Recent Progress in Cluster Weak Lensing

Cluster Lensing And Supernova survey with Hubble

Keiichi Umetsu with T. Broadhurst, E. Medezinski, A. Zitrin, M. Nonino, J. Merten + CLASH
Galaxy Clusters as Cosmological Probe

Cluster counts $n(M,z)$ are exponentially sensitive to cosmology, but also to *mass calibration*!!!
Weak Gravitational Lensing

- **Shear** (Kaiser 92, 93, 95)
  - Shape distortion: $\delta e \sim \gamma$

- **Magnification** (Broadhurst 95)
  - Flux amplification: $\mu F$
  - Area distortion: $\mu \Delta \Omega$

- Sensitive to “modulated” matter density
  \[ \Sigma_c \gamma_+ = \Delta \Sigma(R) \equiv \Sigma(< R) - \Sigma(R) \]

- Sensitive to “total” matter density
  \[ \mu \approx 1 + 2 \kappa; \quad \Sigma_c \kappa = \Sigma(R) \]
Shear fields around Tom’s favorite clusters

Map: RS galaxies
Whiskers: shear

Subaru
Suprime-Cam archival data

Broadhurst, Umetsu, Medezinski+08
A1689 at $z=0.183$, Subaru/S-Cam $BVRiz$ (Umetsu+15)
Shear strength as function of magnitude

\[ \gamma \propto \frac{D_{LS}(z)}{D_S(z)} \]

KSB+ (Umetsu+10) pipeline

Medezinski, Broadhurst, Umetsu+11
Shear strength as function of $z$ (KSB+) 

First detection of WL distance vs. redshift relation!!!
Shear vs. Magnification

Reduced tangential shear

\[ g_+ \approx \gamma_+ = \frac{\Delta \Sigma}{\Sigma_c} \]

Number count depletion due to magnification bias

(Broadhurst, Taylor, & Peacock 95)

\[ n_{\mu} = \bar{n}_{\mu} \mu^{-1 + 2.5 s_{\text{eff}}} \]

Subaru BVRIz data: A1689

(Umetsu et al. 2015)
Combining Shear and Magnification


\[ P(\kappa \mid WL) \propto P(WL \mid \kappa)P(\kappa) = P(g_+ \mid \kappa)P(n_\mu \mid \kappa)P(\kappa) \]

- Mass-sheet degeneracy broken
- Total statistical precision improved by \( \sim 20\%-30\% \)
- Calibration uncertainties marginalized over:

\[ c = \{ \langle W \rangle_s, f_{W,s}, \langle W \rangle_\mu, \bar{n}_\mu, s_{\text{eff}} \}. \]
Multi-probe Lensing Approach
Combining azimuthally-averaged strong and weak lensing observables

\[
\{M_{2D,i}\}^{N_{SL}}_{i=1}, \quad \{g_{+,i}\}^{N_{WL}}_{i=1}, \quad \{n_{\mu,i}\}^{N_{WL}}_{i=1}.
\]

\[
P(\kappa | WL, SL) \propto P(WL, SL | \kappa) P(\kappa) = P(g_+ | \kappa) P(n_\mu | \kappa) P(M_{2D} | \kappa) P(\kappa)
\]

\[
M_{2D}(< R) = \int_{|R'|<R} \Sigma(R') d^2 R'
\]

Multi-probe Lensing Approach

Combining azimuthally-averaged strong and weak lensing observables

\[ \{M_{2D, i}\}_{i=1}^{N_{WL}}, \{\langle g_+, i \rangle\}_{i=1}^{N_{WL}}, \{\langle n_\mu, i \rangle\}_{i=1}^{N_{WL}}. \]

\[ P(\kappa | \text{WL, SL}) \propto P(\text{WL, SL} | \kappa)P(\kappa) = P(\mathbf{g}_+ | \kappa)P(n_\mu | \kappa)P(M_{2D} | \kappa)P(\kappa) \]

\[ M_{2D}(< R) = \int_{|R'|< R} \Sigma(R')d^2 R' \]
Cluster Lensing And Supernova survey with Hubble

PI. Marc Postman (STScI)
http://www.stsci.edu/~postman/CLASH/Home.html
Before CLASH (2010), deep-multicolor Strong (*HST*) + Weak (*Subaru*) lensing data only available for a handful of “super lens” clusters.

- **Umetsu, Broadhurst, Zitrin+11a**
  - $c_{2D} = 6.2 \pm 0.3$
  - 60% superlens bias
  - $<c_{3D}> \sim 3$

**Total mass profile shape**: consistent w self-similar NFW (cf. Newman+13; Okabe+13)

**Degree of concentration**: predicted superlens correction not enough if $<c_{\Lambda CDM}> \sim 3$?
Before CLASH (2010), deep-multicolor Strong (HST) + Weak (Subaru) lensing data only available for a handful of “superlens” clusters

Total mass profile shape: consistent w self-similar NFW (cf. Newman+13; Okabe+13)
Degree of concentration: predicted superlens correction is just enough if $c_{\text{LCDM}} \approx 4$
CLASH X-ray-selected Subsample

- **High-mass clusters with smooth X-ray morphology**
  - $T_x > 5\text{keV} \ (M_{200c} > 5\times10^{14} M_{\text{sun}}/h)$
  - Small BCG/X-ray peak offset, $\sigma_{\text{off}} \sim 10\text{kpc}/h$
  - Smooth regular X-ray morphology

  $\rightarrow$ Optimized for radial-profile analysis

- **CLASH theoretical predictions** (Meneghetti+14)
  - Composite relaxed (70%) and unrelaxed (30%) clusters
  - Mean $<c_{200c}> = 3.9, \ c_{200c} = [3, 6]$
  - Small scatter in $c_{200c}$: $\sigma(\ln c_{200c}) = 0.16$
  - Largely free of orientation bias ($\sim 2\%$ in $<M_{3D}>$
  - $>90\%$ of CLASH clusters to have strong-lensing features
CLASH: Joint Analysis of Strong-lensing, Weak-lensing Shear and Magnification Data for 20 CLASH Galaxy Clusters

High-resolution space imaging with *HST* (ACS/WFC3) for strong lensing

Subaru/Suprime-Cam multi-color imaging for wide-field

34 arcmin
CLASH 

Subaru Weak-lensing Dataset

Joint Analysis of SL & WL shear+magnification

\[ \{ M_{2D,i} \}_{i=1}^{N_{SL}}, \{ \langle g_+\rangle \}_{i=1}^{N_{WL}}, \{ \langle n_\mu\rangle \}_{i=1}^{N_{WL}}. \]

Determination of \( M_{2D}(<R) \) from detailed HST SL modeling (Zitrin+15)

- Effective resolution: \( \Delta R = 10''(<R_{Ein}>/22'')(<N>/17)^{-1/2} \)
- Maximum integration radius: \( R_{max} \sim 2<R_{Ein}> \sim 40'' \)

HST-SL mass integration radii: \( R=(10'', 20'', 30'', 40'') \)

\[ <\chi^2/\text{dof}> = 0.95 \text{ for 20 CLASH clusters} \]
33σ detection of the ensemble-averaged mass profile out to \( \sim 2 r_{200m} \)
Characterizing the Ensemble Mass Profile

\[ \Sigma(R) = \int dl \Delta \rho(r), \]

**Models:**

1. No 2-halo term 
   \( (f_t=1, \rho_{2h}=0) \)
2. With 2-halo term 
   \( (Tinker+10) \)

\[ \Delta \rho(r) = f_t(r) \rho_h(r) + \rho_{2h}(r), \]

\[ f_t(r) = \left[ 1 + \left( \frac{r}{r_t} \right)^2 \right]^{-2}, \]
Comparison of Best-fit Models

Acceptable fits: $p$ values (PTE) > 0.05

Table 4

Best-fit models for the stacked mass profile of the CLASH X-ray-selected subsample

<table>
<thead>
<tr>
<th>Model</th>
<th>$M_{200c}$ ($10^{14} M_\odot h^{-1}$)</th>
<th>$c_{200c}$</th>
<th>Shape/structural parameters</th>
<th>$b_h$</th>
<th>$\chi^2$/dof</th>
<th>PTE$^a$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFW</td>
<td>$14.4^{+1.1}_{-1.0}$</td>
<td>$3.76^{+0.29}_{-0.27}$</td>
<td>$\gamma_c = 1$</td>
<td>—</td>
<td>$11.3/11$</td>
<td>0.419</td>
<td>No truncation</td>
</tr>
<tr>
<td>gNFW</td>
<td>$14.1^{+1.1}_{-1.1}$</td>
<td>$4.04^{+0.53}_{-0.52}$</td>
<td>$\gamma_c = 0.85^{+0.22}_{-0.31}$</td>
<td>—</td>
<td>$10.9/10$</td>
<td>0.366</td>
<td>No truncation</td>
</tr>
<tr>
<td>Einasto</td>
<td>$14.7^{+1.1}_{-1.1}$</td>
<td>$3.53^{+0.36}_{-0.39}$</td>
<td>$\alpha_E = 0.232^{+0.042}_{-0.038}$</td>
<td>—</td>
<td>$11.7/10$</td>
<td>0.306</td>
<td>No truncation</td>
</tr>
<tr>
<td>DARKexp-$\gamma^b$</td>
<td>$14.5^{+1.2}_{-1.1}$</td>
<td>$3.53^{+0.42}_{-0.45}$</td>
<td>$\phi_0 = 3.90^{+0.41}_{-0.45}$</td>
<td>—</td>
<td>$13.5/10$</td>
<td>0.198</td>
<td>No truncation</td>
</tr>
<tr>
<td>Pseudo iso thermal</td>
<td>—</td>
<td>—</td>
<td>$V_c = 1762^{+40}<em>{-39}$ km/s, $r_c = 69^{+7}</em>{-7}$ kpc</td>
<td>—</td>
<td>$23.6/11$</td>
<td>0.015</td>
<td>No truncation</td>
</tr>
<tr>
<td>Burkert</td>
<td>$11.6^{+0.8}_{-0.8}$</td>
<td>—</td>
<td>$r_{200c}/r_0 = 8.81^{+0.42}_{-0.41}$</td>
<td>—</td>
<td>$29.9/11$</td>
<td>0.002</td>
<td>No truncation</td>
</tr>
<tr>
<td>Power-law sphere</td>
<td>$12.5^{+0.8}_{-0.8}$</td>
<td>—</td>
<td>$\gamma_c = 1.78^{+0.02}_{-0.02}$</td>
<td>—</td>
<td>$93.5/11$</td>
<td>0.000</td>
<td>No truncation</td>
</tr>
</tbody>
</table>

Halo model$^c$:

- NFW+LSS (i) $14.1^{+1.0}_{-1.0}$ $3.79^{+0.30}_{-0.38}$ $\gamma_c = 1$ $9.3$ $10.9/10$ 0.450 $\Lambda$CDM $b_h(M)$ scaling
- NFW+LSS (ii) $14.4^{+1.4}_{-1.3}$ $3.74^{+0.33}_{-0.30}$ $\gamma_c = 1$ $7.4^{+4.6}_{-4.7}$ $10.8/10$ 0.377 $b_h$ as a free parameter
- Einasto+LSS (i) $14.3^{+1.1}_{-1.1}$ $3.69^{+0.36}_{-0.42}$ $\alpha_E = 0.248^{+0.051}_{-0.047}$ $9.3$ $10.7/10$ 0.385 $\Lambda$CDM $b_h(M)$ scaling
- Einasto+LSS (ii) $14.5^{+1.9}_{-1.6}$ $3.65^{+0.47}_{-0.51}$ $\alpha_E = 0.245^{+0.053}_{-0.045}$ $8.7^{+4.6}_{-5.6}$ $10.6/9$ 0.301 $b_h$ as a free parameter
- DARKexp+LSS (i) $14.2^{+1.2}_{-1.1}$ $3.64^{+0.44}_{-0.46}$ $\phi_0 = 3.89^{+0.51}_{-0.54}$ $9.3$ $11.7/10$ 0.308 $\Lambda$CDM $b_h(M)$ scaling
- DARKexp+LSS (ii) $14.0^{+1.6}_{-1.1}$ $3.69^{+0.57}_{-0.57}$ $\phi_0 = 3.85^{+0.57}_{-0.61}$ $10.1^{+4.9}_{-5.1}$ $11.6/9$ 0.235 $b_h$ as a free parameter

$^a$ Probability to exceed the observed $\chi^2$ value.

$^b$ We use Dehnen–Tremaine $\gamma$-models with the central cusp slope $\gamma_c = 3 \log_{10} \phi_0 - 0.65$ ($1.7 \leq \phi_0 \leq 6$) as an analytic fitting function for the DARKexp density profile.

$^c$ For halo model predictions, we decompose the total mass overdensity $\Delta \rho(r) = \rho(r) - \rho_{200c}$ as $\Delta \rho = f_t \rho_h + f_{2h}$, where $\rho_h(r)$ is the halo density profile, $\rho_{2h}(r) = \rho_{200c} b_h \xi_{200c}^n(r)$ is the two-halo term, and $f_t(r) = (1 + r^2/r_{200c}^2)^{-3}$ describes the steepening of the density profile in the transition regime around the truncation radius $r_t$, which is assumed to be $r_t = 3 r_{200c}$.

- Consistent with cuspy density profiles (NFW, Einasto, DARKexp)
- Cuspy models that include $\Lambda$CDM 2-halo term ($b_h \sim 9.3$) give improved fits
CLASH Concentration vs. Mass Relation

Predicted (M14):
\[ \langle c_{200c} \rangle = 3.9, \]
\[ 3 \leq c_{200c} \leq 6, \]
\[ \sigma(\ln c_{200c}) = 0.16 \]

Observed:
\[ c_{200c} \big|_{z=0.34} = 3.95 \pm 0.35 \]
at \[ M_{200c} = 10^{15} M_{\odot} / h, \]
\[ \sigma(\ln c_{200c}) = 0.13 \pm 0.06 \]

Normalization, slope, & scatter are all consistent with LCDM when the CLASH selection function based on X-ray morphological regularity and the projection effects are taken into account.
Comparison with LCDM $c(M)$ models

**Table 5**
Comparison of measured and predicted concentrations for the CLASH X-ray-selected subsample

<table>
<thead>
<tr>
<th>Author</th>
<th>Sample</th>
<th>3D/2D</th>
<th>Function$^a$</th>
<th>$c^{(obs)}/c^{(pred)}$</th>
<th>$\chi^2$</th>
<th>PTE$^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Average$^c$</td>
<td>$\sigma^d$</td>
<td></td>
</tr>
<tr>
<td>Theory:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Duffy et al. (2008)</td>
<td>full</td>
<td>3D</td>
<td>$c-M$</td>
<td>1.331 ± 0.108</td>
<td>0.334</td>
<td>22.6</td>
</tr>
<tr>
<td>Duffy et al. (2008)</td>
<td>relaxed</td>
<td>3D</td>
<td>$c-M$</td>
<td>1.165 ± 0.094</td>
<td>0.290</td>
<td>13.6</td>
</tr>
<tr>
<td>Prada et al. (2012)</td>
<td>full</td>
<td>3D</td>
<td>$c-\nu$</td>
<td>0.733 ± 0.065</td>
<td>0.244</td>
<td>24.6</td>
</tr>
<tr>
<td>Bhattacharya et al. (2013)</td>
<td>full</td>
<td>3D</td>
<td>$c-\nu$</td>
<td>1.165 ± 0.095</td>
<td>0.292</td>
<td>14.1</td>
</tr>
<tr>
<td>Bhattacharya et al. (2013)</td>
<td>relaxed</td>
<td>3D</td>
<td>$c-\nu$</td>
<td>1.131 ± 0.092</td>
<td>0.277</td>
<td>12.4</td>
</tr>
<tr>
<td>Dutton &amp; Macciò (2014)</td>
<td>full</td>
<td>3D</td>
<td>$c-M$</td>
<td>1.061 ± 0.086</td>
<td>0.262</td>
<td>10.4</td>
</tr>
<tr>
<td>Meneghetti et al. (2014)</td>
<td>full</td>
<td>3D</td>
<td>$c-M$</td>
<td>1.061 ± 0.089</td>
<td>0.279</td>
<td>10.2</td>
</tr>
<tr>
<td>Meneghetti et al. (2014)</td>
<td>relaxed</td>
<td>3D</td>
<td>$c-M$</td>
<td>0.990 ± 0.083</td>
<td>0.249</td>
<td>9.2</td>
</tr>
<tr>
<td>Diemer &amp; Kravtsov (2015)</td>
<td>full (median)</td>
<td>3D</td>
<td>$c-\nu$</td>
<td>1.021 ± 0.083</td>
<td>0.330</td>
<td>14.4</td>
</tr>
<tr>
<td>Diemer &amp; Kravtsov (2015)</td>
<td>full (mean)</td>
<td>3D</td>
<td>$c-\nu$</td>
<td>1.060 ± 0.086</td>
<td>0.326</td>
<td>13.8</td>
</tr>
<tr>
<td>Meneghetti et al. (2014)</td>
<td>full</td>
<td>2D</td>
<td>$c-M$</td>
<td>1.087 ± 0.092</td>
<td>0.336</td>
<td>13.5</td>
</tr>
<tr>
<td>Meneghetti et al. (2014)</td>
<td>relaxed</td>
<td>2D</td>
<td>$c-M$</td>
<td>1.040 ± 0.086</td>
<td>0.283</td>
<td>10.8</td>
</tr>
<tr>
<td>Meneghetti et al. (2014)</td>
<td>CLASH</td>
<td>2D</td>
<td>$c-M$</td>
<td>0.988 ± 0.078</td>
<td>0.227</td>
<td>9.6</td>
</tr>
</tbody>
</table>

**Observations:**
- Consistent with models that are calibrated for more recent cosmologies (WMAP7 and later)
- Better agreement is achieved when selection effects (overall degree of relaxation) are taken into account
X-ray regular vs. Superlens Clusters

Umetsu+11b: 4 superlenses with $R_{\text{Ein}} > 30''$ at $z_s = 2$ (A1689, A1703, Cl0024, A370)

Higher normalization LCDM cosmology (WMAP7 and later) + “predicted” +60% superlens correction (e.g., Oguri+Blandford09) can explain superlens mass profiles!
Ensemble Calibration of Cluster Masses
Planck13 CMB vs. Cluster Cosmology

- Planck: $3\sigma$ tension between SZ cluster counts and CMB cosmology
- assumes $M_{\text{Planck}} / M_{\text{true}} = (1-b) = 0.8$
- calibrated with XMM hydrostatic masses (Arnaud et al. 2010) + simulations

suggested explanations:
- mass bias underestimated (and no accounting for uncertainties)
- $2.9\sigma$ detection of neutrino masses: $\Sigma m_\nu = (0.58 +/- 0.20)$ eV
  (Planck+WMAPpol+ACT+BAO: $\Sigma m_\nu < 0.23$ eV, 95% CL)

Slide taken from Anja von der Linden’s presentation
Comparison with *Planck* Masses – Not so Simple

Mass-dependent bias (20-45%) observed for *Planck* mass estimates

Mass dependent bias observed for *Planck* mass estimates, with $b \sim 0.2$ and $b \sim 0.45$. The fiducial value assumed by the *Planck* team is $b = \text{const.} = 0.2$.

Sereno, Ettori, & Moscardini 15, CoMaLit II (arXiv:1407.7869)
CLASH Internal Consistency

$M_{3D}(\langle r \rangle)$ de-projected assuming spherical NFW density profiles

WL (U14) and WL+SL (U16) are consistent within 5% at $r = [200, 2000]$ kpc/h

Umetsu+16, arXiv:1507.04385

CLASH ensemble mass calibration uncertainty

- Statistical uncertainty with $N=20$ clusters: $28%/\sqrt{20} = 6.3%$
- Systematic uncertainty: 5.6% (5% shear calibration, 2% dilution)
- Mass modeling bias (deviations from NFW, orientation bias): 3%
- Total calibration uncertainty: 9%
Comparisons with Other WL Surveys

**WtG [Subaru]**
(Applegate+14)

17 clusters

\[
\langle M_{WtG} / M_{CLASH} \rangle = 1.03 \pm 0.09 \quad (\Delta = 200)
\]
\[
= 1.07 \pm 1.12 \quad (\Delta = 500)
\]
\[
= 1.07 \pm 1.12 \quad (\Delta = 1000)
\]

**LoCuSS [Subaru]**
(Okabe & Smith 15)

5 clusters

\[
\langle M_{LoCuSS} / M_{CLASH} \rangle = 1.00 \pm 0.15 \quad (\Delta = \Delta_{\text{vir}})
\]
\[
= 0.98 \pm 0.13 \quad (\Delta = 200)
\]
\[
= 0.93 \pm 0.10 \quad (\Delta = 500)
\]
\[
= 0.84 \pm 0.22 \quad (\Delta = 2500)
\]

**CCCP [CFHT]**
(Hoekstra+15)

6 clusters

\[
\langle M_{CCCP} / M_{CLASH} \rangle = 0.84 \pm 0.10 \quad (\Delta = 500)
\]
\[
= 0.91 \pm 0.24 \quad (\Delta = 2500)
\]

Umetsu+16, arXiv:1507.04385
Summary

– Ensemble mass profile shape
  • Data favor cuspy density profiles predicted for collisionless-DM-dominated halos in gravitational equilibrium (NFW, Einasto, DARKexp)
  • The highest-ranked model is the 2-parameter NFW+LSS model including the 2-halo term using the LCDM $b-M$ relation ($b_h \sim 9.3$)
  • $c_{200c} = 3.8 +/- 0.3$ at $M_{200c}=10^{15}\text{M}_{\text{sun}}/h$, $z=0.34$

– Concentration vs. mass relation
  • Fully consistent with LCDM when the CLASH selection function based on X-ray morphological regularity and projection effects are taken into account

– Ensemble mass calibration
  • Internal consistency (WL vs. WL+SL) at the $\sim5\%$ level
  • Total calibration uncertainty $\sim9\%$ ($\sim6\%$ stat., $\sim6\%$ sys.)
Supplemental Slides
Reionization Lensing Cluster Survey (RELICS)

Newly approved 190-orbit HST survey (7 ACS/WFC3 filters) of 41 high-mass clusters primarily selected from the Planck survey (P.I. Dan Coe; Oct 2015 – Apr 2017)

http://hstrelics.weebly.com
Ensemble-averaged Error Budget

Diagonal elements \((C_{ii})\) averaged over all CLASH clusters

Residual mass-sheet uncertainty (Umetsu+14)

\[
\left(C_{\text{sys}}\right)_{ii} \sim \text{const.} \sim (0.02)^2
\]

Intrinsic profile variations due to triaxiality, substructure, and \(c-M\) scatter (Gruen+15)

\[
\left(C_{\text{int}}\right)_{ii} \approx (0.2)^2 \kappa_i^2
\]
Degree of Mass Concentration

\[ c_{200} \equiv \frac{r_{200}}{r_s} = \frac{\text{(Outer scale radius)}}{\text{(Inner scale radius)}} \]

In hierarchical structure formation, \(<c>\) is predicted to correlate with \(M\).

DM halos that are more massive collapse later on average, when the mean background density of the universe is correspondingly lower (e.g., Bullock+01).
Concentration is sensitive to cosmology

$\sigma_8$ vs. $\Omega_m$

High normalization

$C \sim 2.9$ vs. $3.6$ @ $10^{15} M_{\odot}/h$

Dutton & Maccio 2014
Intrinsic Scatter in $c(M)$: Mass Assembly Histories (MAH)

Scatter is due to another DoF ($\alpha$), related to MAH (Ludlow+13)

- Larger values of $\alpha$ correspond to halos that have been assembled more rapidly than the NFW curve
- Halos with average $c_{200}$ have the NFW-equivalent $\alpha \sim 0.18$
Key Predictions of nonlinear structure formation models

(3) Halo bias: surrounding large-scale structure

\[ \delta(x) := \frac{\rho - \bar{\rho}}{\bar{\rho}} = \int \frac{d^3k}{(2\pi)^3} \tilde{\delta}(k) e^{ik \cdot x} \]

\[ \langle \tilde{\delta}(k) \tilde{\delta}(k') \rangle = (2\pi)^3 \delta_D^3(k + k') P(k) \]
Halo Bias Factor: $b_h$

Clustering of matter around halos with $M$:

$$\xi_{hm}(r \mid M) = \left\langle \delta_h(x \mid M) \delta_m(x + r) \right\rangle$$

$$= \frac{\left\langle \rho_h(r \mid M) \right\rangle}{\bar{\rho}} + b_h(M) \xi_{mm}(r)$$

2h term

Correlated matter distribution (2h term)

Matter correlation function:

$$\xi_{mm}(r) = \left\langle \delta_m(x) \delta_m(x + r) \right\rangle = \int \frac{d^3k}{(2\pi)^3} P(k) e^{ik \cdot r}$$

$$\propto \sigma_8^2$$

Linear halo bias:

$$b_h(\nu) \approx 1 + \frac{\nu^2 - 1}{\delta_c}$$

$$\nu \equiv \frac{\delta_c}{\sigma(M, z)} \sim 3 - 4$$ for clusters
Non-local substructure effect

A substructure at $R \sim r_{\text{vir}}$ of the main halo, modulating $\Delta \Sigma(R) = \Sigma(< R) - \Sigma(R)$

Known 5%-10% negative bias in mass estimates from tangential-shear fitting, inherent to rich substructure in outskirts (Rasia+12)
Magnification bias effects

Flux-limited source counts:

\[ n_{\text{obs}}(>f) = \mu^{-1} n(>\mu^{-1}f) \]

Broadhurst, Taylor & Peacock 1995

Geometric area distortion

\[ n/\mu \]

Flux amplification
Averaged Halo Density Profile $\Sigma(R)$

Stacking lensing signals of individual clusters by

$$\langle \Sigma \rangle = \left( \sum_n W_n \right)^{-1} \left( \sum_n W_n \Sigma_n \right),$$

with individual “sensitivity” matrix

$$(W_n)_{ij} \equiv \Sigma_{(c,\infty)n}^{-2} (C_n^{-1})_{ij},$$

defined with total covariance matrix

$$C = C^{\text{stat}} + C^{\text{sys}} + C^{\text{lss}} + C^{\text{int}},$$

With “trace-approximation”, averaging (stacking) is interpreted as

$$\langle M_{\Delta} \rangle = \frac{\sum_n \text{tr}(W_n) M_{\Delta,n}}{\sum_n \text{tr}(W_n)}$$

Shear doesn't see mass sheet

Averaged lensing profiles in/around LCDM halos (Oguri & Hamana 11)

$$\kappa = \frac{\Sigma(R)}{\Sigma_c}$$

**Total**

$$\gamma_+ = \frac{\Delta \Sigma(R)}{\Sigma_c}$$

**Modulated**

- Tangential shear is a powerful probe of 1-halo term, or intra-halo structure.
- Shear alone cannot recover absolute mass, known as *mass-sheet degeneracy*:

$$\gamma$$ remains unchanged by $$\kappa \rightarrow \kappa + \text{const.}$$
Concentration—Mass Relation of the CLASH X-ray-selected Subsample

Concentration—Mass Scaling Relation

Consider a power-law scaling relation of the form:

\[ c_{200c} = 10^\alpha \left( \frac{M_{200c}}{M_{\text{piv}}} \right)^\beta \left( \frac{1+z}{1+z_{\text{piv}}} \right)^\gamma, \]

with pivot mass and redshift \( M_{\text{piv}} = 10^{15} M_{\odot} / h, z_{\text{piv}} = 0.34 \)

Define new independent \((X)\) and dependent \((Y)\) variables:

\[ Y \equiv \log_{10} \left[ \left( \frac{1+z}{1+z_{\text{piv}}} \right)^{-\gamma} c_{200c} \right], \quad Y(X) = \alpha + \beta X \]

Redshift slope \( \gamma \) is fixed to the theoretical prediction for the CLASH sample, \( \gamma = -0.668 \) (Meneghetti+14)
Bayesian Regression Analysis

We take into account

- Covariance between observed $M$ and $c$
- Intrinsic scatter in $c$
- Non-uniformity in mass probability distribution $P(\log M)$

### Conditional probability $P(y|x)$ with $(x,y) = \text{observed } (X,Y)$

\[
\ln P(y|x) = -\frac{1}{2} \sum_n \left[ \ln (2\pi \sigma_n^2) + \left( \frac{y_n - \langle y_n|x_n \rangle}{\sigma_n} \right)^2 \right],
\]

where $\langle y_n|x_n \rangle$ and $\sigma_n^2 \equiv \text{Var}(y_n|x_n)$ are the conditional mean and variance of $y_n$ given $x_n$, respectively:

\[
\langle y_n|x_n \rangle = \alpha + \beta \mu + \frac{\beta \tau^2 + C_{xy,n}}{\tau^2 + C_{xx,n}} (x_n - \mu),
\]

\[
\sigma_n^2 = \beta^2 \tau^2 + \sigma^2_{Y|X} + C_{yy,n} - \frac{(\beta \tau^2 + C_{xy,n})^2}{\tau^2 + C_{xx,n}},
\]

where $\sigma^2_{Y|X}$ is the intrinsic scatter in the $Y-X$ relation;
Marginalized Posterior Distributions

\[ c_{200c} = 10^\alpha \left( \frac{M_{200c}}{M_{\text{piv}}} \right)^\beta \left( \frac{1 + z}{1 + z_{\text{piv}}} \right)^\gamma \]

- \( \alpha \): intercept
- \( \beta \): slope
- \( \sigma_{\gamma|X} \): scatter
- \( \mu \): Gaussian mean of \( P(\ln M) \)
- \( \tau \): Gaussian width of \( P(\ln M) \)

High \( \beta \) tail associated with small \( \tau \): i.e., localized \( P(\ln M) \)
Einasto Shape Parameter vs. Halo Mass

$\alpha_E$: degree of curvature of the Einasto density profile

$$\frac{d \ln \rho(r)}{d \ln r} = -2 \left( \frac{r}{r-2} \right)^{\alpha_E}$$

$\alpha_E \approx 0.155 + 0.0095 \nu^2$ (Gao +08)

$$\nu = \frac{\delta_c}{\sigma(M)}$$

Preliminary results
Einasto Shape Parameter vs. Halo Mass

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$\nu = \frac{\delta_c}{\sigma(M)}$

Preliminary results
Einasto Shape Parameter vs. Halo Peak Height

\( \alpha_E \): degree of curvature of the Einasto density profile

\[
\frac{d \ln \rho(r)}{d \ln r} = -2 \left( \frac{r}{r_2} \right)^{\alpha_E}
\]

\( \alpha_E \approx 0.155 + 0.0095 \nu^2 \) (Gao + 08)

\[
\nu = \frac{\delta_c}{\sigma(M)}
\]

Preliminary results
CLASH HST Lensing Dataset

Cluster Gravitational Lensing

Key Objectives

**Intra-halo structure**
- Density profile, $\rho(r)$
- Halo mass, $M_\Delta$
- Concentration, $c = r_\Delta/r_{-2}$
- Halo asphericity

**Surrounding LSS**
- Halo bias $b_h(M)$
- DM clustering strength $\sigma_8$
- Assembly bias

Diemer & Mansfield
Cluster Gravitational Lensing

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Diemer & Mansfield