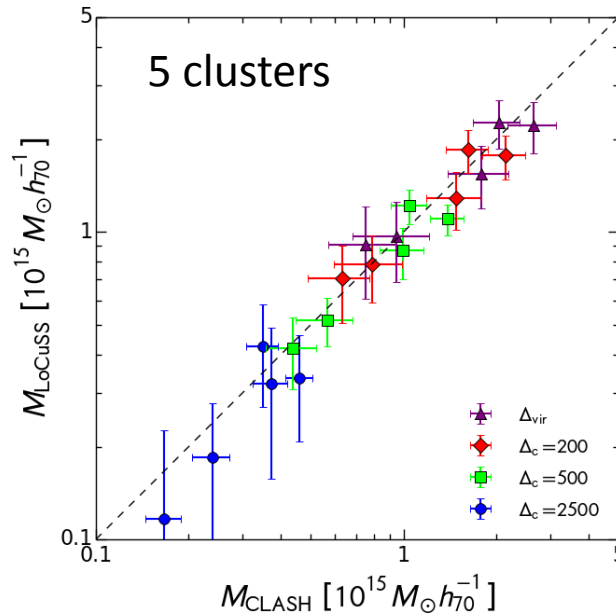
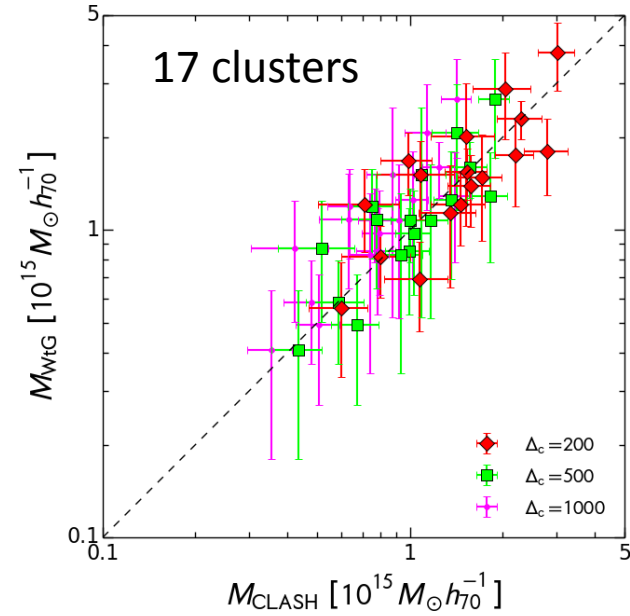


# Comparisons with Other WL Surveys

**WtG [Subaru]**  
(Applegate+14)

**LoCuSS [Subaru]**  
(Okabe & Smith 15)

**CCCP [CFHT]**  
(Hoekstra+15)



$$\begin{aligned} \langle M_{\text{WtG}} / M_{\text{CLASH}} \rangle &= 1.03 \pm 0.09 \quad (\Delta = 200) \\ &= 1.07 \pm 1.12 \quad (\Delta = 500) \\ &= 1.07 \pm 1.12 \quad (\Delta = 1000) \end{aligned}$$

$$\begin{aligned} \langle M_{\text{LoCuSS}} / M_{\text{CLASH}} \rangle &= 1.00 \pm 0.15 \quad (\Delta = \Delta_{\text{vir}}) \\ &= 0.98 \pm 0.13 \quad (\Delta = 200) \\ &= 0.93 \pm 0.10 \quad (\Delta = 500) \\ &= 0.84 \pm 0.22 \quad (\Delta = 2500) \end{aligned}$$

$$\begin{aligned} \langle M_{\text{CCCP}} / M_{\text{CLASH}} \rangle &= 0.84 \pm 0.10 \quad (\Delta = 500) \\ &= 0.91 \pm 0.24 \quad (\Delta = 2500) \end{aligned}$$

Umetsu+16,  
arXiv:1507.04385



# Summary

## – Ensemble mass profile shape

- Data favor cuspy density profiles predicted for collisionless-DM-dominated halos in gravitational equilibrium (NFW, Einasto, DARKexp)
- The highest-ranked model is the 2-parameter NFW+LSS model including the 2-halo term using the LCDM  $b$ - $M$  relation ( $b_h \sim 9.3$ )
- $c_{200c} = 3.8 \pm 0.3$  at  $M_{200c} = 10^{15} M_{\text{sun}}/h$ ,  $z=0.34$

## – Concentration vs. mass relation

- Fully consistent with LCDM when the CLASH selection function based on X-ray morphological regularity and projection effects are taken into account

## – Ensemble mass calibration

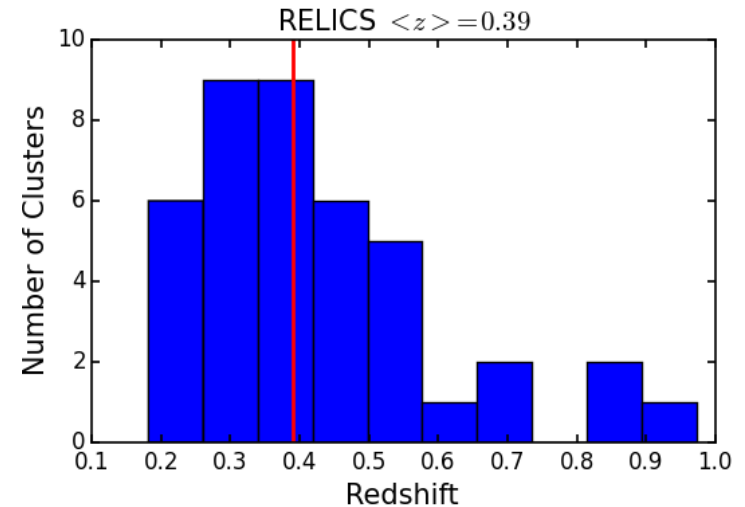
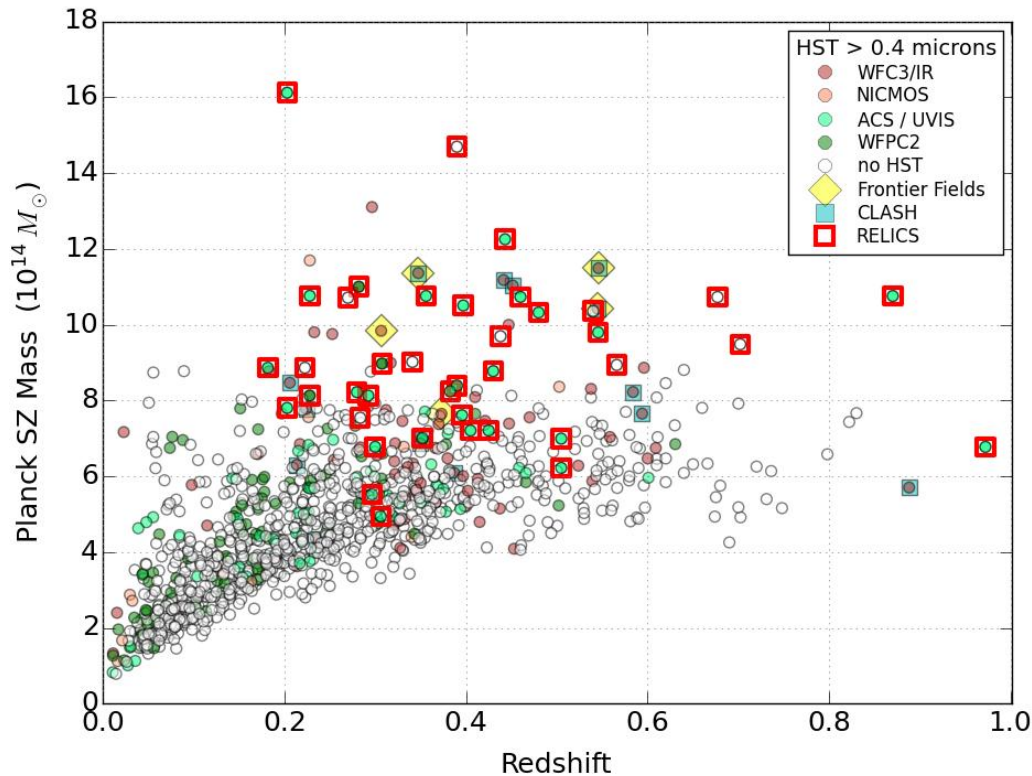
- Internal consistency (WL vs. WL+SL) at the  $\sim 5\%$  level
- Total calibration uncertainty  $\sim 9\%$  ( $\sim 6\%$  stat.,  $\sim 6\%$  sys.)

# Supplemental Slides



# Reionization Lensing Cluster Survey (RELICS)

Newly approved 190-orbit *HST* survey (7 ACS/WFC3 filters) of 41 high-mass clusters primarily selected from the *Planck* survey (P.I. Dan Coe; Oct 2015 – Apr 2017)

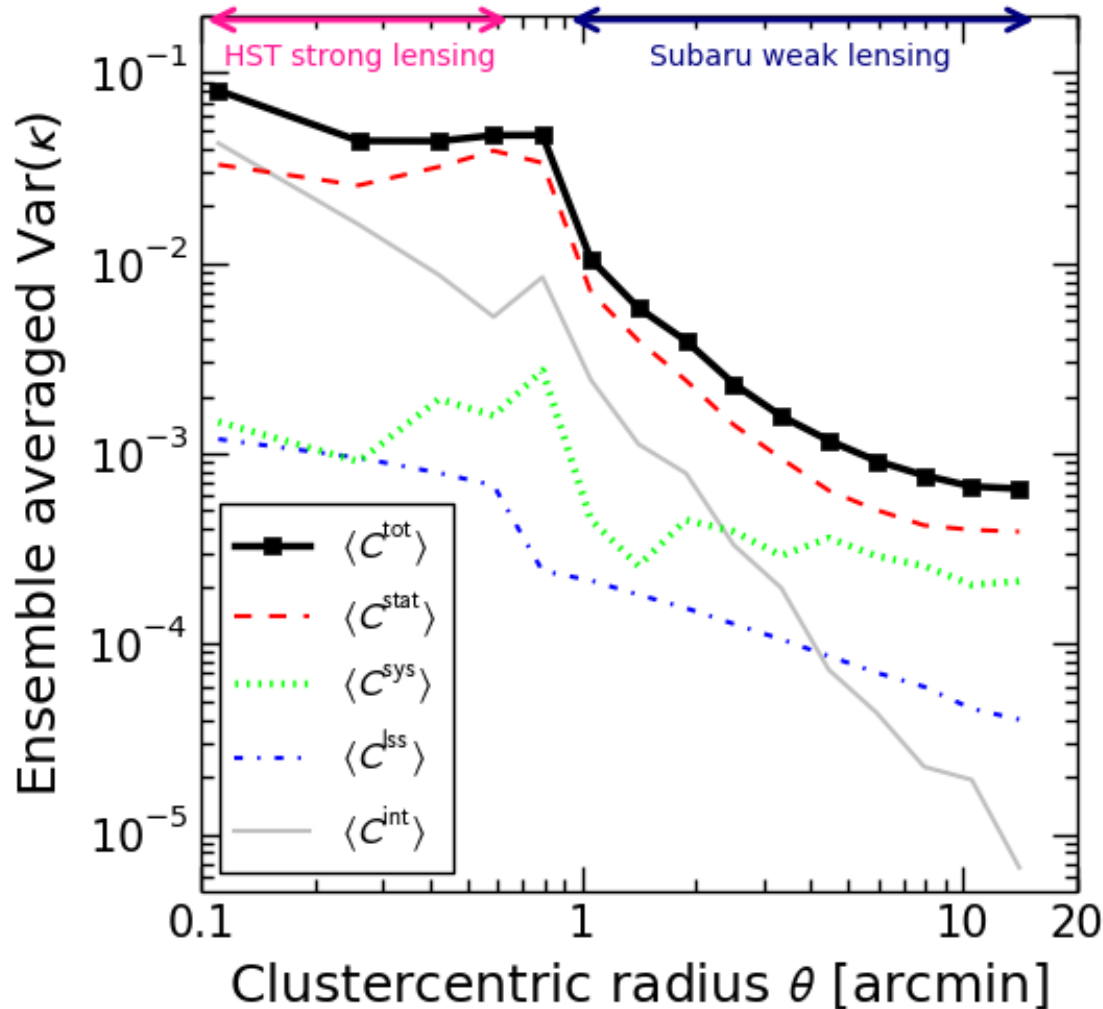


<http://hstrelics.weebly.com>



# Ensemble-averaged Error Budget

Diagonal elements ( $C_{ii}$ ) averaged over all CLASH clusters



Residual mass-sheet uncertainty (Umetsu+14)

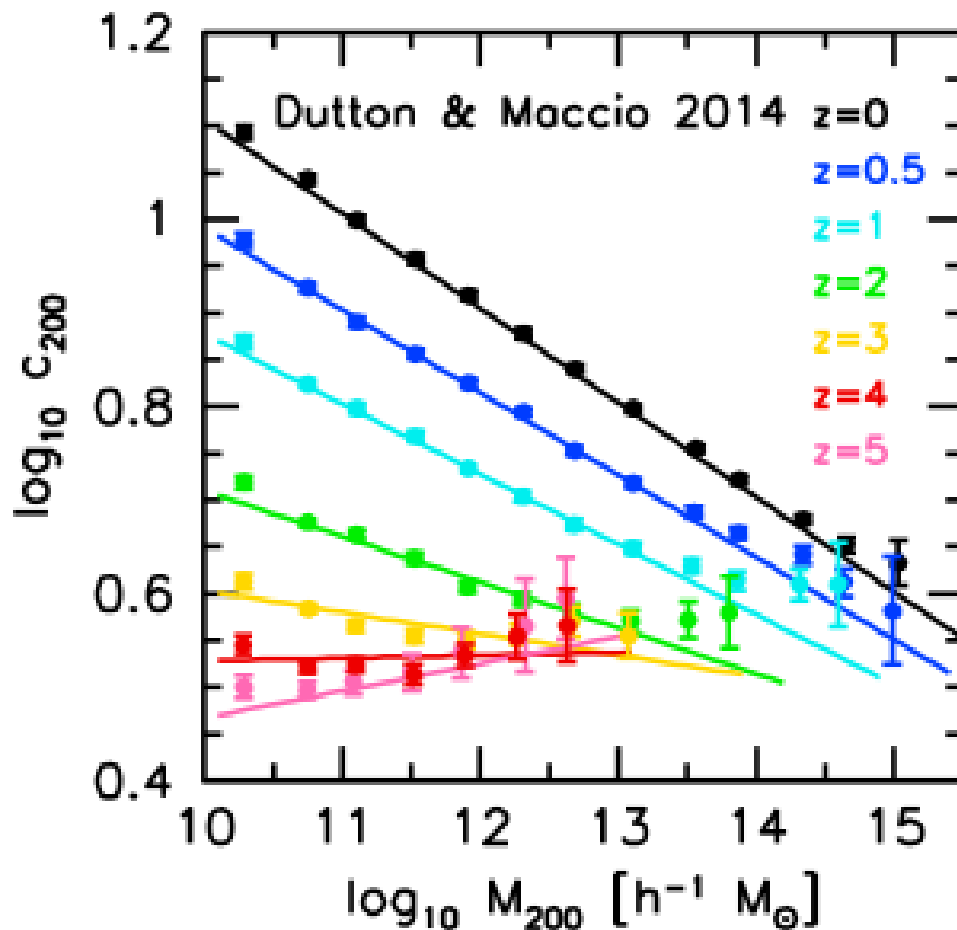
$$\langle C_{\text{sys}} \rangle_{ii} \sim \text{const.} \sim (0.02)^2$$

Intrinsic profile variations due to triaxiality, substructure, and  $c$ - $M$  scatter (Gruen+15)

$$\langle C_{\text{int}} \rangle_{ii} \approx (0.2)^2 K_i^2$$

# Degree of Mass Concentration

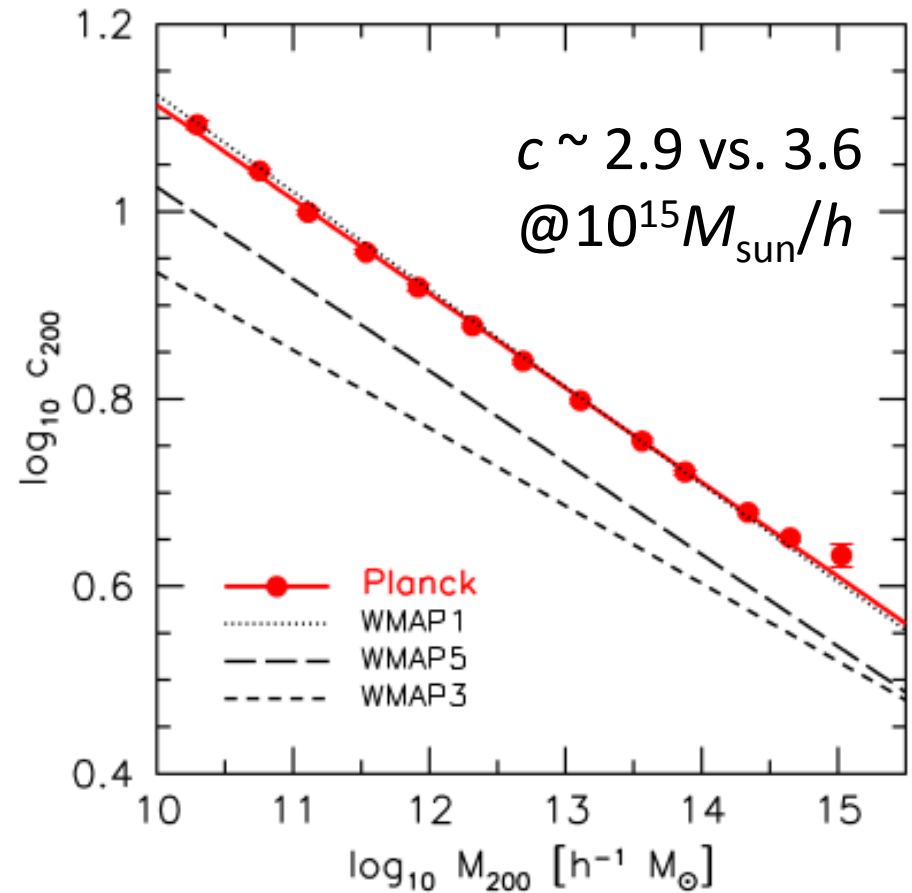
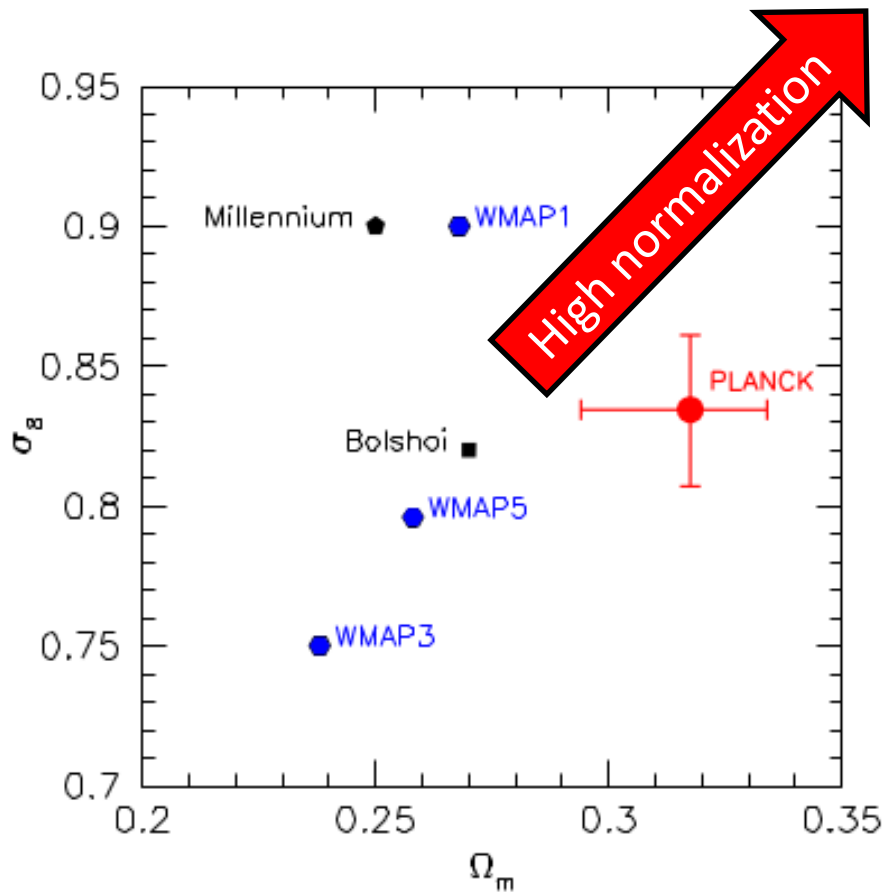
$$c_{200} \equiv \frac{r_{200}}{r_s} = \frac{\text{(Outer scale radius)}}{\text{(Inner scale radius)}}$$



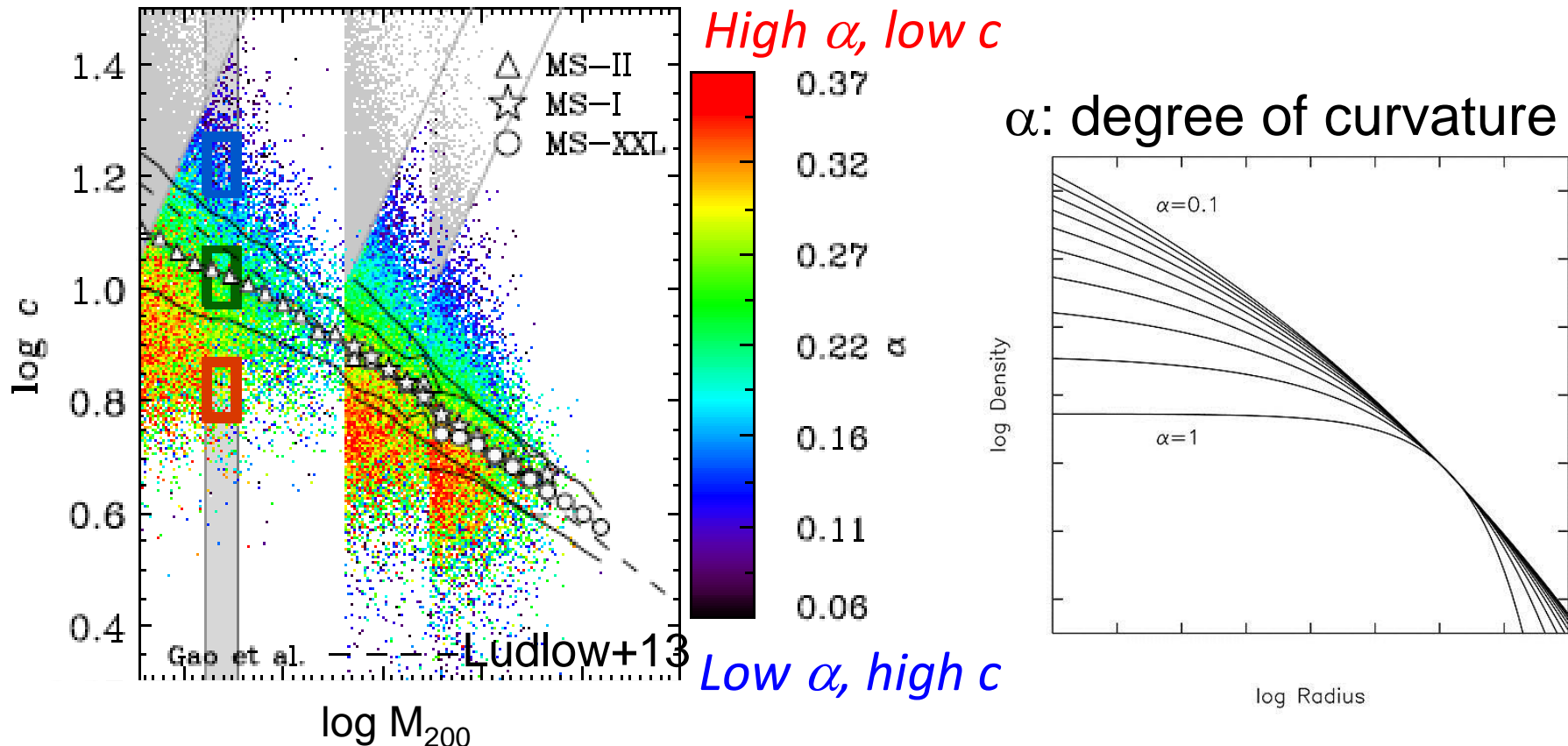
In hierarchical structure formation,  $\langle c \rangle$  is predicted to correlate with  $M$

DM halos that are more massive collapse later on average, when the mean background density of the universe is correspondingly lower (e.g., Bullock+01)

# Concentration is sensitive to cosmology



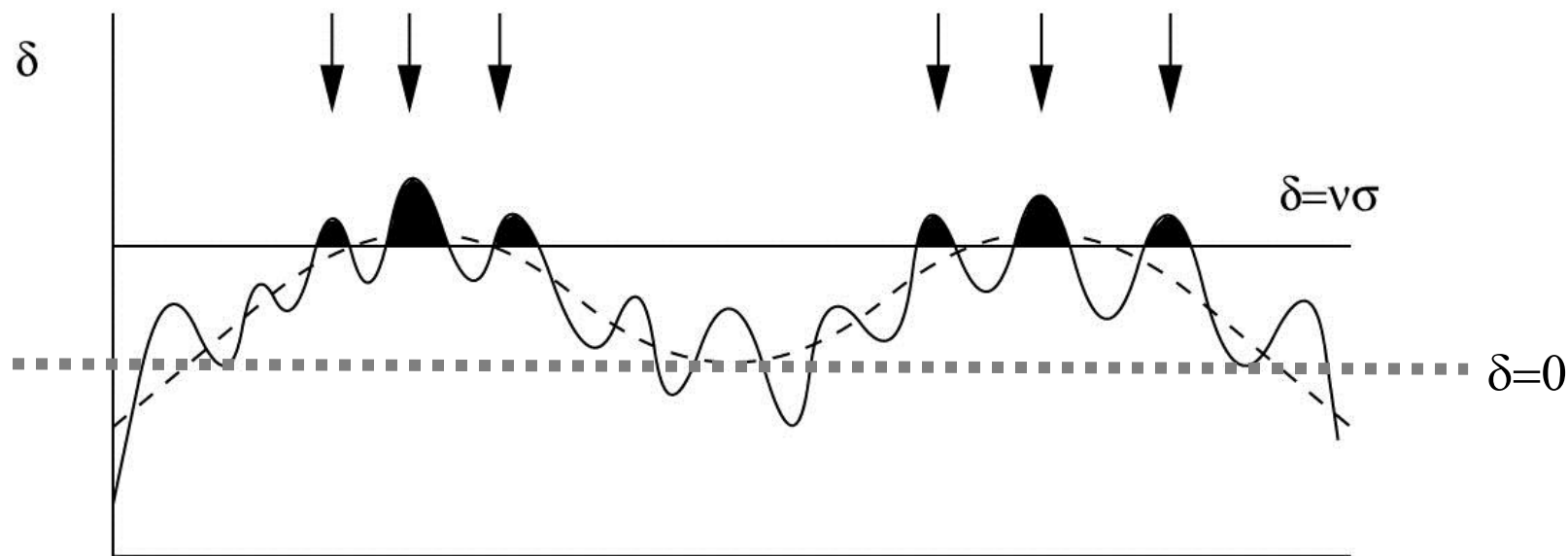
# Intrinsic Scatter in $c(M)$ : Mass Assembly Histories (MAH)



- Scatter is due to another DoF ( $\alpha$ ), related to MAH (Ludlow+13)
- Larger values of  $\alpha$  correspond to halos that have been assembled more rapidly than the NFW curve
- Halos with average  $c_{200}$  have the NFW-equivalent  $\alpha \sim 0.18$

# Key Predictions of nonlinear structure formation models

## (3) Halo bias: surrounding large-scale structure



$$\delta(\mathbf{x}) := \frac{\rho - \bar{\rho}}{\bar{\rho}} = \int \frac{d^3k}{(2\pi)^3} \tilde{\delta}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} \quad \mathbf{x}$$

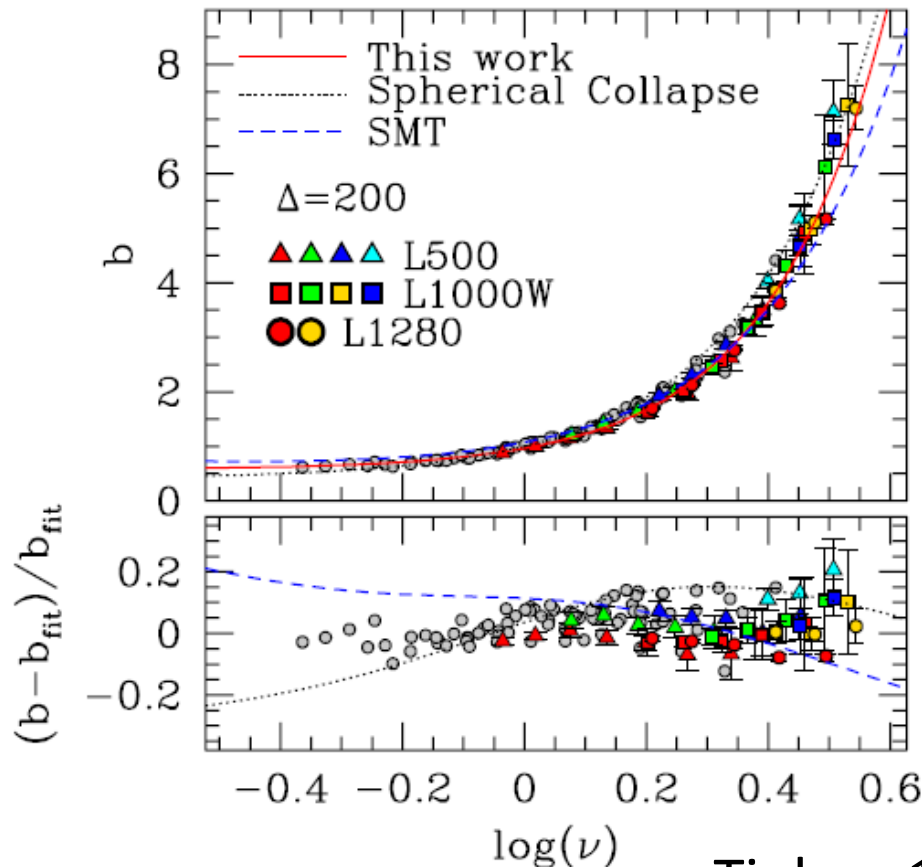
$$\langle \tilde{\delta}(\mathbf{k}) \tilde{\delta}(\mathbf{k}') \rangle = (2\pi)^3 \delta_D^3(\mathbf{k} + \mathbf{k}') P(k)$$

# Halo Bias Factor: $b_h$

Clustering of matter  
around halos with  $M$ :

$$\xi_{\text{hm}}(r | M) \equiv \langle \delta_h(\mathbf{x} | M) \delta_m(\mathbf{x} + \mathbf{r}) \rangle$$

$$= \frac{\langle \rho_h(r | M) \rangle}{\bar{\rho}} + \underline{b_h(M) \xi_{\text{mm}}(r)} \quad \text{2h term}$$



**Correlated matter distribution (2h term)**

**Matter correlation function:**

$$\xi_{\text{mm}}(\mathbf{r}) \equiv \langle \delta_m(\mathbf{x}) \delta_m(\mathbf{x} + \mathbf{r}) \rangle = \int \frac{d^3k}{(2\pi)^3} P(k) e^{i\mathbf{k} \cdot \mathbf{r}}$$

$$\propto \sigma_8^2$$

**Linear halo bias:**

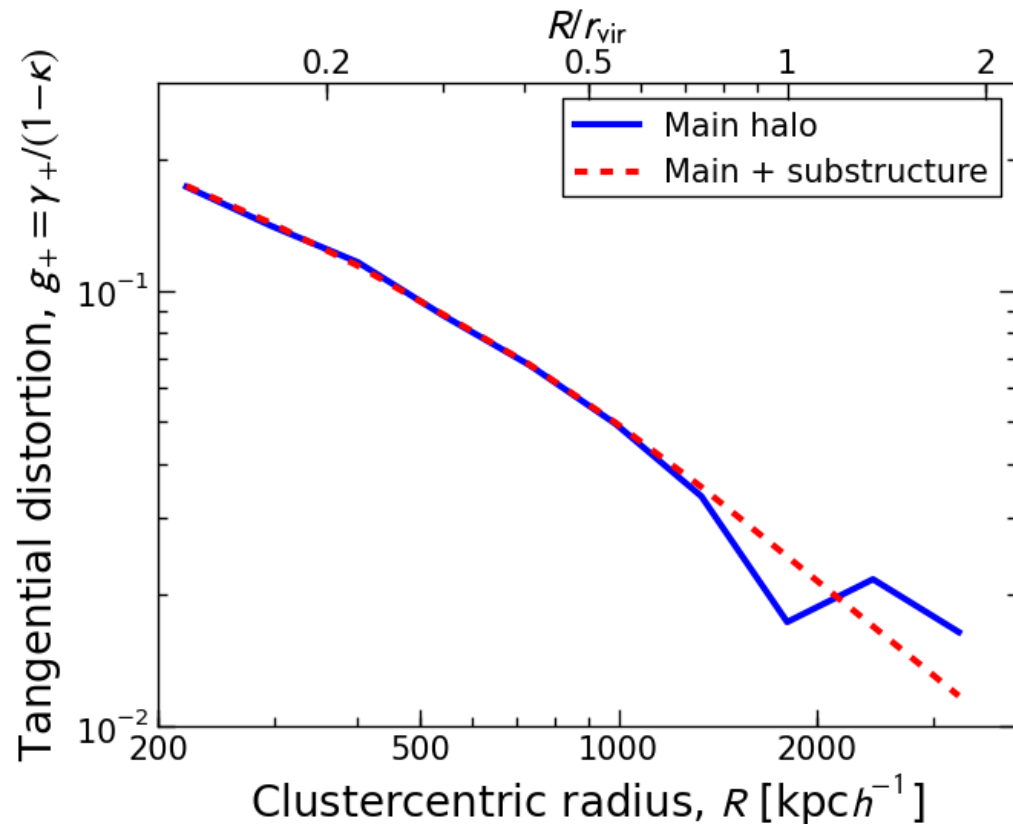
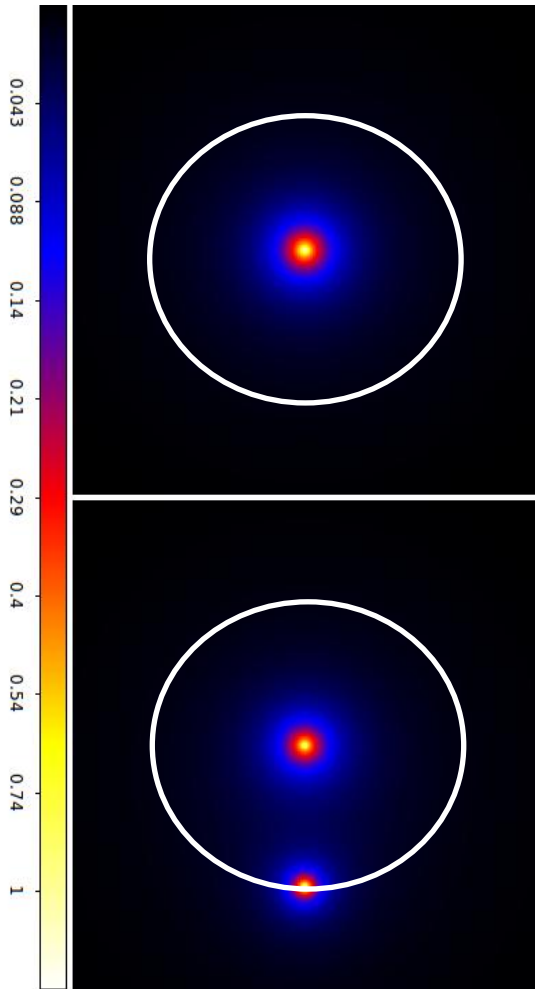
$$b_h(\nu) \approx 1 + \frac{\nu^2 - 1}{\delta_c}$$

$$\nu \equiv \frac{\delta_c}{\sigma(M, z)} \sim 3 - 4 \text{ for clusters}$$

Tinker+10 LCDM simulations

# Non-local substructure effect

A substructure at  $R \sim r_{\text{vir}}$  of the main halo, modulating  $\Delta\Sigma(R) = \Sigma(< R) - \Sigma(R)$



Known 5%-10% negative bias in mass estimates from tangential-shear fitting, inherent to rich substructure in outskirts (Rasia+12)

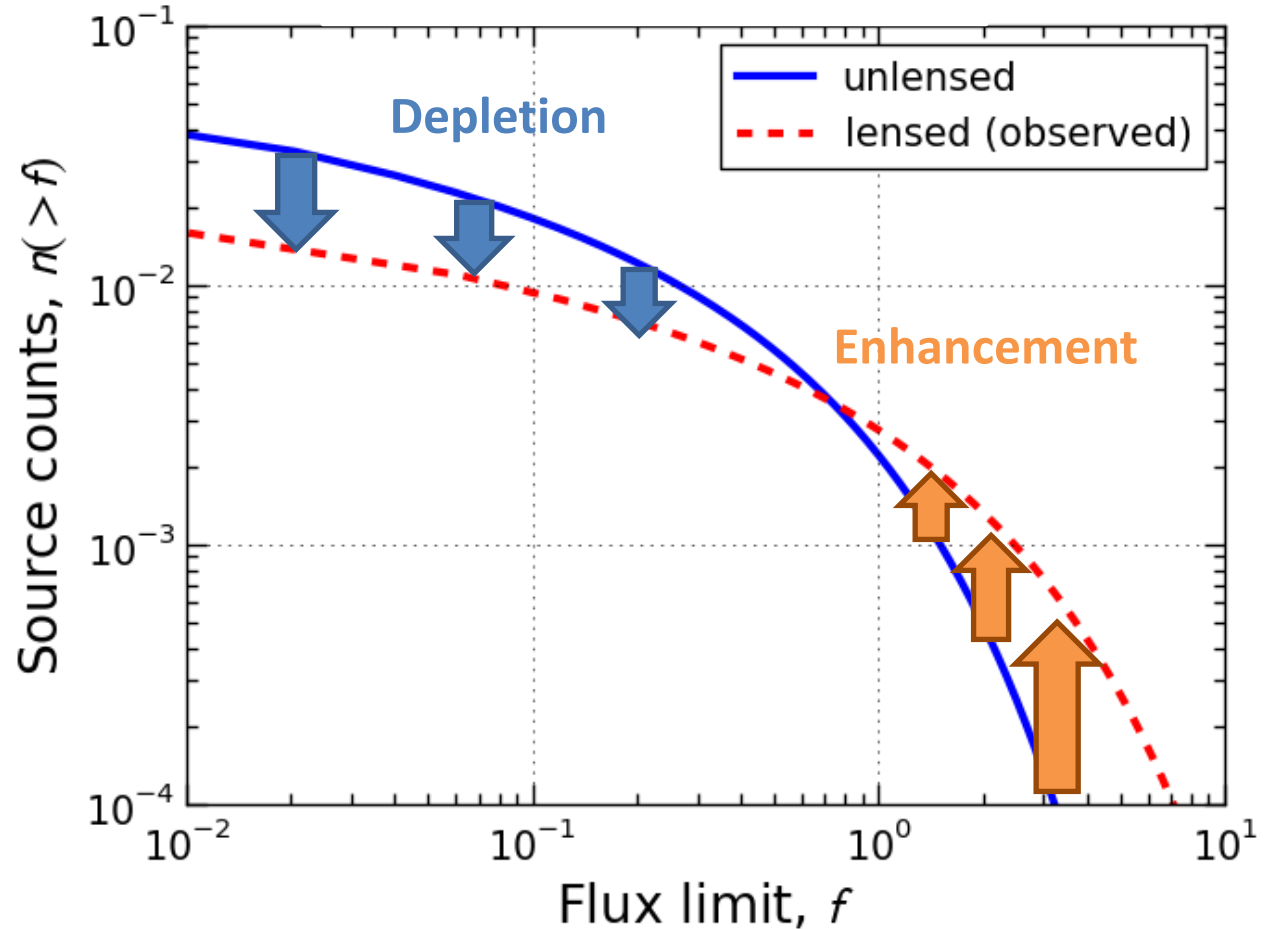


# Magnification bias effects

Flux-limited  
source counts:

$$n_{\text{obs}}(> f) = \mu^{-1} n(> \mu^{-1} f)$$

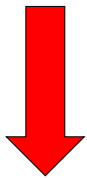
Broadhurst, Taylor &  
Peacock 1995



Flux amplification



Geometric area  
distortion



$n/\mu$



# Averaged Halo Density Profile $\Sigma(R)$

Stacking lensing signals of individual clusters by

$$\langle\langle \Sigma \rangle\rangle = \left( \sum_n \mathcal{W}_n \right)^{-1} \left( \sum_n \mathcal{W}_n \Sigma_n \right),$$

*Summing over clusters ( $n=1, 2, \dots$ )*

with individual “sensitivity” matrix

$$(\mathcal{W}_n)_{ij} \equiv \Sigma_{(c, \infty)n}^{-2} (C_n^{-1})_{ij},$$

defined with total covariance matrix

$$C = C^{\text{stat}} + C^{\text{sys}} + C^{\text{lss}} + C^{\text{int}},$$

**With “trace-approximation”, averaging (stacking) is interpreted as**

$$\langle\langle M_\Delta \rangle\rangle = \frac{\sum_n \text{tr}(\mathcal{W}_n) M_{\Delta,n}}{\sum_n \text{tr}(\mathcal{W}_n)}$$

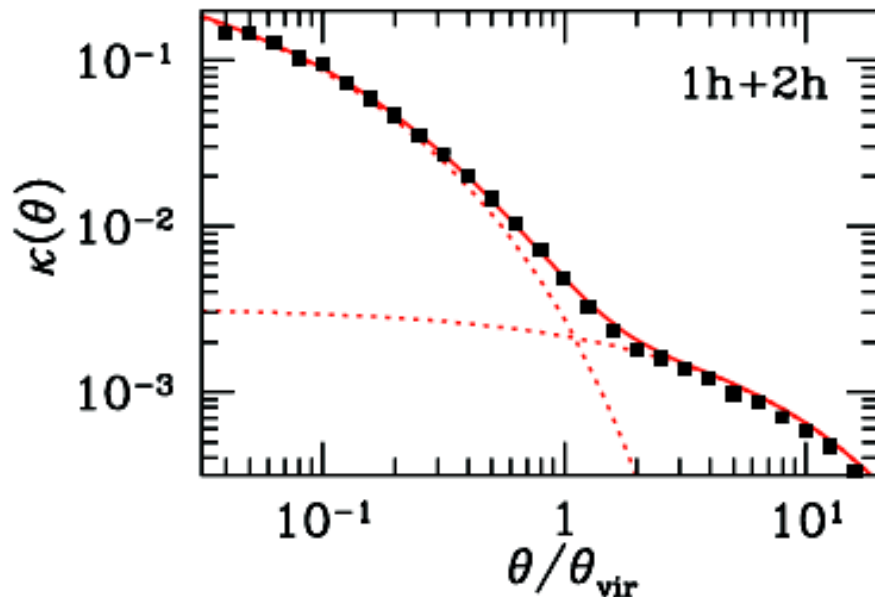
Umetsu et al. 2014,  
*ApJ*, 795, 163

# Shear doesn't see mass sheet

Averaged lensing profiles in/around LCDM halos (Oguri & Hamana 11)

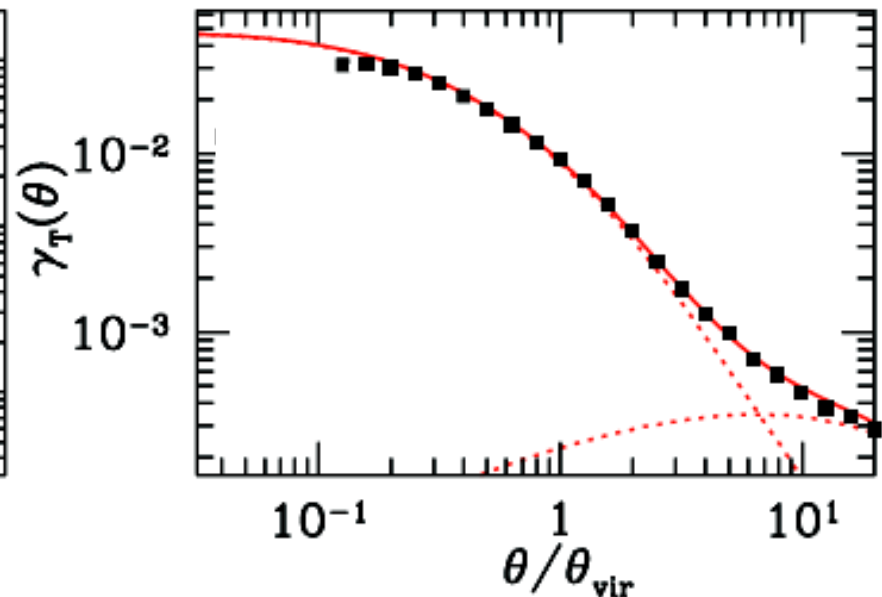
Total

$$\kappa = \Sigma(R) / \Sigma_c$$



Modulated

$$\gamma_+ = \Delta\Sigma(R) / \Sigma_c$$



- Tangential shear is a powerful probe of **1-halo term**, or **intra-halo structure**.
- Shear alone cannot recover absolute mass, known as **mass-sheet degeneracy**:

$\gamma$  remains unchanged by  $\kappa \rightarrow \kappa + \text{const.}$



# Concentration—Mass Relation of the CLASH X-ray-selected Subsample

Umetsu et al. 2016, arXiv:1507.04385



# Concentration—Mass Scaling Relation

Consider a power-law scaling relation of the form:

$$c_{200c} = 10^\alpha \left( \frac{M_{200c}}{M_{\text{piv}}} \right)^\beta \left( \frac{1+z}{1+z_{\text{piv}}} \right)^\gamma,$$

with pivot mass and redshift  $M_{\text{piv}} = 10^{15} M_{\text{sun}} / h$ ,  $z_{\text{piv}} = 0.34$

Define new independent ( $X$ ) and dependent ( $Y$ ) variables:

$$Y \equiv \log_{10} \left[ \left( \frac{1+z}{1+z_{\text{piv}}} \right)^{-\gamma} c_{200c} \right], \quad Y(X) = \alpha + \beta X$$
$$X \equiv \log_{10} (M_{200c} / M_{\text{piv}}).$$

Redshift slope  $\gamma$  is fixed to the theoretical prediction for the CLASH sample,  $\gamma = -0.668$  (Meneghetti+14)



# Bayesian Regression Analysis

We take into account

- Covariance between observed  $M$  and  $c$
- Intrinsic scatter in  $c$
- Non-uniformity in mass probability distribution  $P(\log M)$

**Conditional probability**  $P(y|x)$  with  $(x,y) = \text{observed } (X,Y)$

$$\ln \mathcal{P}(\mathbf{y}|\mathbf{x}) = -\frac{1}{2} \sum_n \left[ \ln (2\pi\sigma_n^2) + \left( \frac{y_n - \langle y_n|x_n \rangle}{\sigma_n} \right)^2 \right], \quad (35)$$

where  $\langle y_n|x_n \rangle$  and  $\sigma_n^2 \equiv \text{Var}(y_n|x_n)$  are the conditional mean and variance of  $y_n$  given  $x_n$ , respectively:

$$\begin{aligned} \langle y_n|x_n \rangle &= \alpha + \beta\mu + \frac{\beta\tau^2 + C_{xy,n}}{\tau^2 + C_{xx,n}}(x_n - \mu), \\ \sigma_n^2 &= \beta^2\tau^2 + \sigma_{Y|X}^2 + C_{yy,n} - \frac{(\beta\tau^2 + C_{xy,n})^2}{\tau^2 + C_{xx,n}}, \end{aligned} \quad (36)$$

where  $\sigma_{Y|X}$  is the intrinsic scatter in the  $Y-X$  relation;



# Marginalized Posterior Distributions

$$c_{200c} = 10^\alpha \left( \frac{M_{200c}}{M_{\text{piv}}} \right)^\beta \left( \frac{1+z}{1+z_{\text{piv}}} \right)^\gamma$$

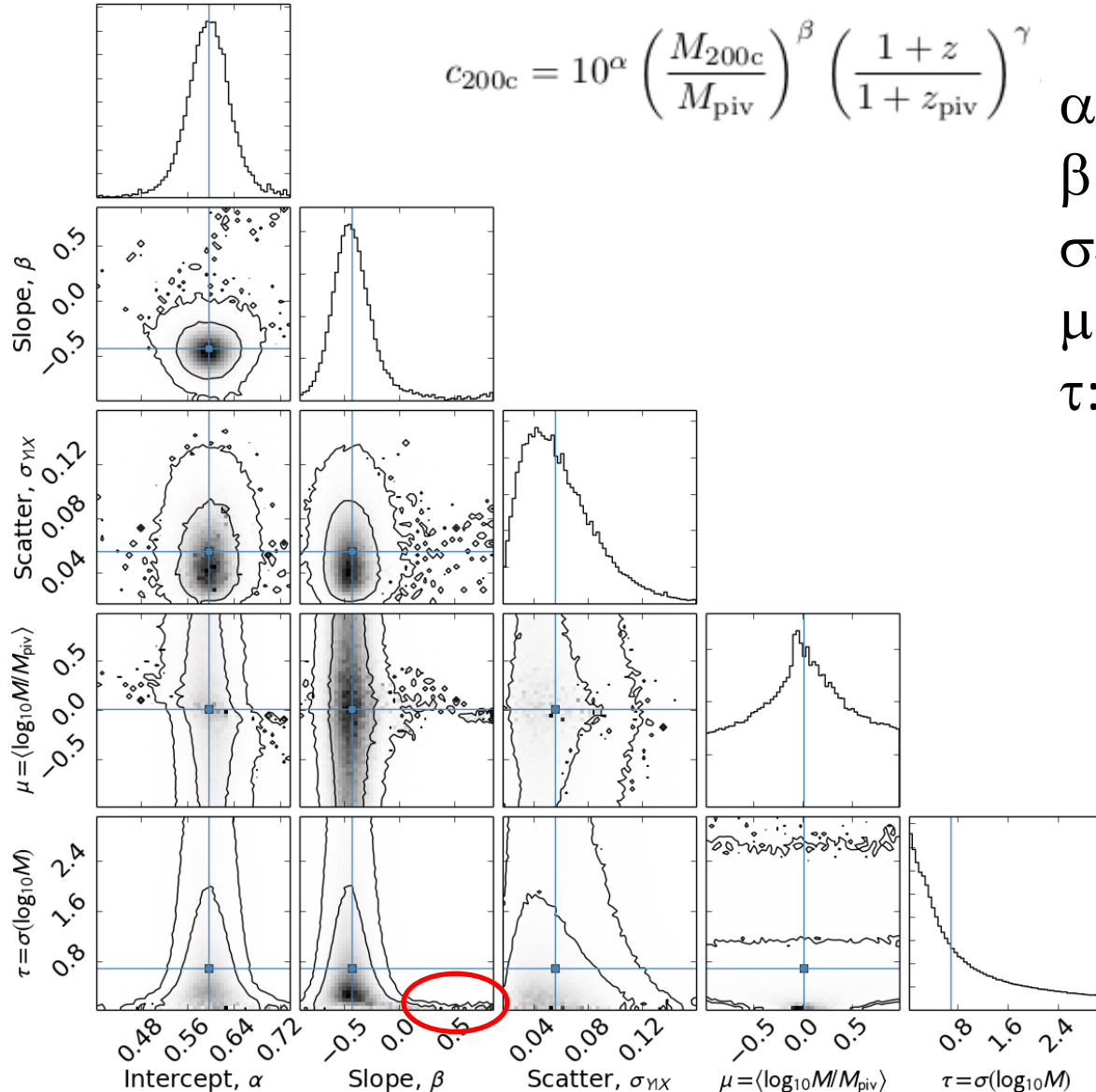
$\alpha$ : intercept

$\beta$ : slope

$\sigma_{Y|X}$ : scatter

$\mu$ : Gaussian mean of  $P(\ln M)$

$\tau$ : Gaussian width of  $P(\ln M)$



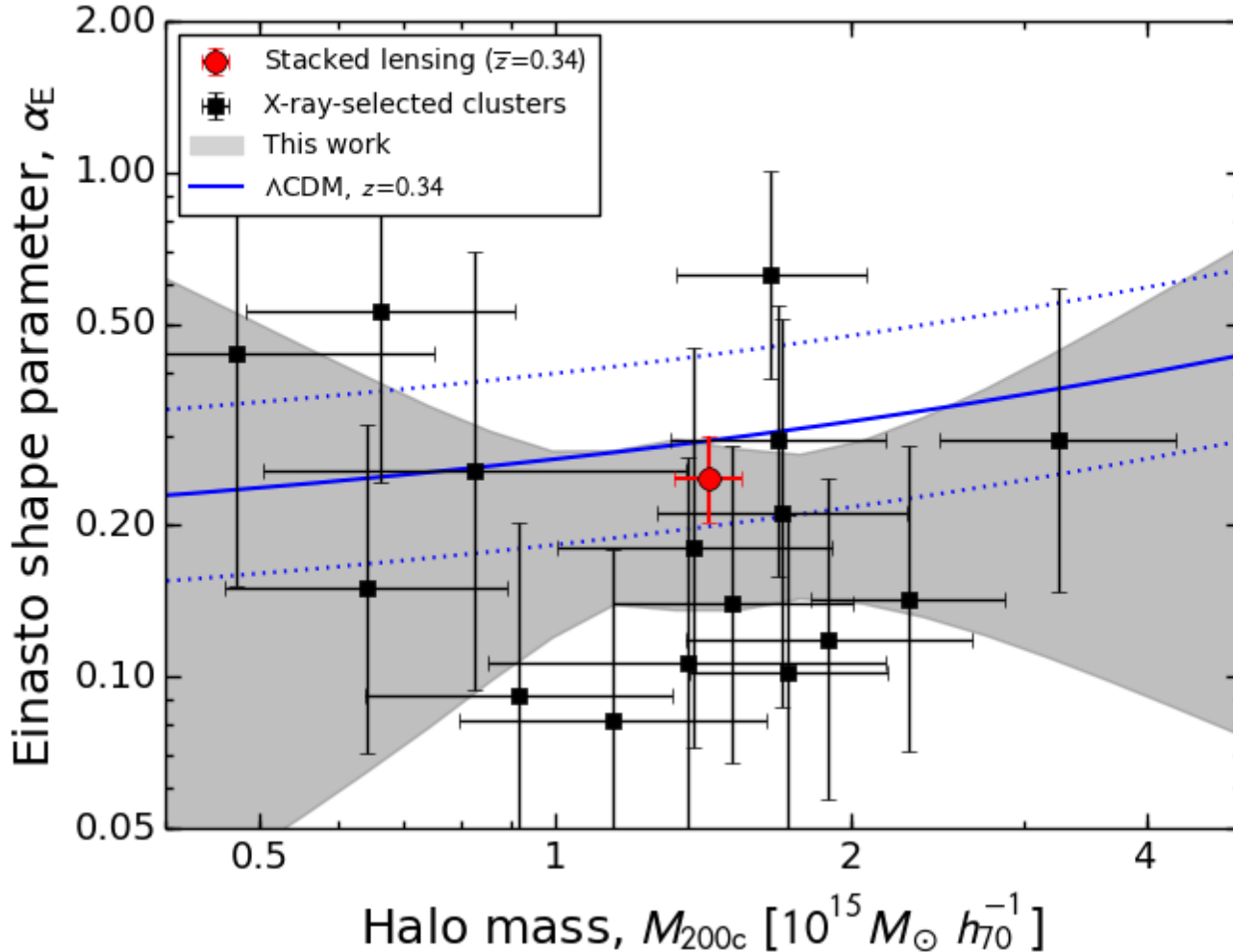
High  $\beta$  tail associated with small  $\tau$ : i.e., localized  $P(\ln M)$



# Einasto Shape Parameter vs. Halo Mass

$\alpha_E$ : degree of curvature of the Einasto density profile

$$\frac{d \ln \rho(r)}{d \ln r} = -2 \left( \frac{r}{r_{-2}} \right)^{\alpha_E}$$



$$\alpha_E \approx 0.155 + 0.0095v^2 \quad (\text{Gao} + 08)$$

$$v = \frac{\delta_c}{\sigma(M)}$$

**Preliminary results**

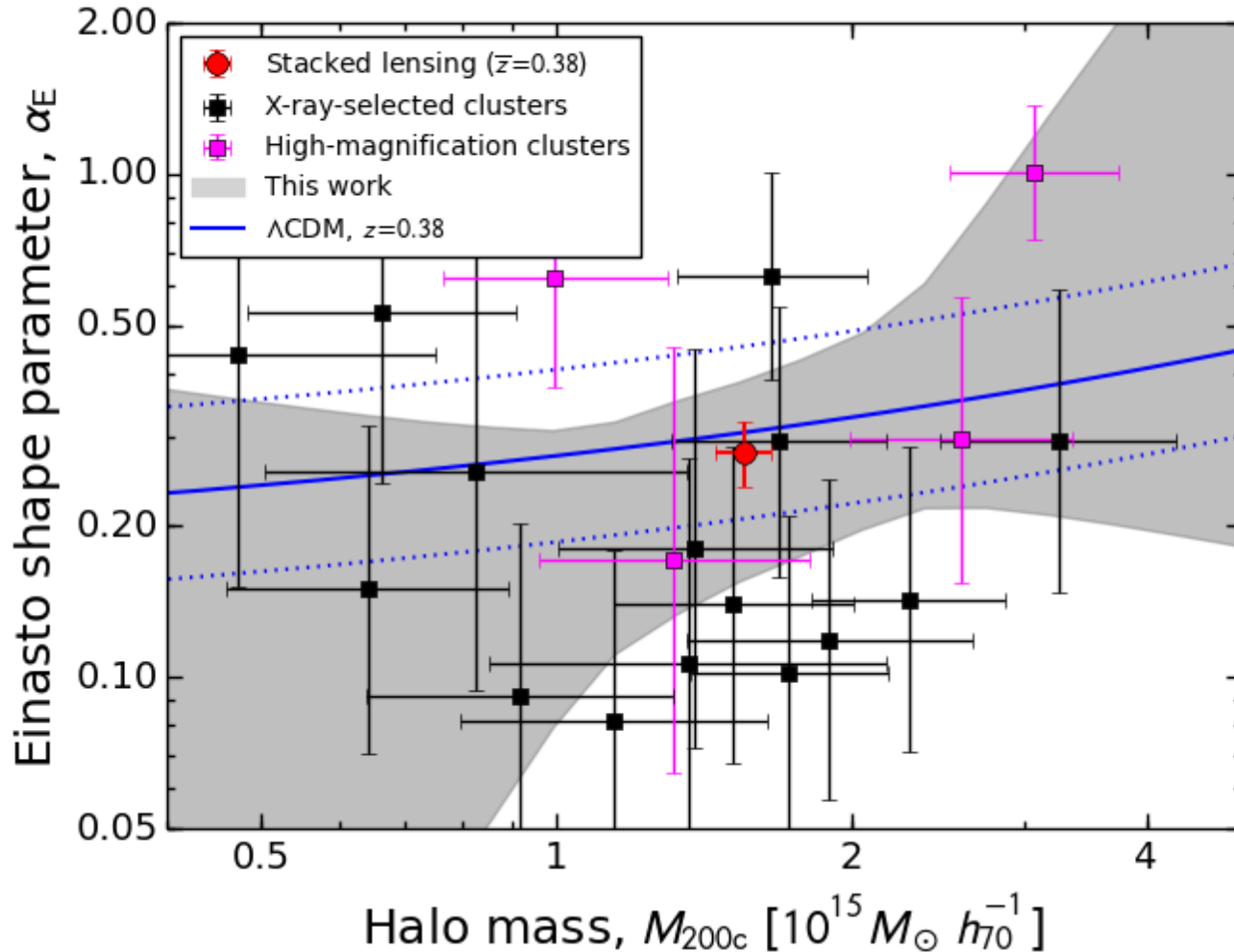




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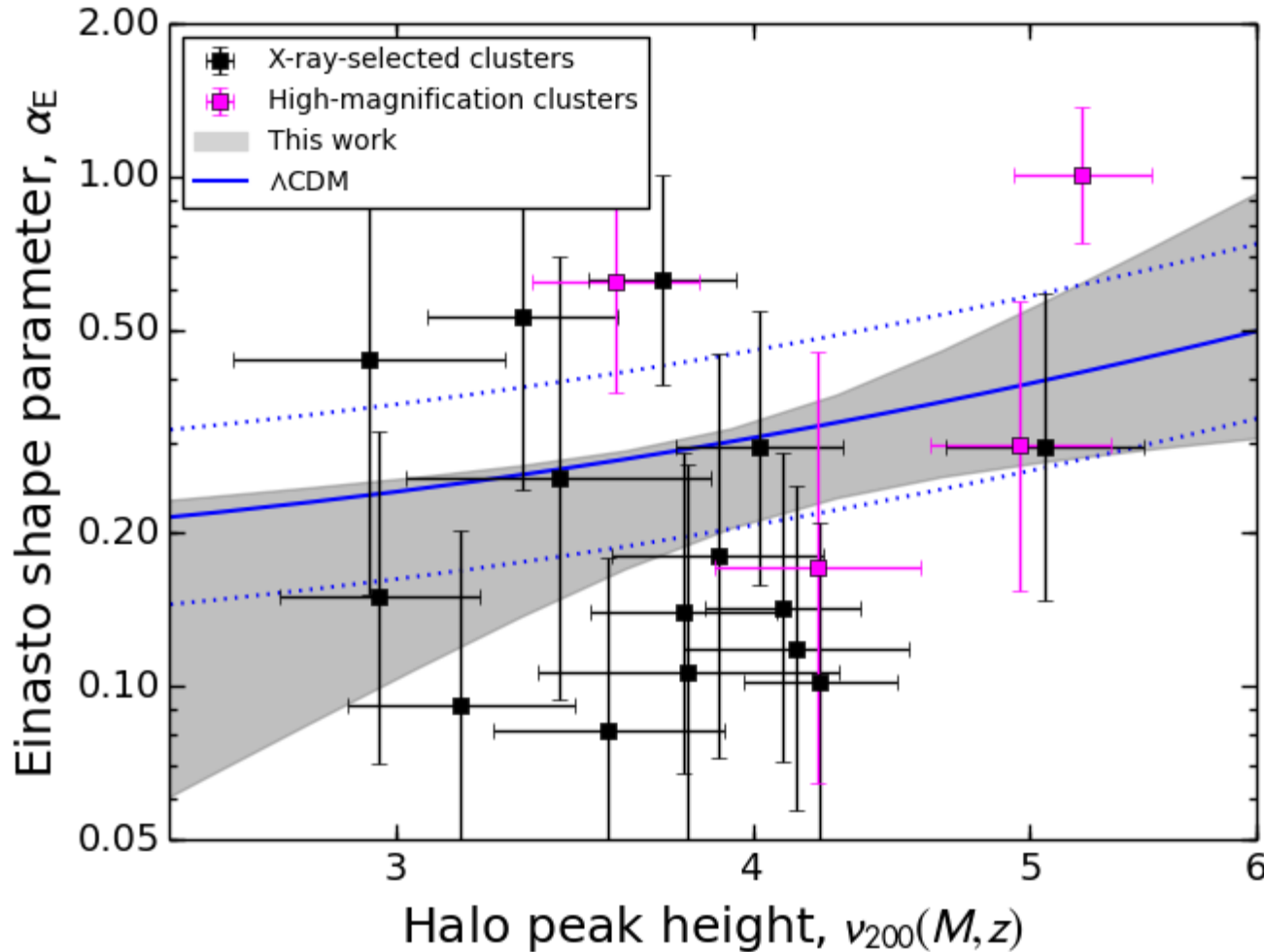
**Preliminary  
results**



# Einasto Shape Parameter vs. Halo Peak Height

$\alpha_E$ : degree of curvature of the Einasto density profile

$$\frac{d \ln \rho(r)}{d \ln r} = -2 \left( \frac{r}{r_{-2}} \right)^{\alpha_E}$$

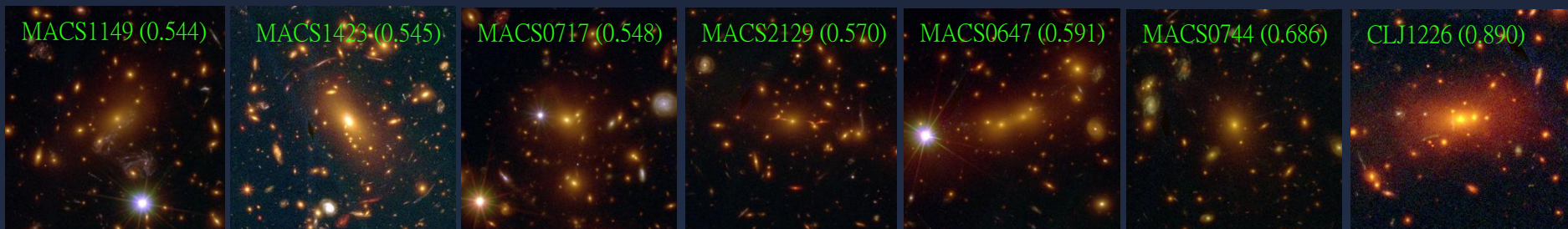
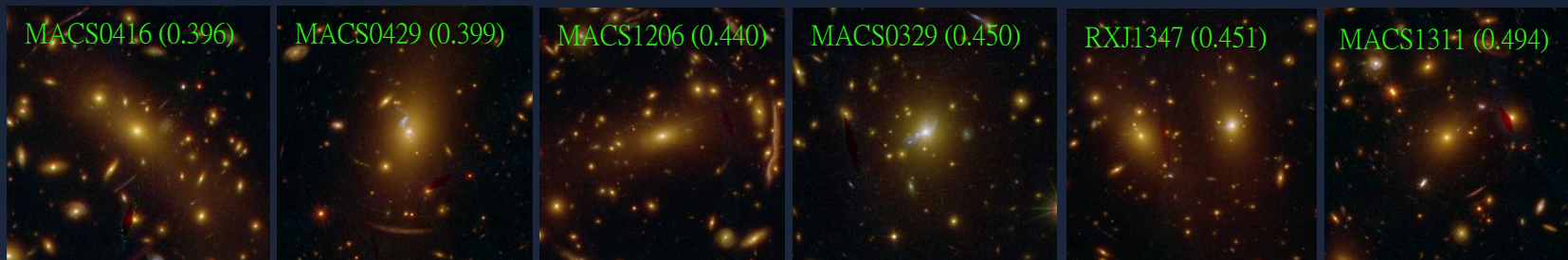


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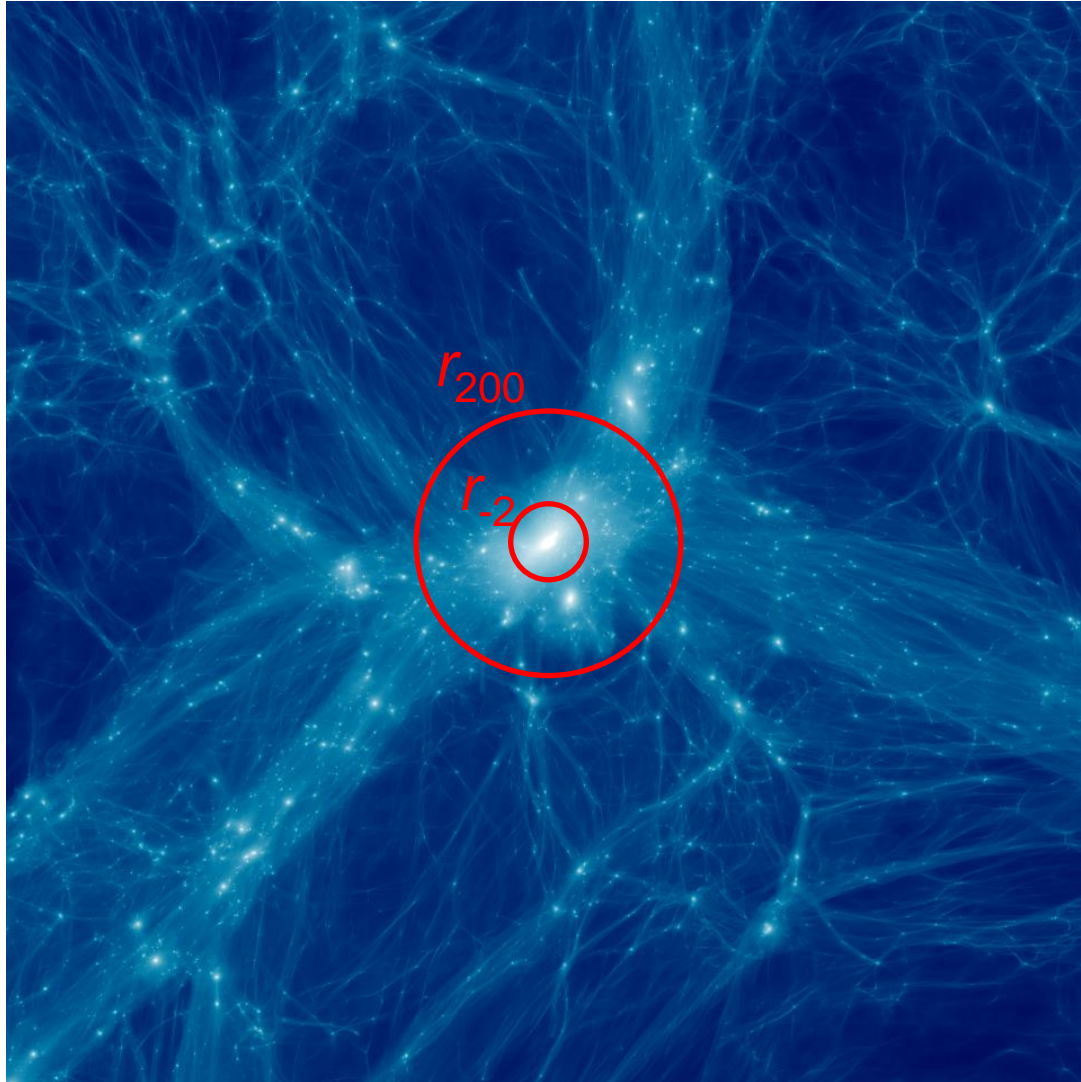
$$v = \frac{\delta_c}{\sigma(M)}$$

**Preliminary results**

# CLASH *HST* Lensing Dataset



# Cluster Gravitational Lensing



Diemer & Mansfield

## Key Objectives

### **Intra-halo structure**

Density profile,  $\rho(r)$

Halo mass,  $M_{\Delta}$

Concentration,  $c = r_{\Delta}/r_{-2}$

Halo asphericity

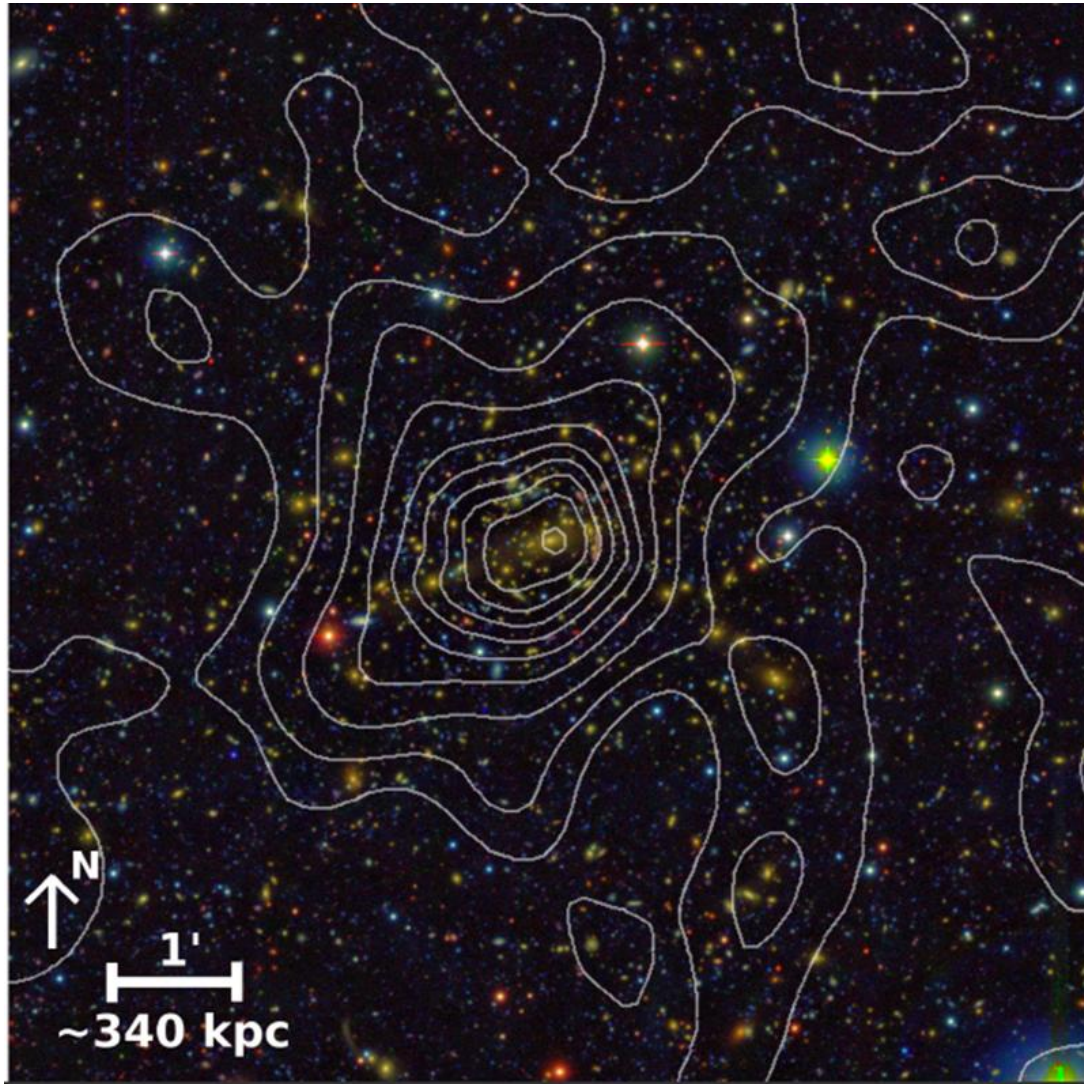
### **Surrounding LSS**

Halo bias  $b_h(M)$

DM clustering strength  $\sigma_8$

Assembly bias

# Cluster Gravitational Lensing



Diemer & Mansfield

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