Informal Seminar @ Physics, HKU
August 27, 2015

Weak Gravitational Lensing by Clusters of Galaxies

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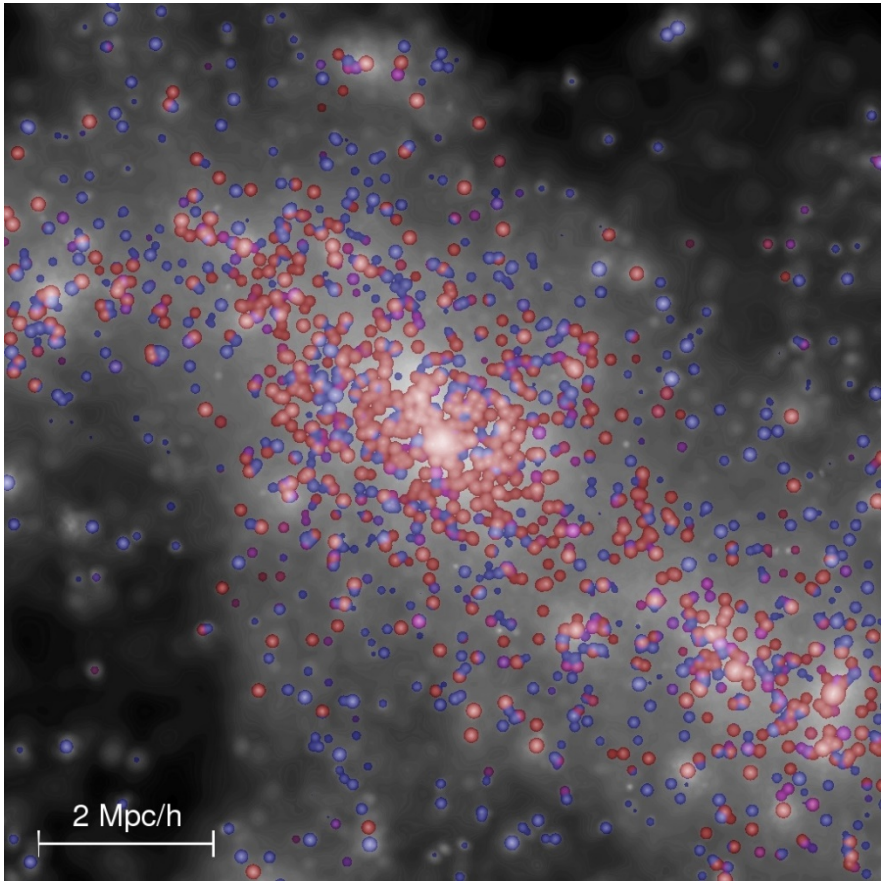
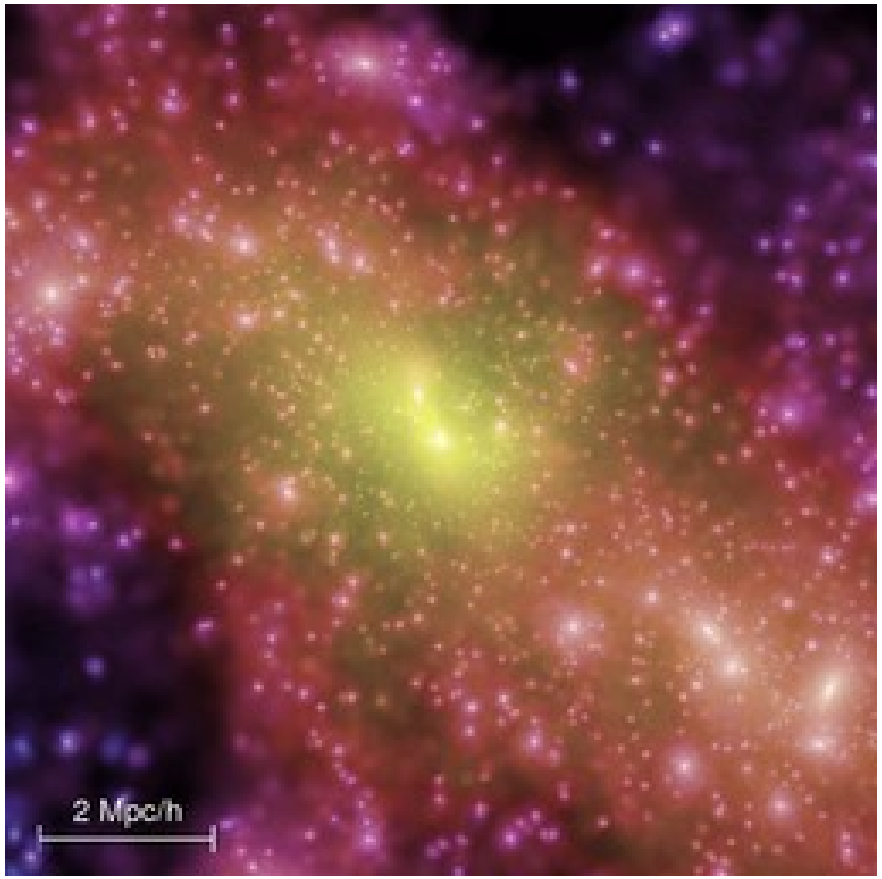
Galaxy Clusters

Discrete galaxies



Underlying dark matter (~mass)

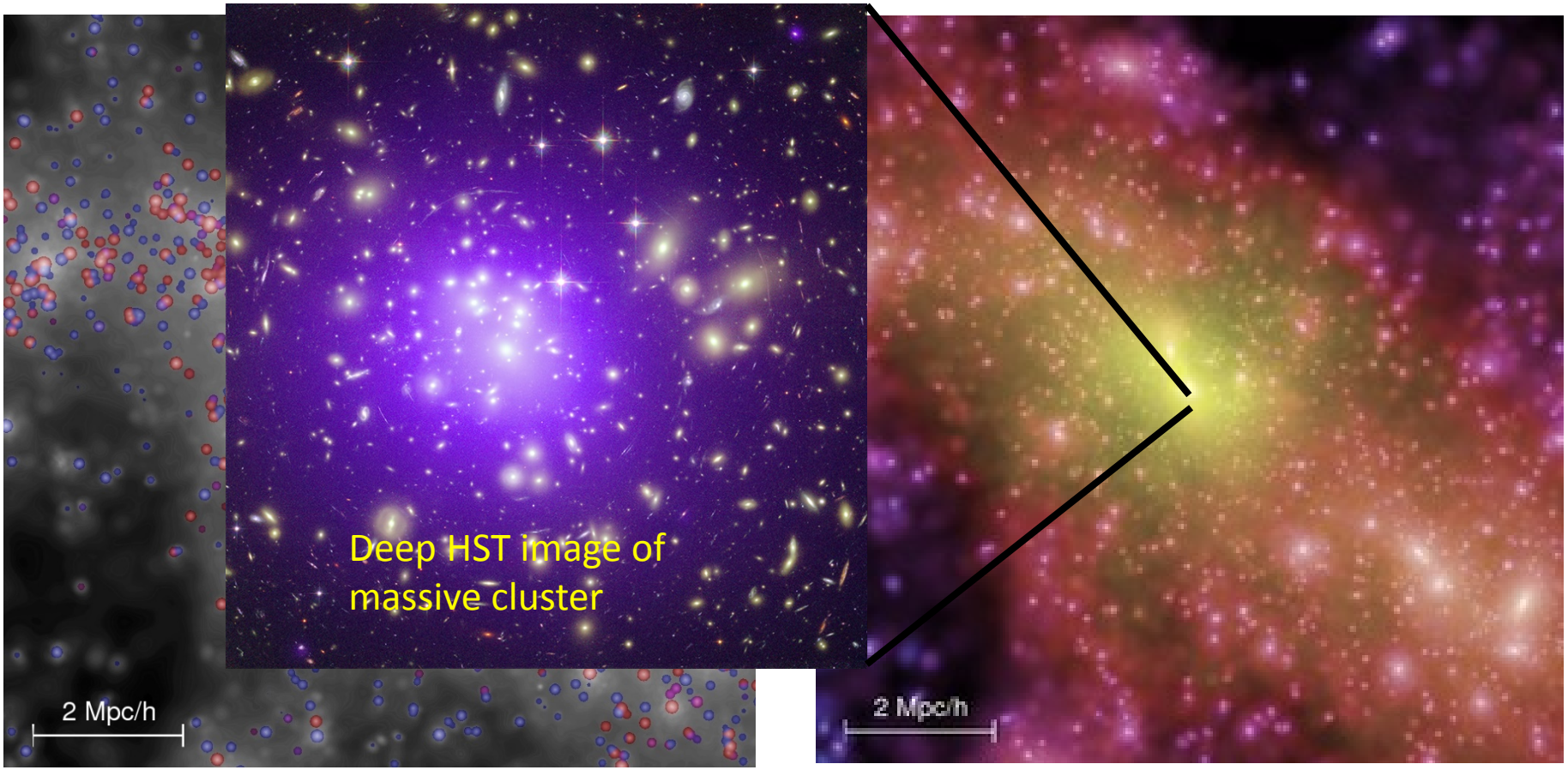


Millennium Simulation

Galaxy Clusters

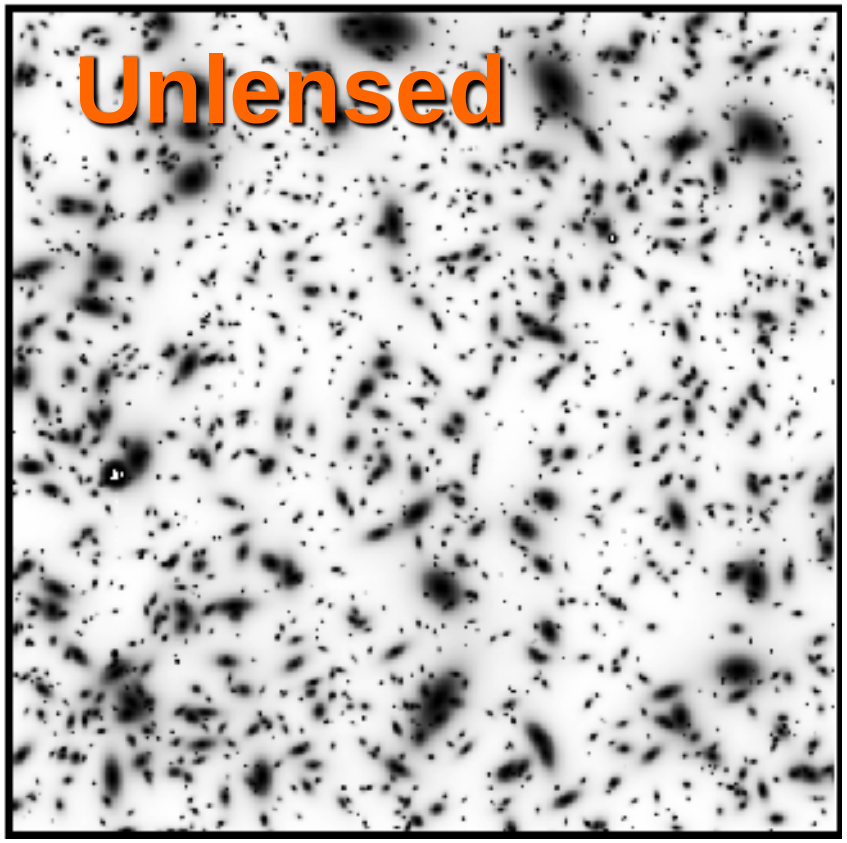
Discrete galaxies

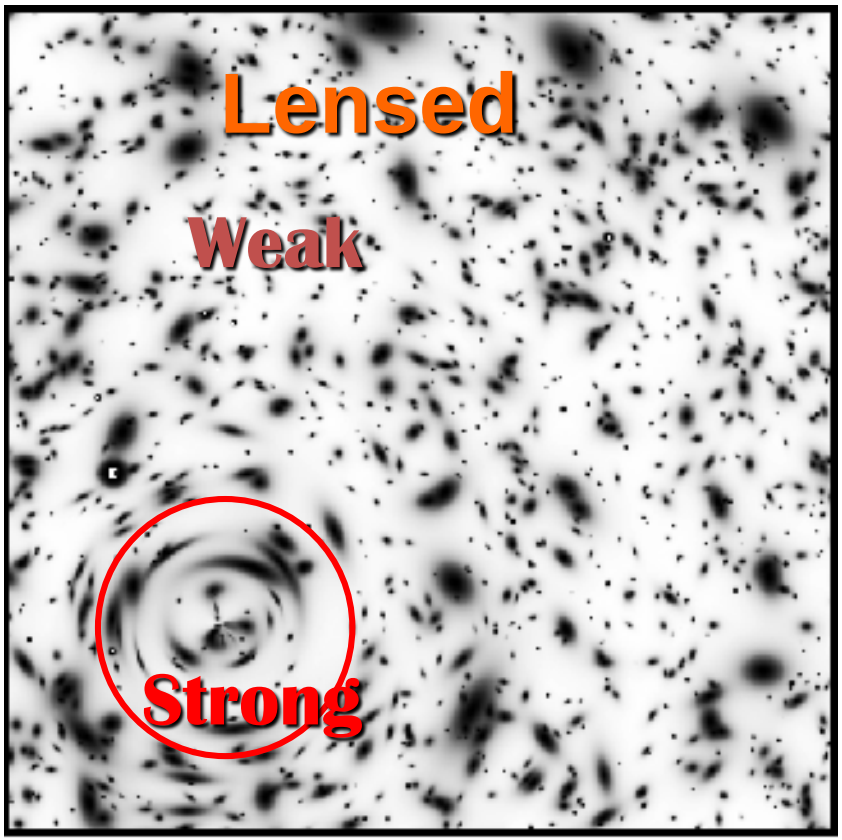
Underlying dark matter (~mass)



Millennium Simulation

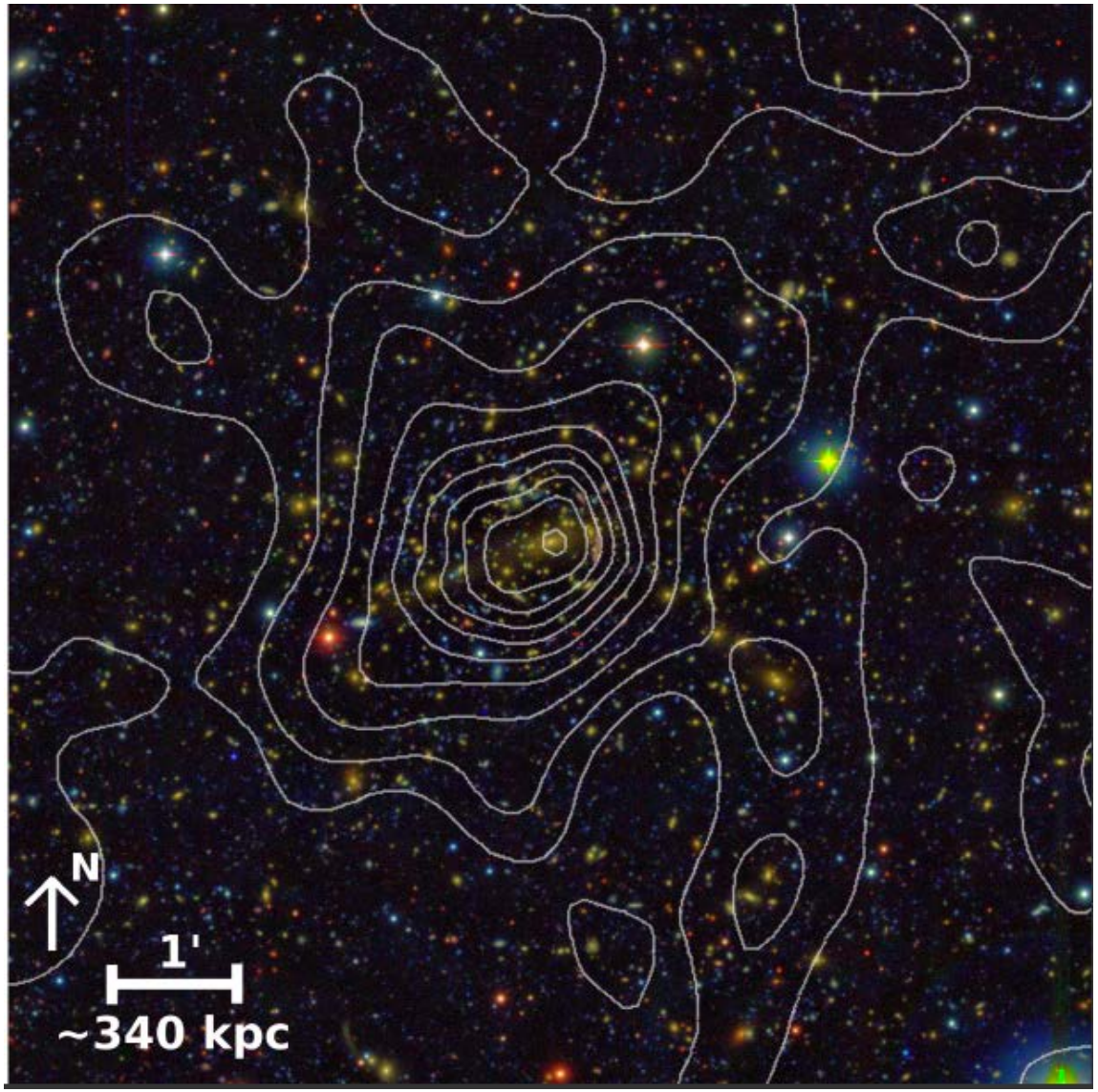
Gravitational Lensing





Fort & Mellier

Cluster Mass Reconstruction



MACSJ1206 (z=0.44)

Strong-and-Weak lensing analysis (SaWLens: J. Merten) of CLASH *HST* + *Subaru* data

Umetsu et al. 2012

Gravitational Bending of Light

Lightrays propagating in an inhomogeneous universe will undergo **small transverse excursions** along the photon path

Bending angle: small transverse excursion of photon momentum ($|\Psi|/c^2 \ll 1$)

$$\delta\hat{\alpha} \approx \frac{\delta p_\perp}{p_\parallel} = -\frac{2}{c^2}\nabla_\perp \Psi(\chi_\parallel, \chi_\perp)\delta\chi_\parallel$$

Gravitational field of deflecting matter

$$\hat{\alpha}^{\mathrm{GR}} = 2\hat{\alpha}^{\mathrm{Newton}} \rightarrow \frac{4GM}{c^2 r} = 1.''75\left(\frac{M}{M_{sun}}\right)\left(\frac{r}{R_{sun}}\right)^{-1}$$

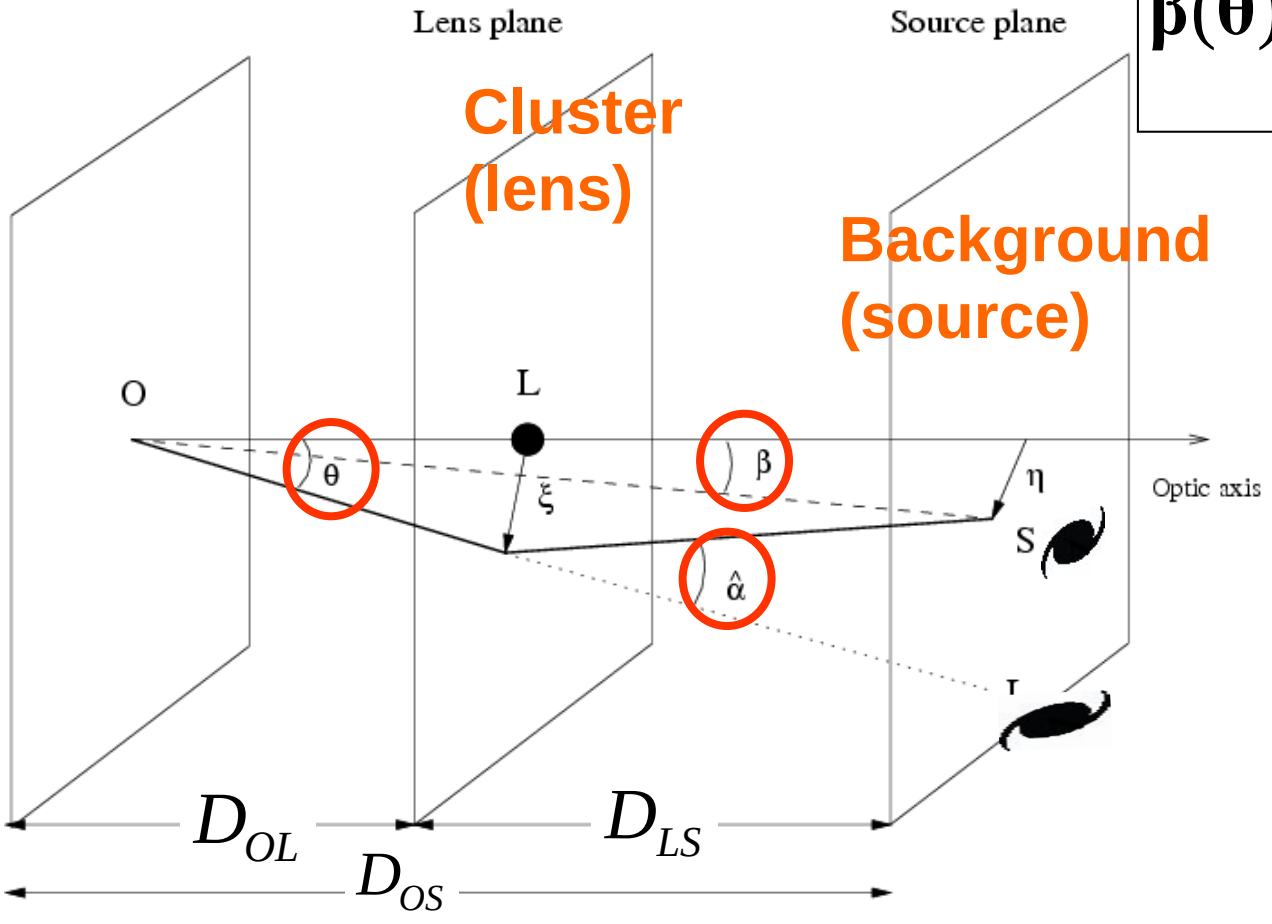
Cluster Lens Equation

Cosmological lens equation + thin-lens approximation

β: true (but unknown) source position

θ: apparent image position

$$\boldsymbol{\beta}(\boldsymbol{\theta}) - \boldsymbol{\theta} = \frac{D_{LS}}{D_{OS}} \int \delta\hat{\boldsymbol{\alpha}}(\boldsymbol{\theta})$$



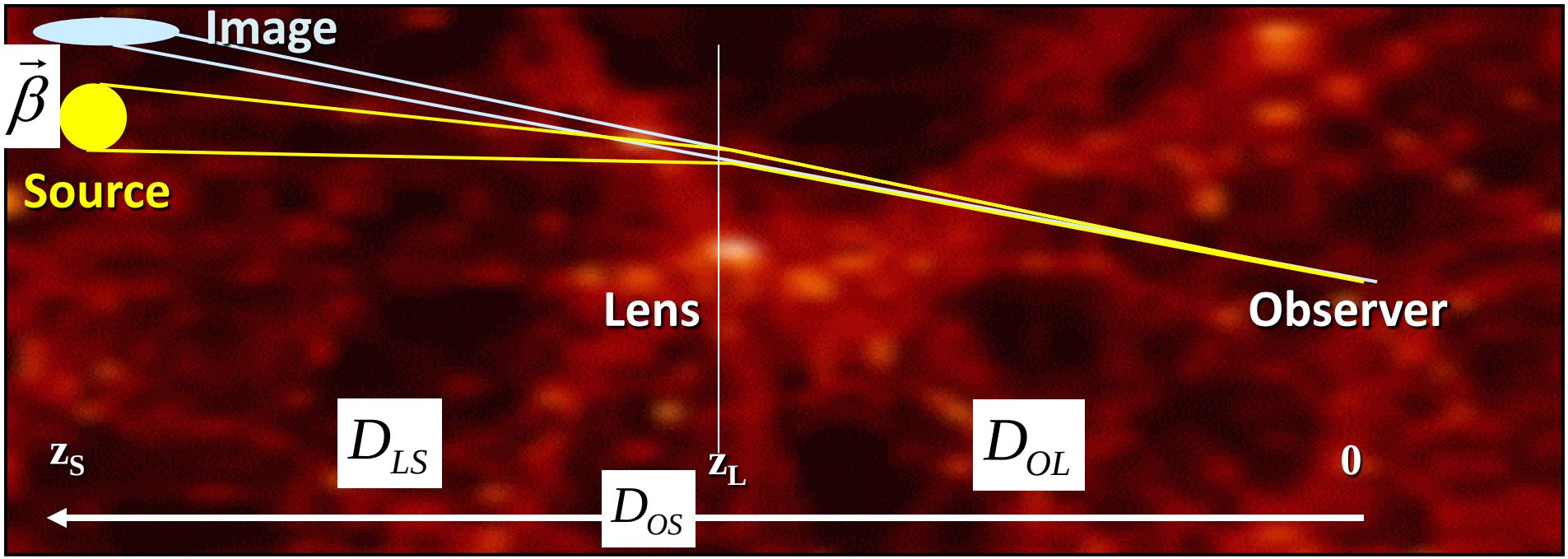
Angular diameter distances:

D_{OL}, D_{LS}, D_{OS} ~ $O(c/H_0)$

For a rigid derivation of cosmological lens eq., see, e.g., Futamase 95

Deflection and Distortion

$$\boldsymbol{\beta}(\boldsymbol{\theta}) - \boldsymbol{\theta} = \frac{D_{\mathrm{LS}}}{D_{\mathrm{OS}}} \int_{\mathrm{Observer}}^{\mathrm{Source}} \delta\hat{\boldsymbol{\alpha}}(\boldsymbol{\theta}) \equiv -\boldsymbol{\nabla}\psi(\boldsymbol{\theta})$$



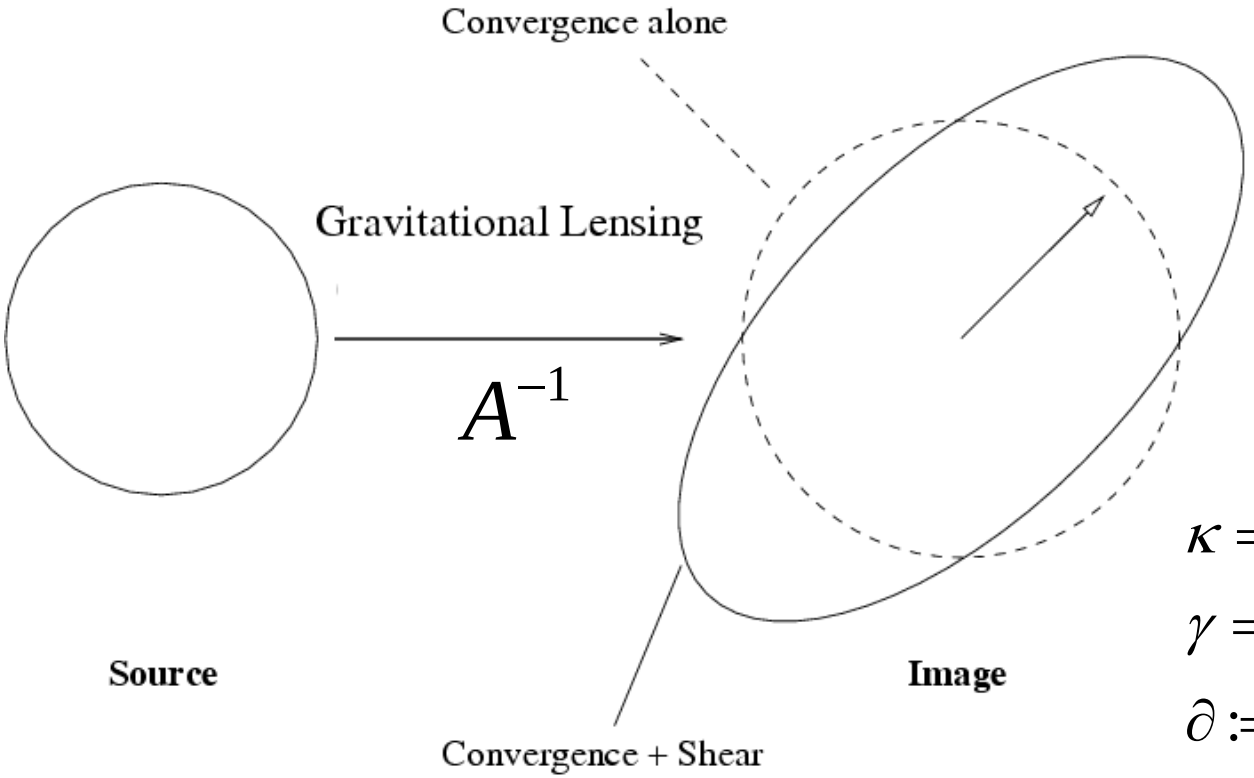
Deformation of an image

$$\delta\beta_i = (\delta_{ij} - \psi_{,ij})\delta\theta_j + O(\delta\theta^2)$$

Magnification, μ

$$\mu^{-1} = \det\left(\frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}}\right) = |1 - \nabla\nabla\psi|$$

Convergence (κ) and Shear (γ)



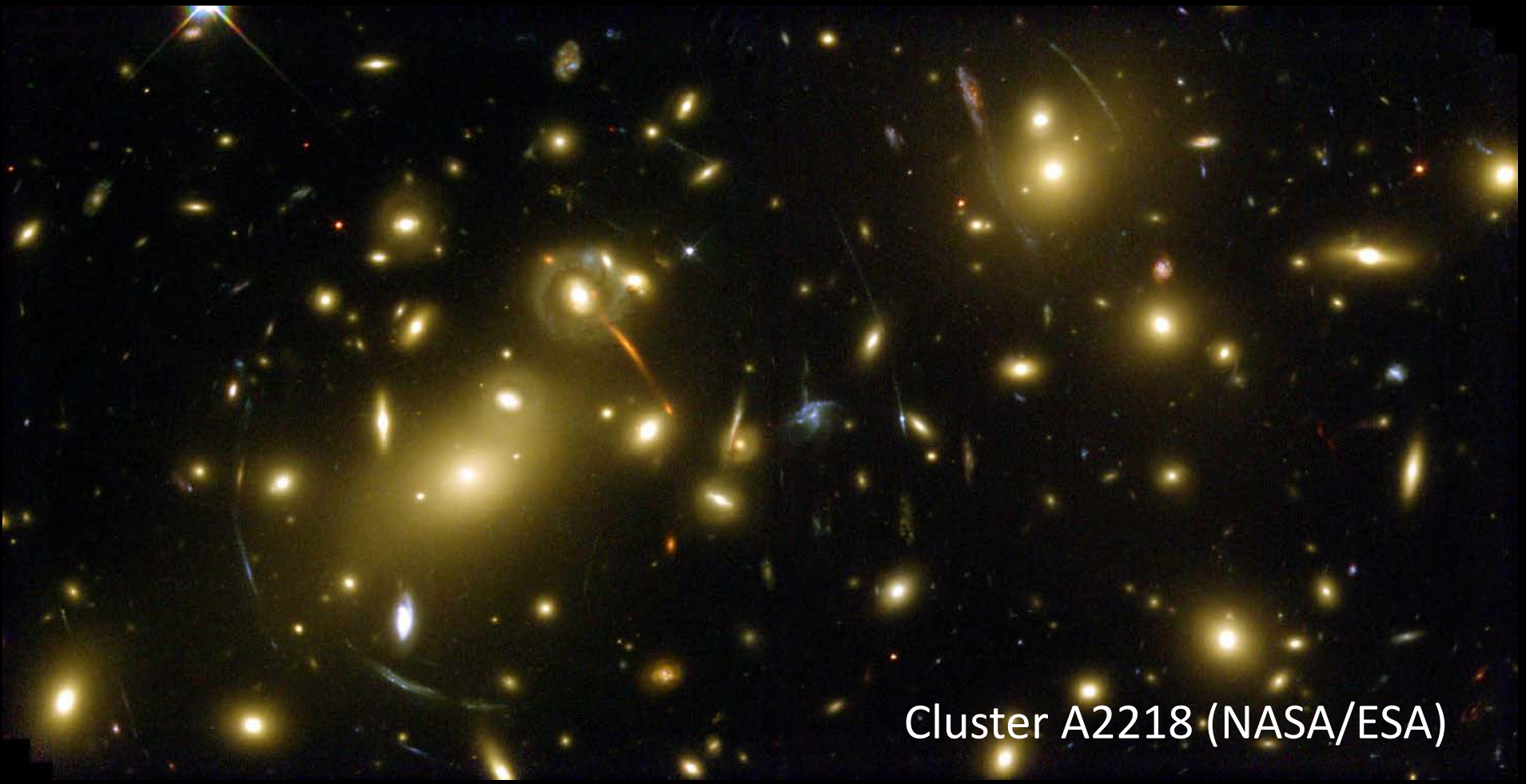
$$\kappa = \partial\partial^{*}\Psi / 2 = \Delta\Psi / 2$$

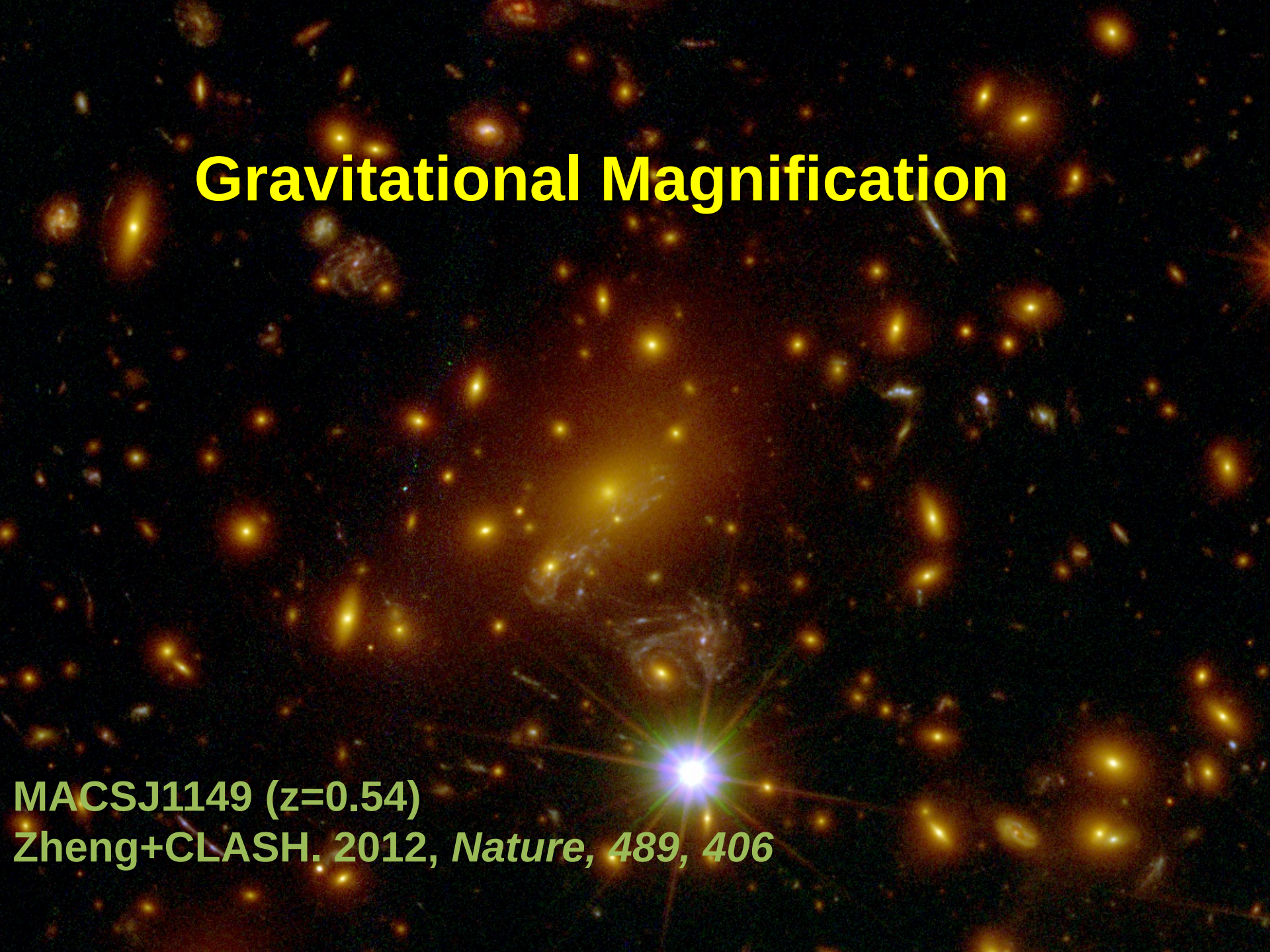
$$\gamma = \partial\partial\Psi / 2$$

$$\partial := \partial_x + i\partial_y = e^{i\phi}\partial_r$$

$$\mathcal{A}(\boldsymbol{\theta}) = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} = (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix},$$

Gravitational Shear




Gravitational Magnification
MACSJ1149 (z=0.54)
Zheng+CLASH. 2012, *Nature, 489, 406*

Convergence,κ

κ: weighted line-of-sight projection of density contrast $\delta=\delta\rho/\rho$

$$\kappa = \frac{3H_0^2\Omega_m}{2c^2}\int_0^{\chi_s} d\chi\, \frac{r(\chi)r(\chi_s-\chi)}{r(\chi_s)}\frac{\delta}{a} = \int_{\rm Observer}^{\rm Source} d\Sigma\, \Sigma_{\rm crit}^{-1}$$

Surface mass density field

$$\Sigma(\boldsymbol{\chi}_\perp) = \int_0^{\chi_s} d\chi\, a(\rho-\overline{\rho}) = \int_{\rm Observer}^{\rm Source} dl\, \delta\rho$$

Critical surface mass density

$$\Sigma_{\rm crit} = \frac{c^2}{4\pi G}\frac{D_{\rm OS}}{D_{\rm OL}D_{\rm LS}}$$

- **Strong lensing**: $\Sigma \sim \Sigma_{\rm crit}$ @ cluster cores
- **Weak lensing**: $\Sigma \sim 0.1\,\Sigma_{\rm crit}$ @ outside cores
- **Cosmic lensing**: $|\Sigma| \lesssim 0.01\,\Sigma_{\rm crit}$ @ LSS

Shear to Convergence (mass)

Shear tensor

Linear Stokes parameters

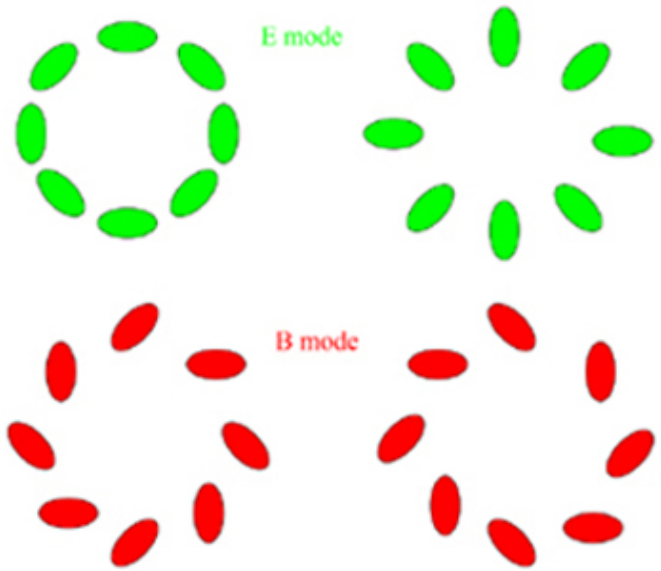
$$\Gamma_{ij} = \begin{bmatrix} +\gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} +Q & U \\ U & -Q \end{bmatrix}$$

Shear-to-mass relation

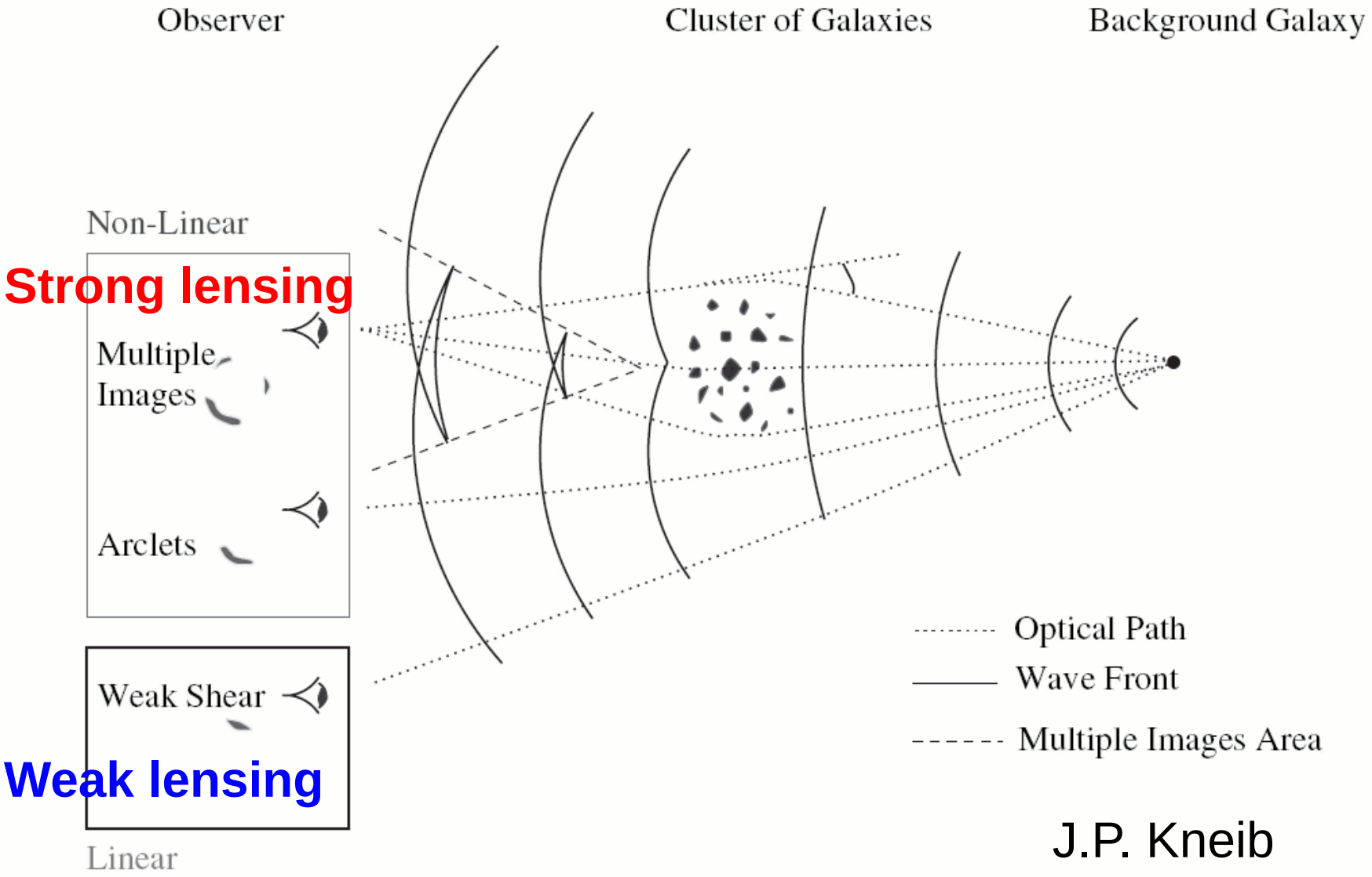
E mode

$$\Delta\kappa = \partial_i \partial_j \Gamma_{ij} \Leftrightarrow E$$

Kaiser & Squires (1993)



Strong and Weak Lensing



J.P. Kneib

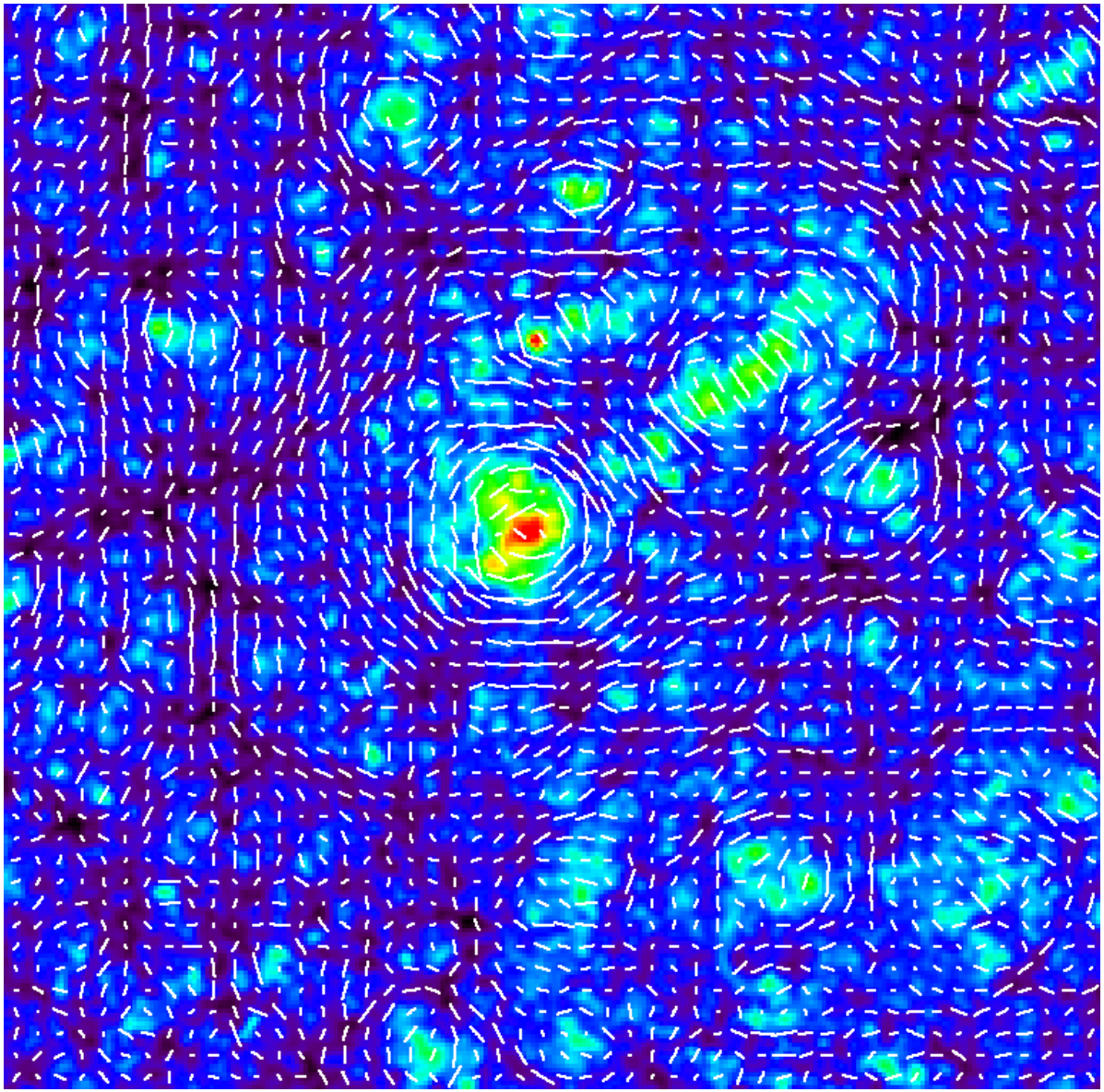
Weak Shear Field

Weak shear is observable

$$\gamma \approx \frac{a-b}{a+b} e^{2i\phi}$$

Cosmic shear: a few %

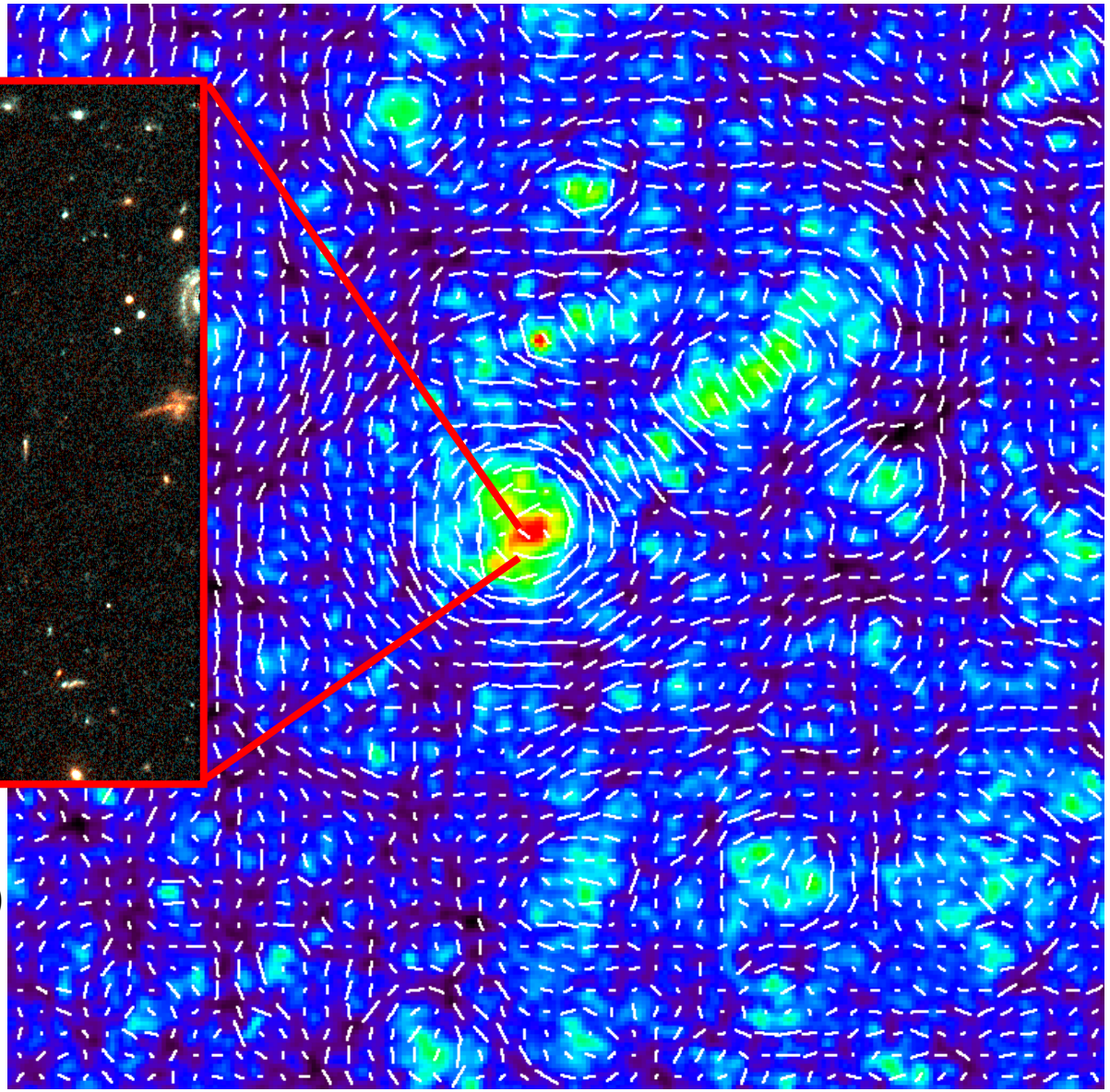
Cluster shear: 10-20%



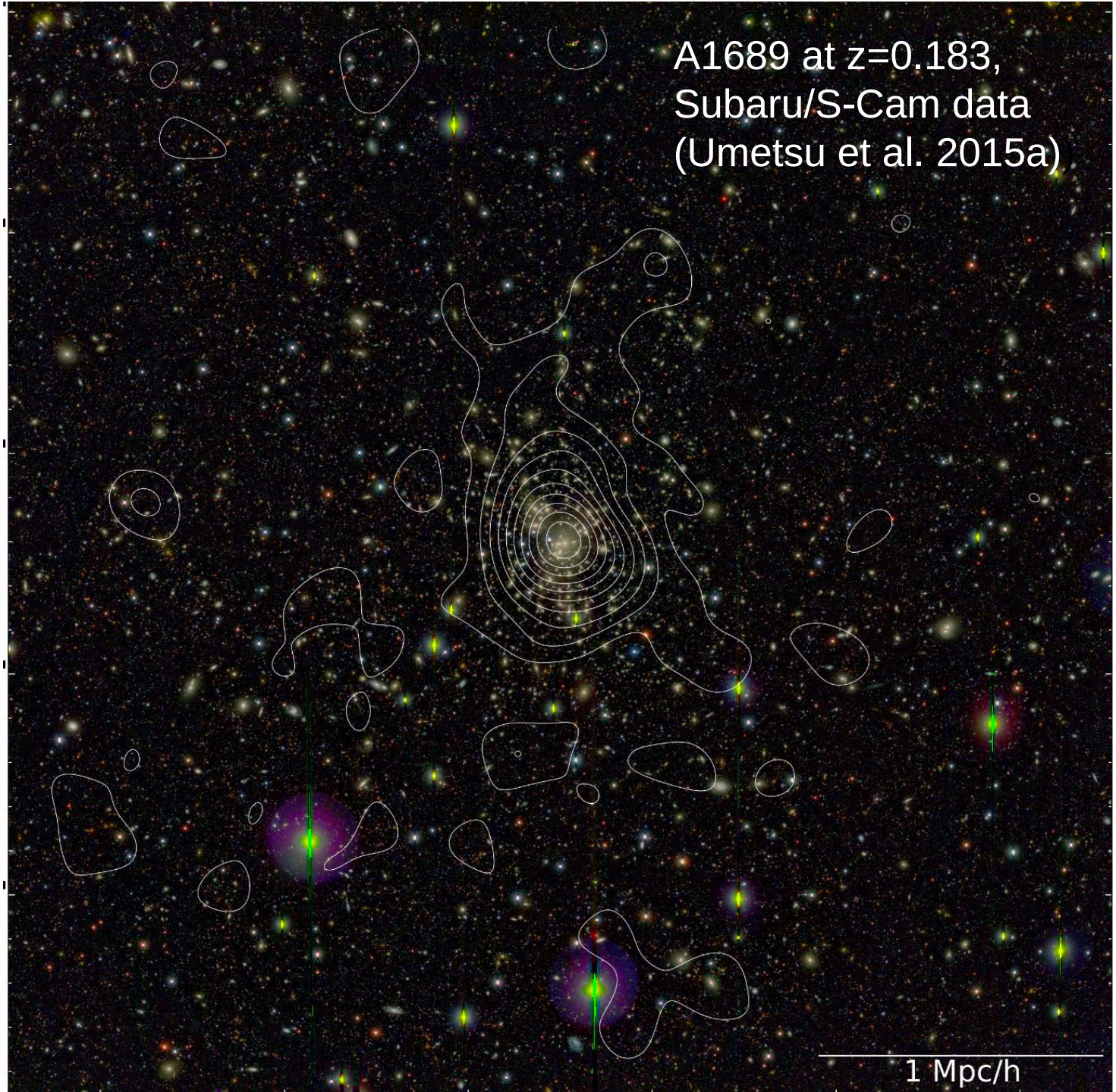
Simulated 3x3 degree field (Hamana 02)

Weak Shear Field

Cluster z = 0.77; Arc z = 4.89:
Photo from H. Yee (HST/ACS)



Simulated 3x3 degree field (Hamana 02)


A1689 at z=0.183,
Subaru/S-Cam data
(Umetsu et al. 2015a)
1 Mpc/h

Weak Lensing is noisy!

Practical shear estimate **unbiased!!**

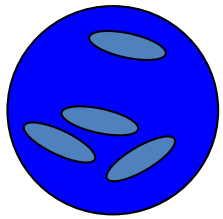
$$\gamma_{\rm obs} \approx \gamma + \gamma_{\rm intrinsic} + \Delta\gamma_{\rm noise} \Rightarrow \langle \gamma_{\rm obs} \rangle = \gamma = |\gamma| e^{2i\phi}$$

Uncertainty per galaxy

$$\sigma_g = \sqrt{{\sigma_{\rm int}}^2 + {\sigma_{\rm noise}}^2} \approx 0.3 - 0.4$$

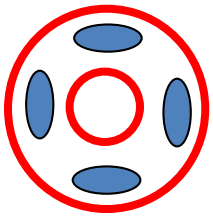
S/N of shear amplitude averaged in an aperture

$$\mathrm{S/N} = |\gamma| \Big/ \left(\sigma_g / \sqrt{N}\right) \approx 2.0 \left(\frac{|\gamma|}{0.1}\right) \left(\frac{\sigma_g}{0.4}\right)^{-1} \left(\frac{n_g}{20\mathrm{arcmin}^{-2}}\right)^{1/2} \left(\frac{\Delta\theta}{1'}\right)$$

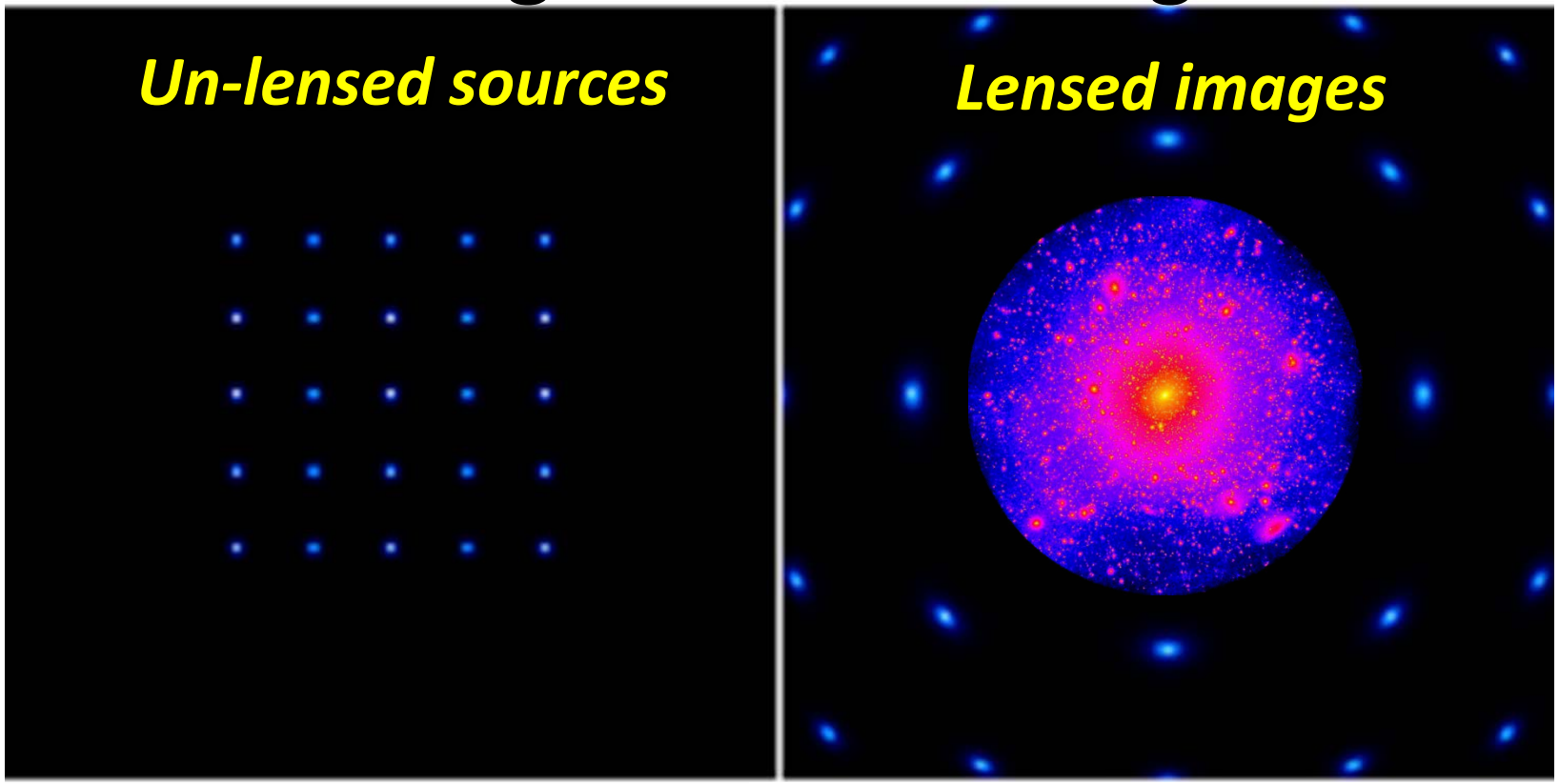


S/N of tangential shear averaged over the cluster

$$\mathrm{S/N} \sim \left(\gamma_+ \Big/ \sigma_g / \sqrt{2N}\right) \approx 10 \left(\frac{\sigma_v}{1000\mathrm{km/s}}\right)^2 \left(\frac{\sigma_g}{0.4}\right)^{-1} \left(\frac{n_g}{20\mathrm{arcmin}^{-2}}\right)^{1/2} \left(\frac{\ln(\theta_{\rm max}/\theta_{\rm min})}{\ln 10}\right)^{1/2} \left(\frac{\langle D_{ds}/D_s \rangle}{0.7}\right)$$



Weak-Lensing Shear and Magnification



- **Shear**
  - ✓ Geometric shape dist: $\delta e_+ \sim \gamma_+$

Sensitive to "modulated" matter density

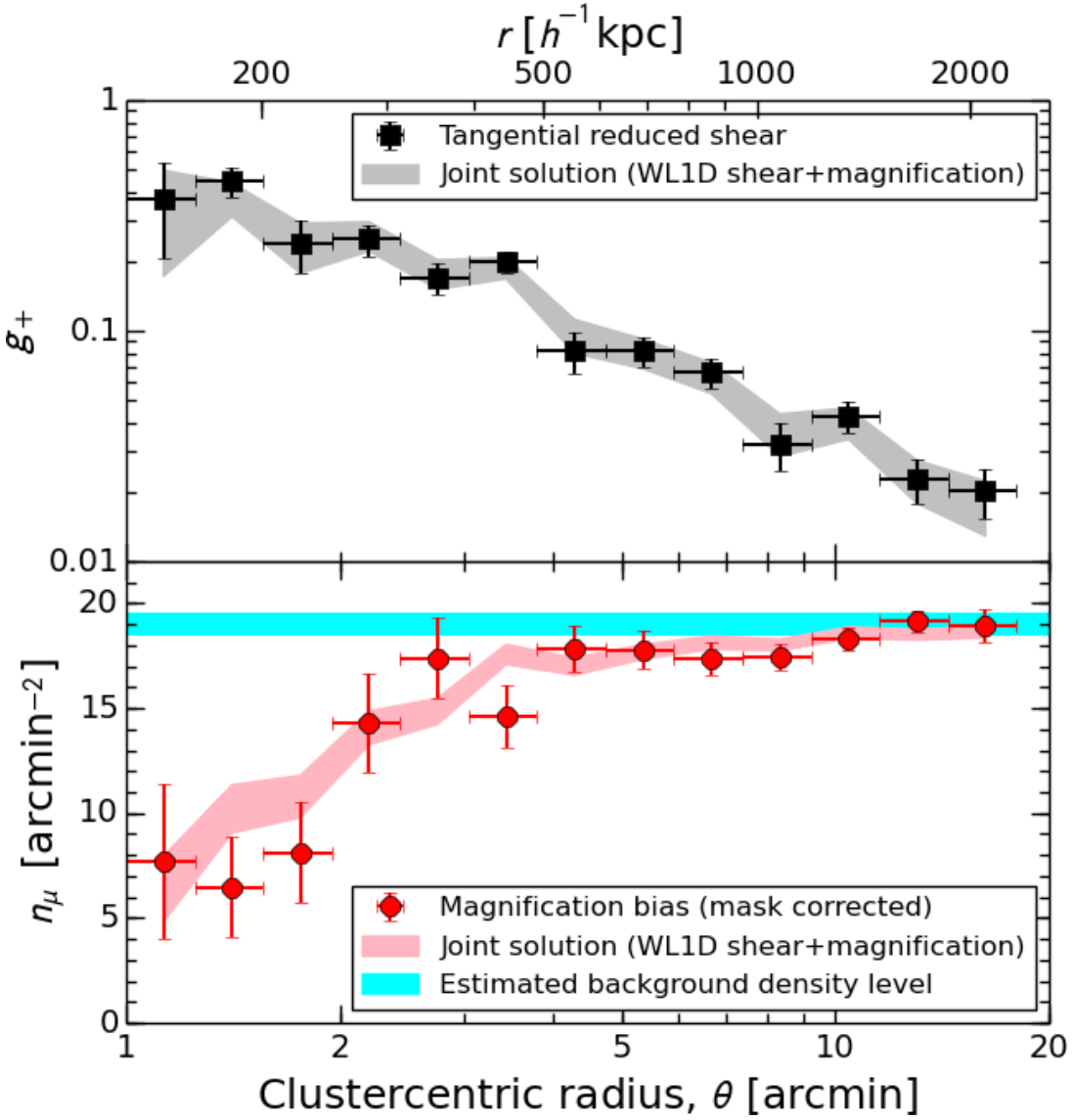
$$\Sigma_{\rm crit}\gamma_+ = \Delta\Sigma \equiv \Sigma(<R) - \Sigma(R)$$

- **Magnification**
  - ✓ Flux amplification: μF
  - ✓ Geometric area dist: $\mu\Delta\Omega$

Sensitive to "total" matter density

$$\mu \approx 1 + 2\kappa; \quad \Sigma_{\rm crit}\kappa = \Sigma$$

Shear vs. Magnification



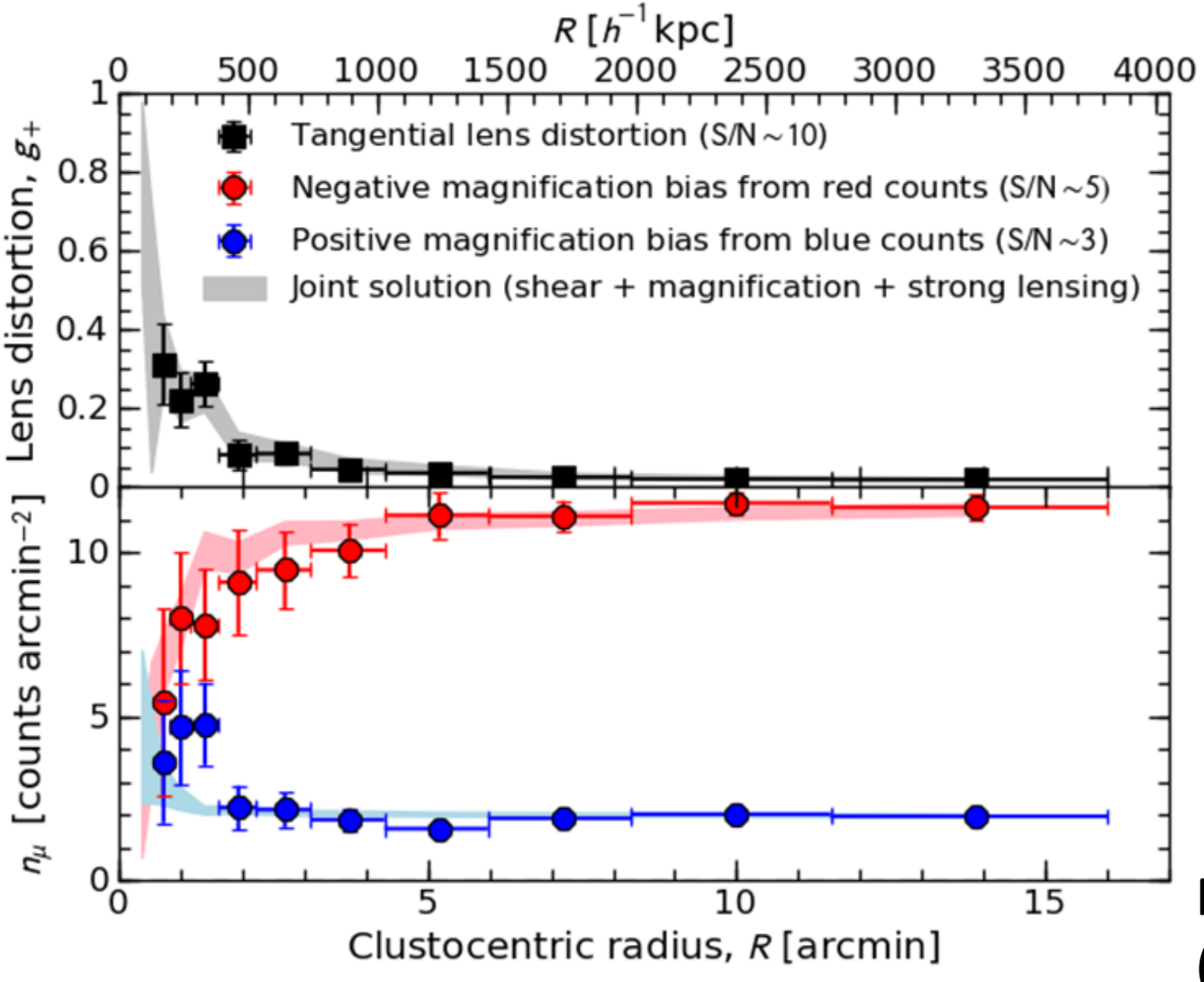
Tangential reduced shear

$$g_+ \approx \gamma_+ = \Delta\Sigma / \Sigma_{\mathrm{crit}}$$

Number count depletion due to magnification bias (Broadhurst, Taylor, & Peacock 1995)

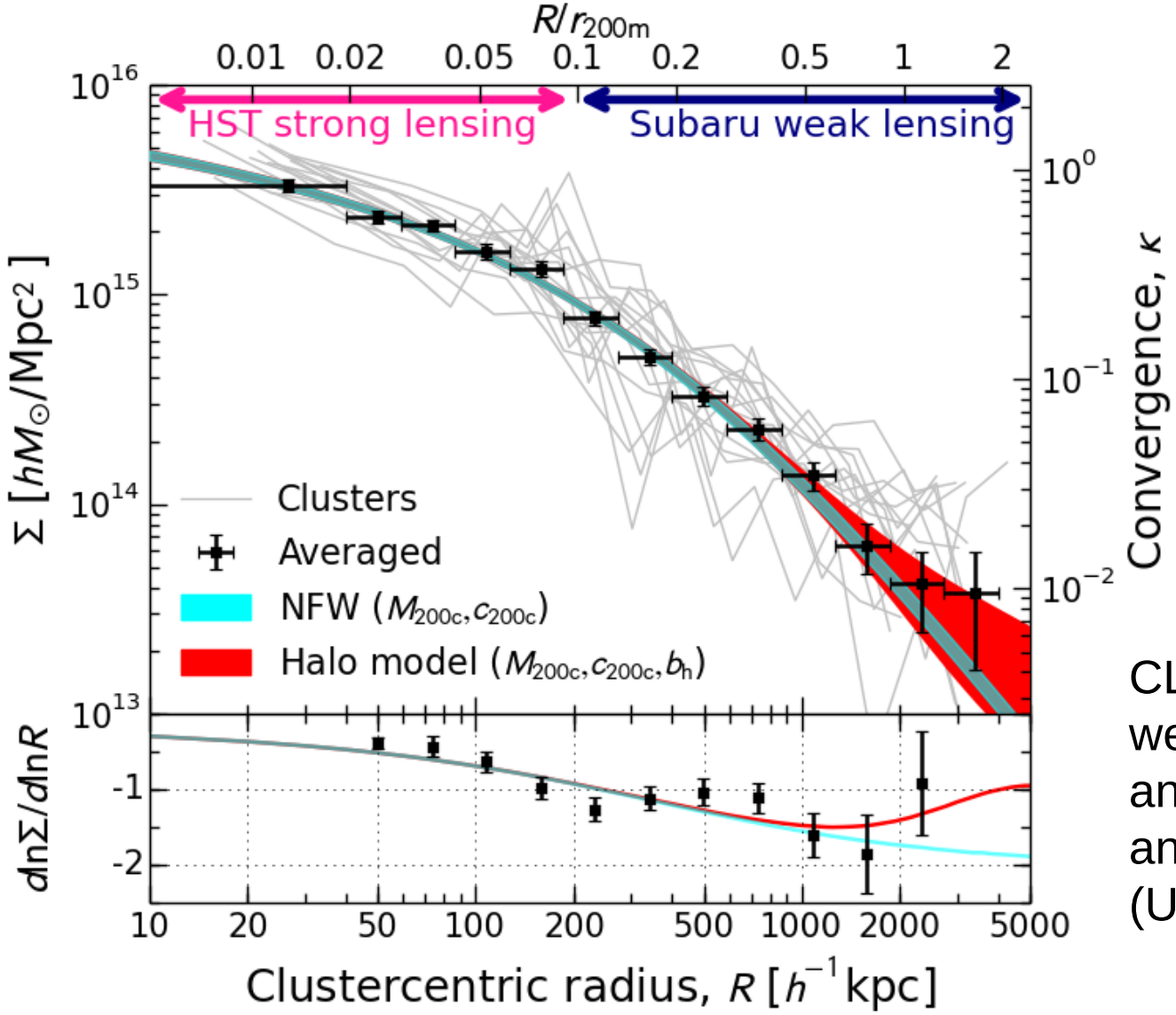
A1689 Subaru/S-Cam data (Umetsu et al. 2015a)

Multi-probe Cluster Lensing Analysis



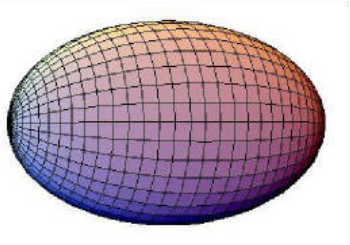
MACSJ1206 CLASH
(Umetsu 2013)

Multi-probe Stacked Lensing Analysis



CLASH strong-lensing, weak-lensing shear and magnification analysis of 16 clusters (Umetsu et al. 2015b)

Cluster Weak Lensing Applications



- 2D mass reconstruction
  - Halo asphericity
  - Mass distribution in merging clusters
    - DM properties from X-ray/SZE-WL offsets (*Bullet* cluster)
    - Cluster infall velocity (pairwise halo velocity)
    - Cluster physics (shock heating, particle acceleration, etc)
- Intra-halo mass profiles
  - Mass measurements/calibration for cluster cosmology
  - Mass vs. concentration relation
- Outskirt and large-scale mass profiles
  - Halo bias, matter power spectrum ($P(k)$, σ_8)
  - Screening mechanisms in modified gravity theory