Gravitational Lensing by Clusters of Galaxies

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Galaxy Clusters: Building Blocks of the Universe

Large-scale structure of DM in the present-day universe

DM structure around a cluster-sized DM halo

$\Lambda$ cold dark matter ($\Lambda$CDM) simulations by Diemer & Kravtsov (2014)
Galaxies and Dark Matter in Clusters

Discrete galaxies

Underlying dark matter (~mass)

Millennium Simulation
Galaxies and Dark Matter in Clusters

Discrete galaxies
Underlying dark matter (~mass)

Deep HST image of massive cluster

Millennium Simulation
Galaxy Cluster MACS J1206 (z=0.44)

Sunyaev-Zel’dovich Effect

Cluster member galaxies (HST)

Umetsu et al. (2012)
Hubble Frontier Fields Cluster A2744

Cluster member galaxies (*HST*)

Intra-cluster Stellar Light

Credit: NASA (https://www.spacetelescope.org/images/archive/category/cosmology/)
Gravitational Lensing

Unlensed

Lensed

Weak

Strong

Fort & Mellier
Cluster Mass Reconstruction

MACS J1206 (z=0.44)

Strong-and-Weak lensing analysis (SaWLens: J. Merten) of CLASH HST + Subaru data

Umetsu et al. 2012
Gravitational Bending of Light

Lightrays propagating in an inhomogeneous universe will undergo small transverse excursions along the photon path.

Bending angle: small transverse excursion of photon momentum ($|\Psi|/c^2 << 1$)

$$\delta \hat{\alpha} \approx \frac{\delta p_\perp}{p_\parallel} = -\frac{2}{c^2} \nabla_\perp \Psi(\chi_\parallel, \chi_\perp) \delta \chi_\parallel$$

Gravitational field of deflecting matter

$$\hat{\alpha}^{GR} = 2 \hat{\alpha}^{Newton} \rightarrow \frac{4GM}{c^2 r} = 1.75 \left( \frac{M}{M_{sun}} \right) \left( \frac{r}{R_{sun}} \right)^{-1}$$
Cluster Lens Equation

Cosmological lens equation + thin-lens approximation

\[ \beta(\theta) - \theta = \frac{D_{LS}}{D_{OS}} \int \delta \hat{\alpha}(\theta) \]

\[ \frac{dD}{D_{OS}} \int \delta \hat{\alpha}(\theta) \]

\( \beta \): true (but unknown) source position

\( \theta \): apparent image position

For a rigid derivation of cosmological lens eq., see, e.g., Futamase 95

Angular diameter distances:

\( D_{OL}, D_{LS}, D_{OS} \sim O(c/H_0) \)
Deflection and Distortion

\[ \beta(\theta) - \theta = \frac{D_{LS}}{D_{OS}} \int_{\text{Observer}}^{\text{Source}} \delta \hat{\alpha}(\theta) \equiv -\nabla \psi(\theta) \]

Deformation of an image

\[ \delta \beta_i = (\delta_{ij} - \psi_{,ij}) \delta \theta_j + O(\delta \theta^2) \]

Magnification, \( \mu \)

\[ \mu^{-1} = \det \left( \frac{\partial \beta}{\partial \theta} \right) = |1 - \nabla \nabla \psi| \]
Convergence ($\kappa$) and Shear ($\gamma$)

\[ \kappa = \partial \partial^* \Psi / 2 = \Delta \Psi / 2 \]
\[ \gamma = \partial \partial \Psi / 2 \]
\[ \partial := \partial_x + i \partial_y = e^{i\phi} \partial_r \]

\[ A(\theta) = \begin{pmatrix} 1 - \kappa & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix} = (1 - \kappa) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix}, \]
Multiple Imaging

165 multiple images of 61 galaxies strongly lensed by A1689 (HST/ACS GTO)

- Broadhurst et al. 2005
- Coe et al. 2010
- Diego et al. 2015
- Umetsu et al. 2015a
Gravitational Magnification

MACSJ1149 (z=0.54)
Convergence, $\kappa$

$\kappa$: weighted line-of-sight projection of density contrast $\delta = \delta \rho / \rho$

\[
\kappa = \frac{3 H_0^2 \Omega_m}{2c^2} \int_0^{\chi_s} d\chi \frac{r(\chi) r(\chi_s - \chi) \delta}{r(\chi_s)} = \int_{\text{Source}} d\Sigma \sum_{\text{crit}}^{-1}
\]

Surface mass density field

$\Sigma(\chi_{\perp}) = \int_0^{\chi_s} d\chi \rho(\rho - \overline{\rho}) = \int_{\text{Observer}} dl \delta \rho$

Critical surface mass density

$\Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_{\text{OS}}}{D_{\text{OL}} D_{\text{LS}}}$

- **Strong lensing**: $\Sigma \sim \Sigma_{\text{crit}}$ @ cluster cores
- **Weak lensing**: $\Sigma \sim 0.1 \Sigma_{\text{crit}}$ @ outside cores
- **Cosmic lensing**: $|\Sigma| \lesssim 0.01 \Sigma_{\text{crit}}$ @ LSS
Shear to Convergence (mass)

Shear tensor

$$\Gamma_{ij} = \begin{bmatrix} +\gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} +Q & U \\ U & -Q \end{bmatrix}$$

Linear Stokes parameters

Shear-to-mass relation

$$\Delta\kappa = \partial_i \partial_j \Gamma_{ij} \Leftrightarrow E$$

Kaiser & Squires (1993)
Strong and Weak Lensing

Strong lensing

Weak lensing

Observer
Cluster of Galaxies
Background Galaxy

Non-Linear
Multiple Images
Arclets

Weak Shear
Linear

Optical Path
Wave Front
Multiple Images Area

J.P. Kneib
Weak Shear Field

Weak shear is observable

\[ \gamma := \gamma_1 + i\gamma_2 = \frac{a - b}{a + b} e^{2i\phi} \]

Cosmic shear: a few %
Cluster shear: 10-20%
Weak Shear Field

Cluster $z = 0.77$; Arc $z = 4.89$:
Photo from H. Yee (HST/ACS)

Simulated 3x3 degree field (Hamana 02)
High-resolution space imaging with HST (ACS/WFC3) for strong lensing

SUBARU (S-Cam) multi-color imaging for wide-field weak
A1689 at $z=0.183$, Subaru/S-Cam data (Umetsu et al. 2015a)
Weak-Lensing Shear and Magnification

Shear

- Geometric shape dist: $\delta e_+ \sim \gamma_+$

Magnification

- Flux amplification: $\mu F$
- Geometric area dist: $\mu \Delta \Omega$

Sensitive to “modulated” matter density

$$\Sigma_{\text{crit}} \gamma_+ = \Delta \Sigma \equiv \Sigma(< R) - \Sigma(R)$$

Sensitive to “total” matter density

$$\mu \approx 1 + 2 \kappa; \quad \Sigma_{\text{crit}} \kappa = \Sigma$$
Shear vs. Magnification

Tangential reduced shear
\[ g_+ \approx \gamma_+ = \frac{\Delta \Sigma}{\Sigma_{\text{crit}}} \]

Number count depletion due to magnification bias
(Broadhurst, Taylor, & Peacock 1995)

A1689 Subaru/S-Cam data
(Umetsu et al. 2015a)
Multi-probe Cluster Lensing Analysis

MACSJ1206 CLASH (Umetsu 2013)
Multi-probe Stacked Lensing Analysis

CLASH strong-lensing, weak-lensing shear and magnification analysis of 16 clusters (Umetsu et al. 2015b)
Cluster Weak Lensing Applications

• 2D mass reconstruction
  – Halo asphericity
  – Mass distribution in merging clusters
    • DM properties from X-ray/SZE-WL offsets (Bullet cluster)
    • Cluster infall velocity (pairwise halo velocity)
    • Cluster physics (shock heating, particle acceleration, etc)

• Intra-halo mass profiles
  – Mass measurements/calibration for cluster cosmology
  – Mass vs. concentration relation

• Outskirt and large-scale mass profiles
  – Halo bias, matter power spectrum \( (P(k), \sigma_8) \)
  – Screening mechanisms in modified gravity theory