CosPA 2007 (2007/11/14)

A Moment Method for Measuring the Higher Order Weak Lensing Effects

References:

Okura, Umetsu, Futamase 2007, ApJ, 660, 995

Okura, Umetsu, Futamase 2007 (astro-ph/0710.2262)

(Umetsu & Broadhurst 2007)

Keiichi Umetsu

(Academia Sinica IAA)

Lensing Collaborators

Weak Lensing Fexion / HOLICs

- Toshifumi Futamase (Tohoku Univ)
- Yuki Okura (Tohoku Univ)

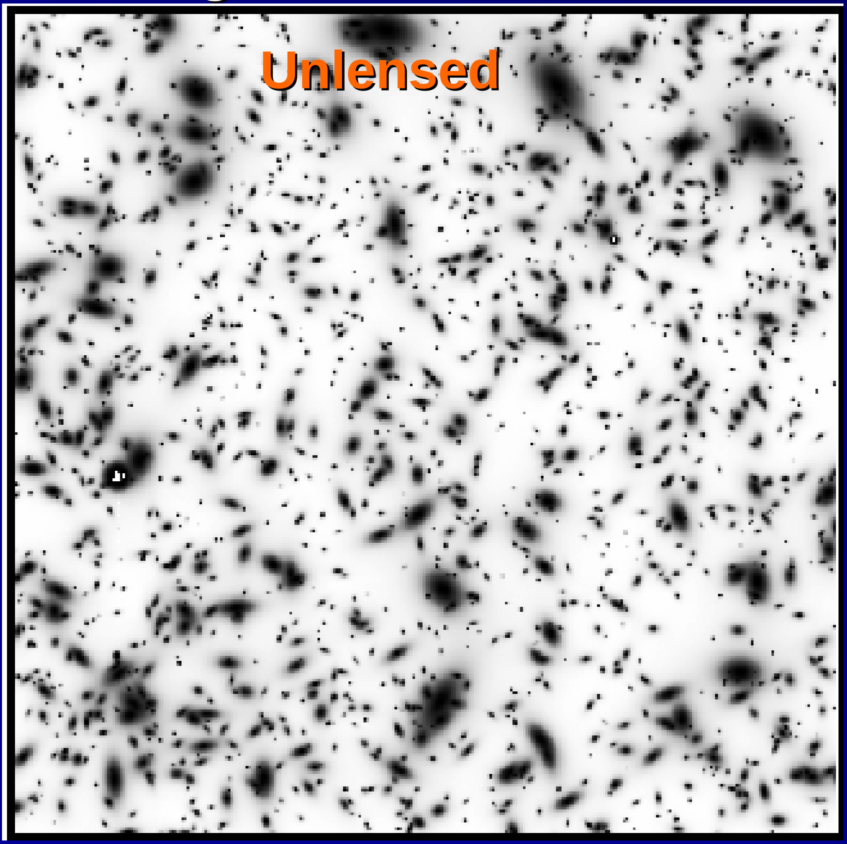
Cluster Weak Lensing

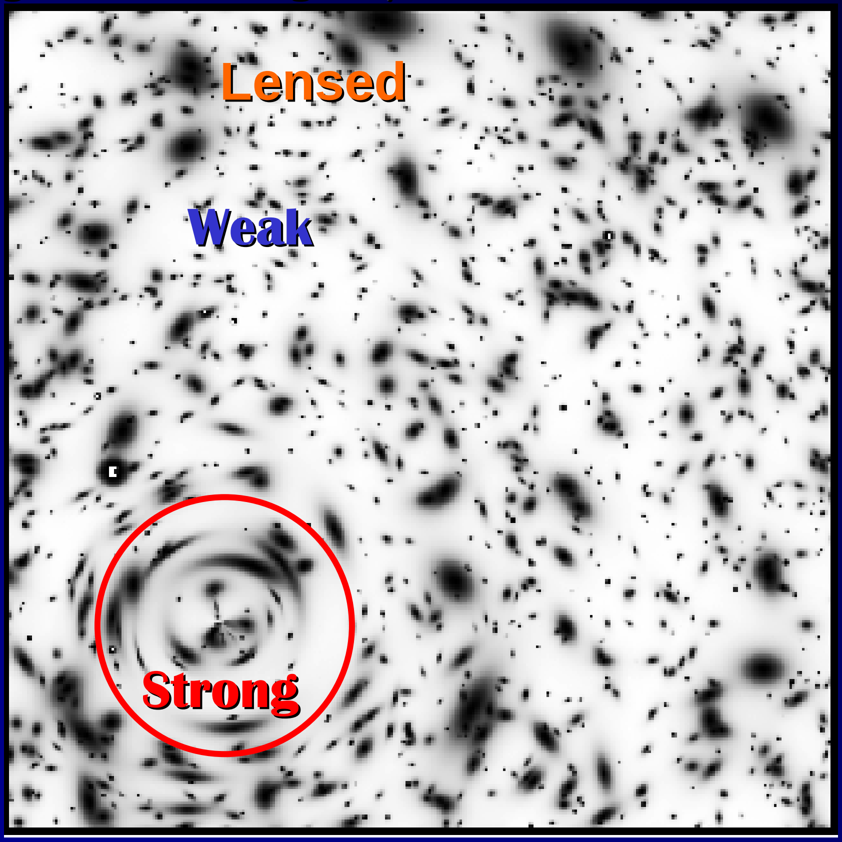
- Tom Broadhurst (Tel Aviv)
- Elinor Medezinski (Tel Aviv)
- Yoel Rephaeli (Tel Aviv)
- Masahiro Takada, Nobuhiro Okabe (Tohoku)
- AMiBA science team (ASIAA, NTU/Phys)
- Tzihong Chiueh (NTU/Phys)
- Mark Birkinshaw & Katy Lancaster (Bristol)

Gravitational Lensing

The images of background sources carry the imprint of the gravitational potential of the intervening matter.

Gravitational Lensing is essentially a coordinate mapping from a foreground (source) to the background (image) plane:

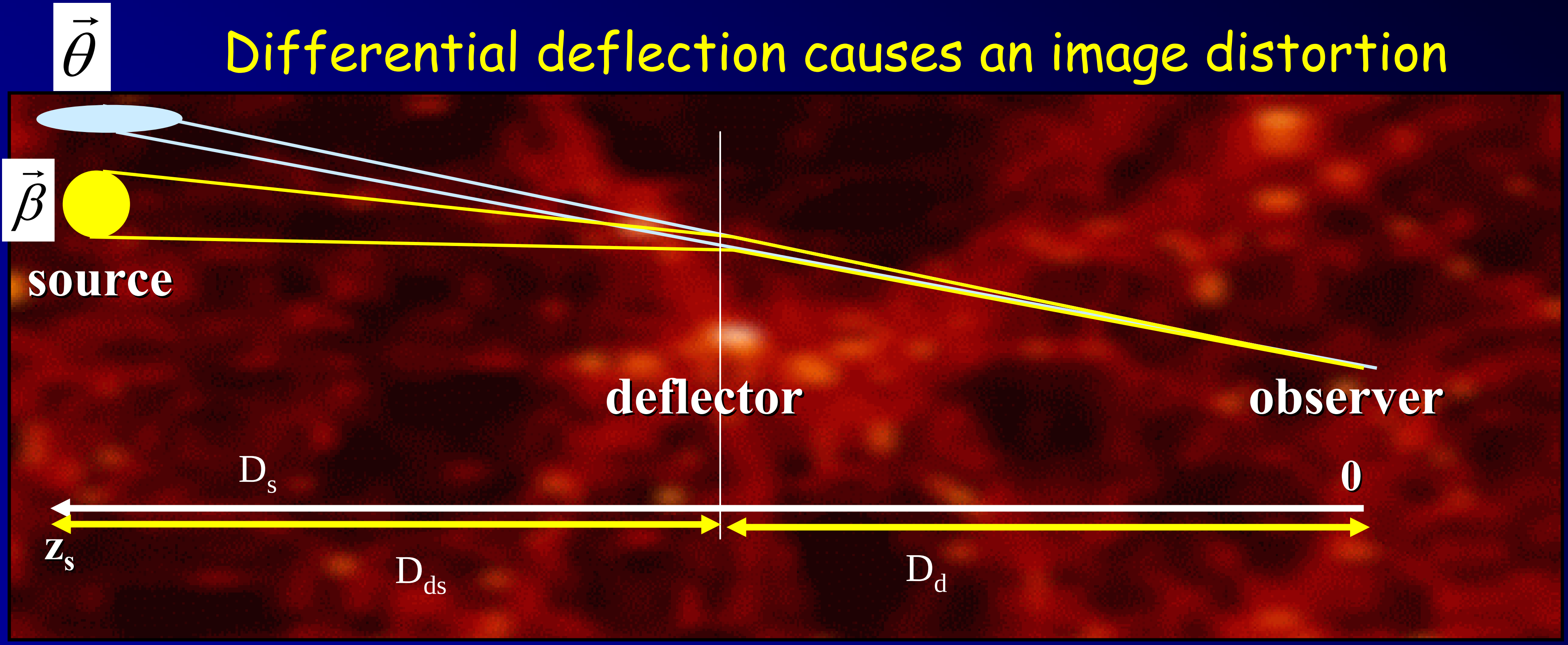




Light-Ray Bending and Image Deformation

First derivatives of ϕ gives a deflection

Differential deflection causes an image distortion



For an infinitesimal light source:

$$d^2\vec{\beta} = \mathbf{A}d^2\vec{\theta} \quad \text{with} \quad \mathbf{A} \equiv \mathbf{1} - \nabla\nabla\phi \neq \mathbf{1}$$

A: Jacobian matrix of the lens equation

Lens mapping Eq. in general:

$$\delta\beta_i = D_{ij}{}^{(1)}\delta\theta_j + \frac{1}{2}D_{ijk}{}^{(2)}\delta\theta_j\delta\theta_k + \ldots$$

Ordinal (1st order) Weak Lensing

2nd derivatives of ϕ:
Area distortion (magnification) and
Quadrupole shape deformation

Jacobian matrix of lens eq.

$$\mathbf{A}_{ij}(\boldsymbol{\theta}) = \delta_{ij} - \nabla_i \nabla_j \phi$$

$$\mathbf{A}(\boldsymbol{\theta}) = (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix}$$

$$\partial := \partial_1 + i\partial_2$$

complex differential operator

2D Poission Eq.

$$\kappa = \frac{1}{2}\partial\partial^*\phi = \frac{\Sigma_m}{\Sigma_{\rm crit}}$$

$$\gamma = \frac{1}{2}\partial\partial\phi \equiv \gamma_1 + i\gamma_2$$

$$(\partial\partial\phi, \partial\partial^*\phi)$$

dof=2+1

Lensing convergence ::
Dimensionless surface mass density of the lens (spin-0)

Complex Gravitational Shear:: spin-2 quadrupole image distoriton

2^{nd} Order Weak Lensing: Flexion

Flexion is 3^{rd} order derivatives of the potential:

$$\mathbf{D}_{ijk}^{(2)} = \mathbf{A}_{ij,k} = -\nabla_i \nabla_j \nabla_k \phi$$

dof=2+2

$$(\partial\partial\partial\phi, \partial\partial\partial^*\phi)$$

Complex First flexion

$$F = \frac{1}{2}\partial^*\partial\partial\phi = \partial\kappa$$

Spin-1

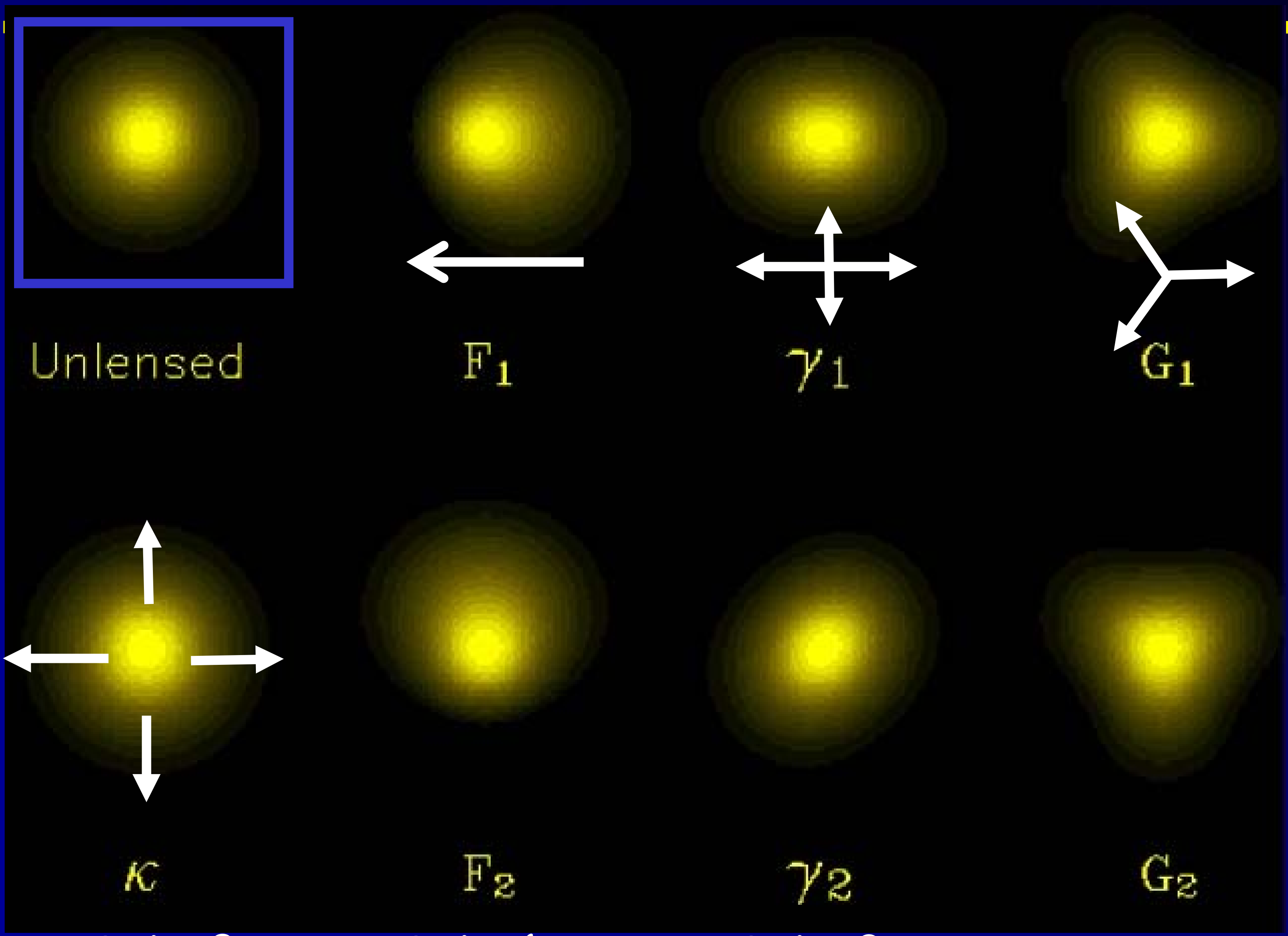
Complex Second flexion

$$G = \frac{1}{2}\partial\partial\partial\phi = \partial\gamma$$

Spin-3

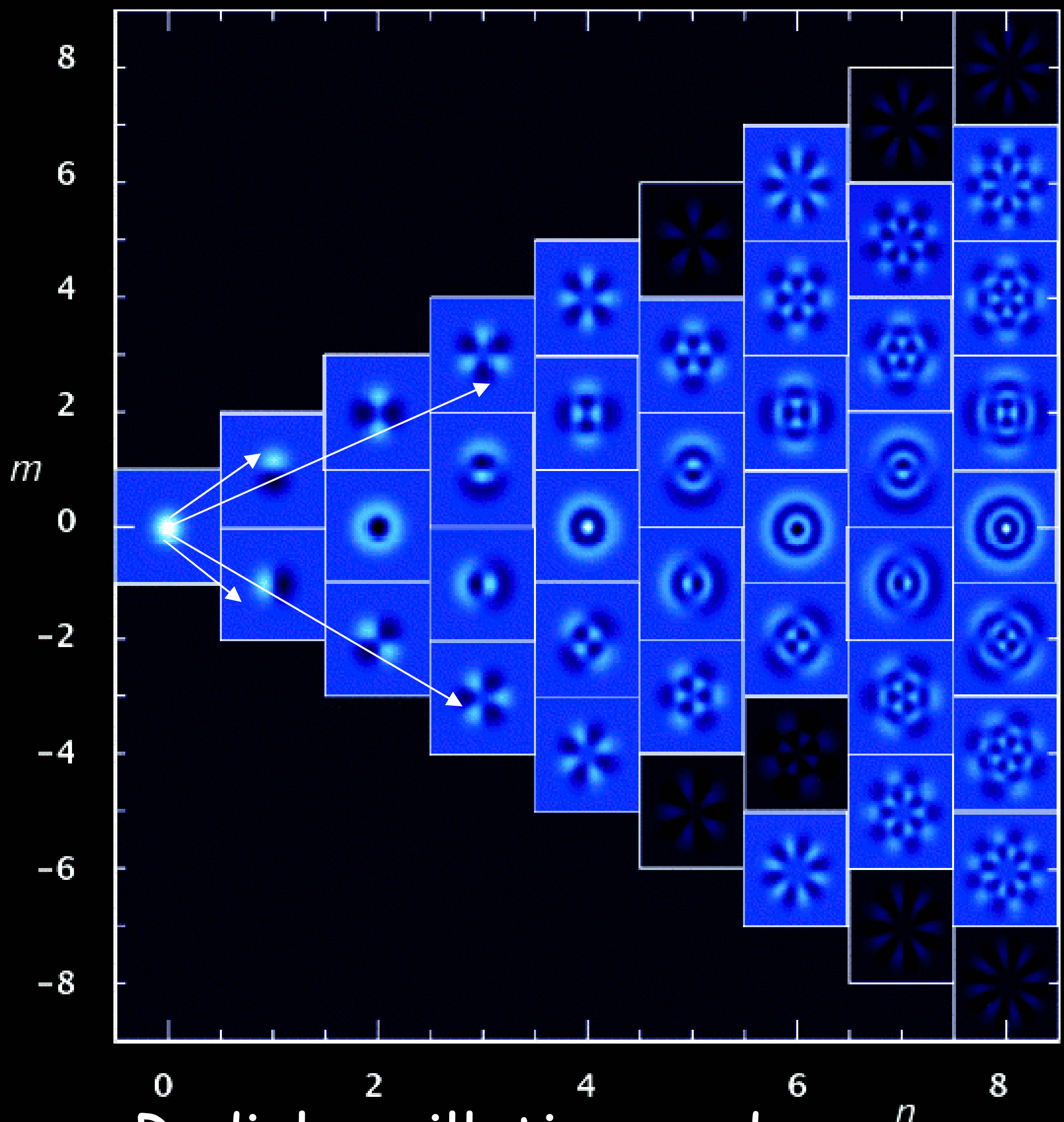
(Goldberg & Natarajan 2002, Bacon et al. 2005)

Effects of Convergence, Shear, Flexion



Spin-0 Spin-1 Spin-2 Spin-3

Flexion and Shear in Shapelets



Tangential oscillation mode

Radial oscillation mode

Decompose an image into a set of orthogonal 2D-basis functions (Gaussian-weighted Hermite polynomial)

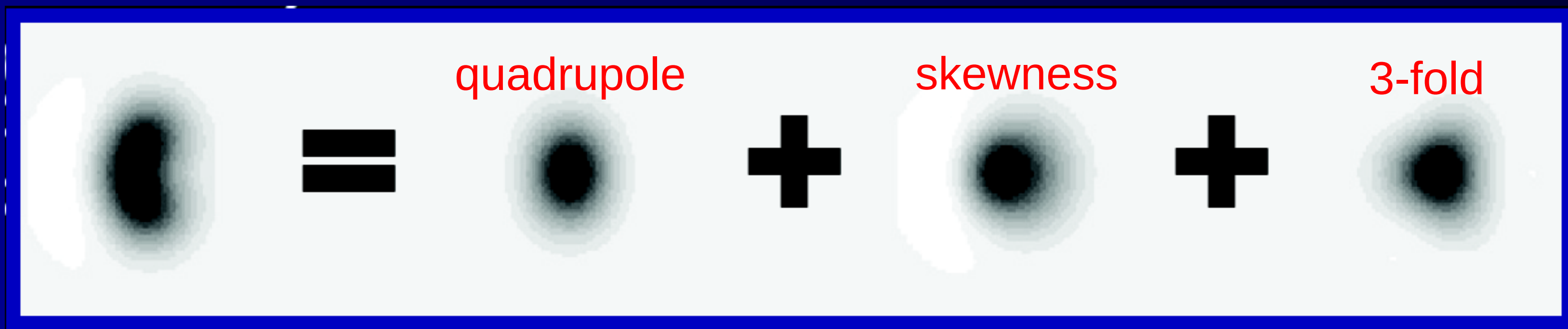
Cf. Massey & Refregier 2005

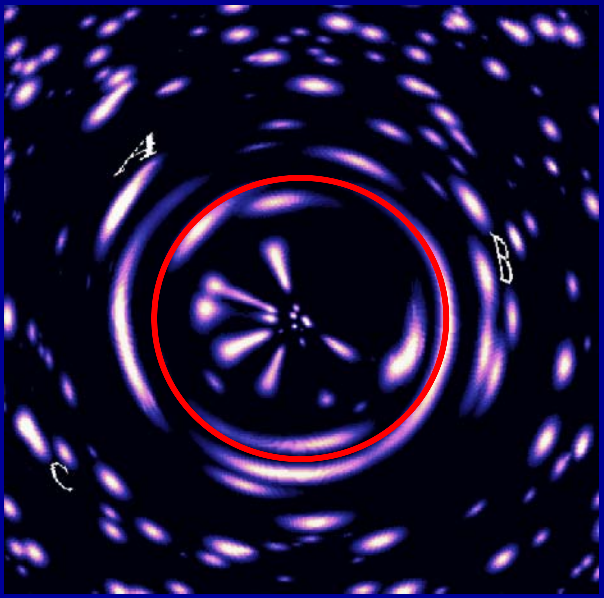
By Richard Massey

Where is Flexion useful?

An intermediate regime between WEAK and STRONG lensing can be well described by shearing and flexing effects:

Arclets = lensed images with slight curvatures





$$\gamma = \frac{1}{2}\partial\partial\phi$$

Spin-2

$$F = \frac{1}{2}\partial^{*}\partial\partial\phi$$

Spin-1

$$G = \frac{1}{2}\partial\partial\partial\phi$$

Spin-3

Goldberg & Natarajan 02, Bacon et al. 2005

Shear vs. Flexion

Resolution limit in ordinal (=Spin-2) weak lensing with ground based telescopes (Subaru, CFHT, etc.):

$$\mathrm{FWHM} = 0.'9 \left(\frac{\mathrm{S/N}}{5}\right)\left(\frac{\kappa}{0.2}\right)^{-1} \boxed{\left(\frac{\sigma_\gamma}{0.4}\right)^{-1}\left(\frac{n_g}{30\mathrm{arcmin}^{-2}}\right)^{1/2}}$$

Ordinal WL is sensitive to structures of <u>1'-10',</u> which is dominated by clusters of galaxies

Flexion measures the gradient of shear; so is relatively sensitive to small-scale structures (e.g., galaxies, groups of galaxies)

$$\frac{S(F)}{S(\gamma)} \sim \frac{L\phi / r^3}{\phi / r^2} \sim \frac{L}{r}$$

L: image size

r: distance from the lens

Even though the higher-order effect is small, at small scales (r), for large images (L), Flexion signal might dominate over Shear signal

How to Measure Flexion?

Higher Order Lensing Image Characteristics (HOLICs)

Observable brightness-weighted shape moments of galaxy images:

$$Q_{ijk} = < x_i x_j x_k >$$

$$Q_{ijkl} = < x_i x_j x_k x_l >$$

Okura, Umetsu, Futamase (2007a) find natural combinations of higher-order moments to define HOLICs, and have shown explicit relations between HOLICs and Flexion:

Spin-1 HOLICs

$$\varsigma := \frac{Q_{111} + Q_{122} + i(Q_{112} + Q_{222})}{Q_{1111} + Q_{2222} + 2Q_{1122}} \sim \frac{e^{i\varphi}}{L}$$

Spin-3 HOLICs

$$\delta := \frac{Q_{111} - 3Q_{122} + i(3Q_{112} - Q_{222})}{Q_{1111} + Q_{2222} + 2Q_{1122}} \sim \frac{e^{3i\varphi}}{L}$$

$$\zeta^{(s)} \approx \zeta - \frac{9}{4}F, \quad \delta^{(s)} \approx \delta - \frac{3}{4}G$$

$$e \approx e^{(s)} + 2\gamma$$

Okura, Umetsu, Futamase 2007a

Flexion to Surface Mass Density $\kappa=\Sigma/\Sigma_{crit}$

Observable Flexion measures the 1st derivative of shear

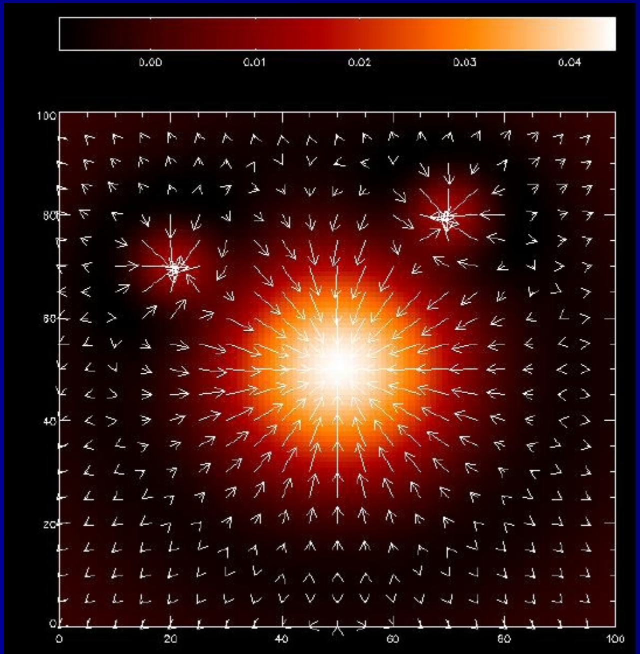
Non-parametric inversion from flexion to mass density

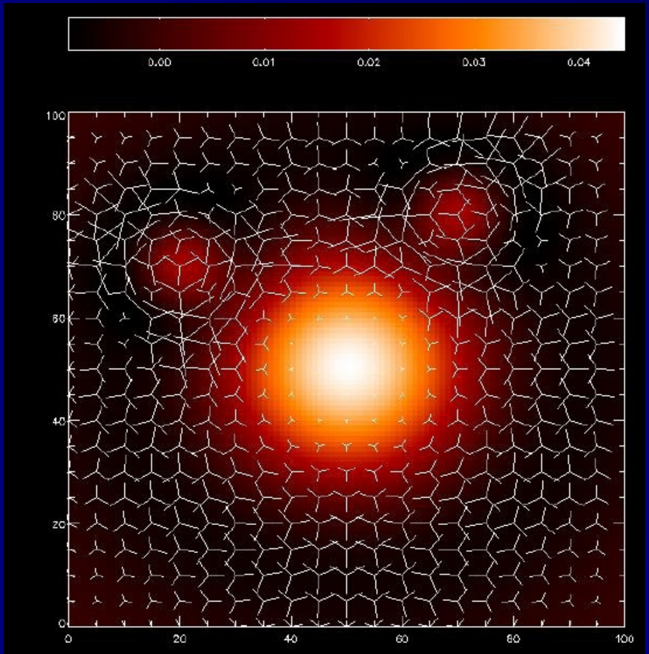
$$F=\frac{1}{2}\partial\partial\partial^{*}\phi=\partial\kappa$$

$$G=\frac{1}{2}\partial\partial\partial\phi=\partial\gamma$$

$$\kappa=\Delta^{-1}(\partial^{*}F)$$

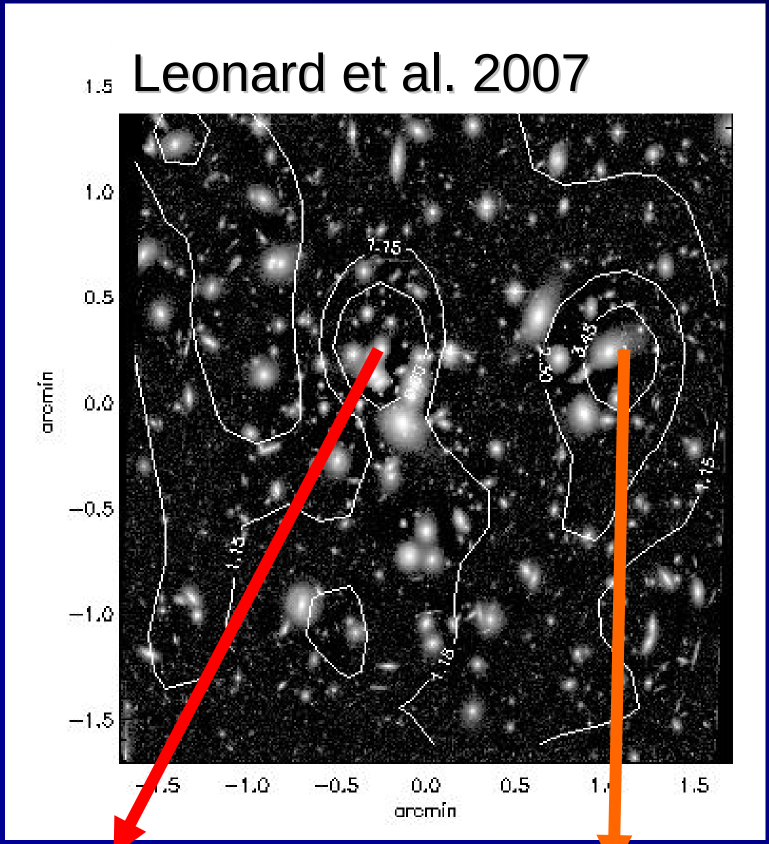
$$\kappa=\Delta^{-1}(\partial^{*}\partial^{*}\partial^{*}G)$$





Bacon et al. 2005

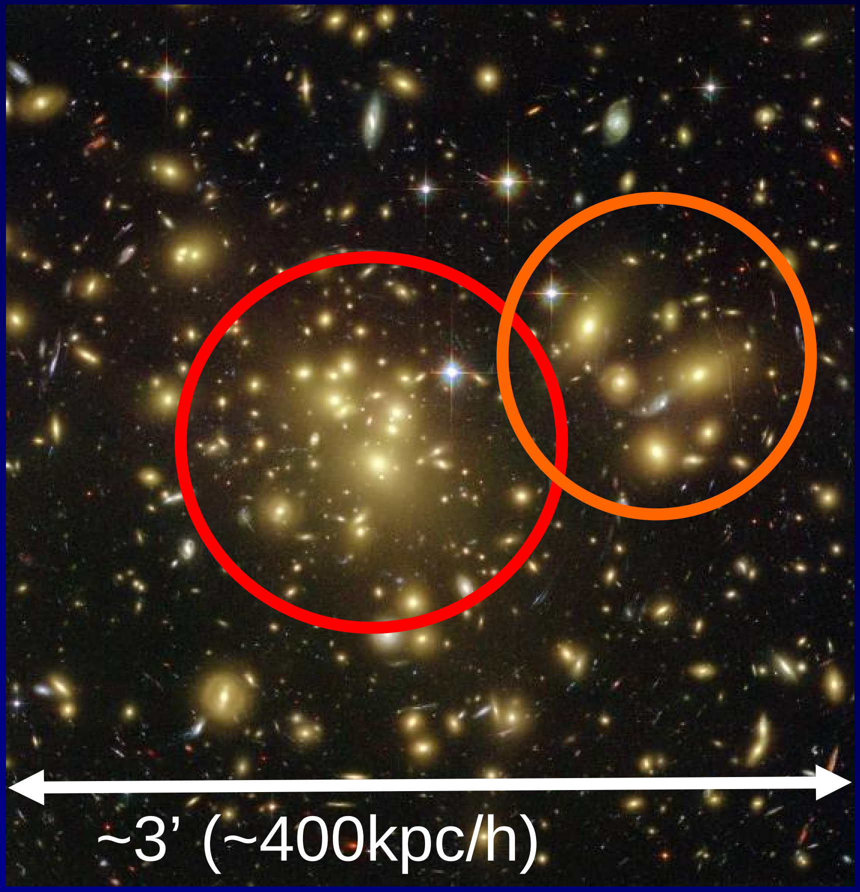
Application to a Real Cluster?



Negative peak at cD location!!

Substructure detected by 1st Flexion

A1689 HST/ACS data



No significant detection using a large # of galaxies:
ng ~ 70 galaxies/arcmin^2

Core regon of A1689 (z=0.18)

Problems and Motivations

- **Shapelets**:: Leonard et al. (2007) found HOLICs technique is less sensitive than Shapelets to contamination by bright wings of cluster (=lens) galaxies and other neighboring sources. So HOLICs technique seems to be suitable for cluster lensing.
- **HOLICs+** :: Goldberg & Leonard (2007) extended our HOLICs method to include isotropic PSF corrections but ignoring (1) spin-1 and spin-3 PSF anisotropy corrections, (2) additional correction terms for Gaussian-weighted moment calculations, etc..

→ **Fully generalize "KSB formalism" for ordinal spin-2 weak lensing, by including higher-order Gaussian-weighted moment calculations (HOLICs) and relevant corrections for spin-1 and spin-3 PSF anisotropies**

Okura, Umetsu, Futamase 2007b

HOLICs Moment Method

KSB formalism (Kaiser, Squares, Broadhurst 1995):
Adds in Lensing and PSF induced response to intrinsic ellipticity that is randomly oriented

Spin-2

$$e_\alpha = e_\alpha^{(s)} + P_{\alpha\beta}^{\gamma}\gamma_\beta + P_{\alpha\beta}^{sm}q_\beta$$

Higher-order generalization **of KSB moment method by**
Okura, Umetsu, Futamase 2007b

Spin-1

$$\varsigma_\alpha = \varsigma_\alpha^{(s)} + (C^F)_{\alpha\beta}F_\beta + (C^q)_{\alpha\beta}(\varsigma_q)_\beta$$

Spin-3

$$\delta_\alpha = \delta_\alpha^{(s)} + (D^G)_{\alpha\beta}G_\beta + (D^q)_{\alpha\beta}(\delta_q)_\beta$$

Unbiased estimator for Flexion

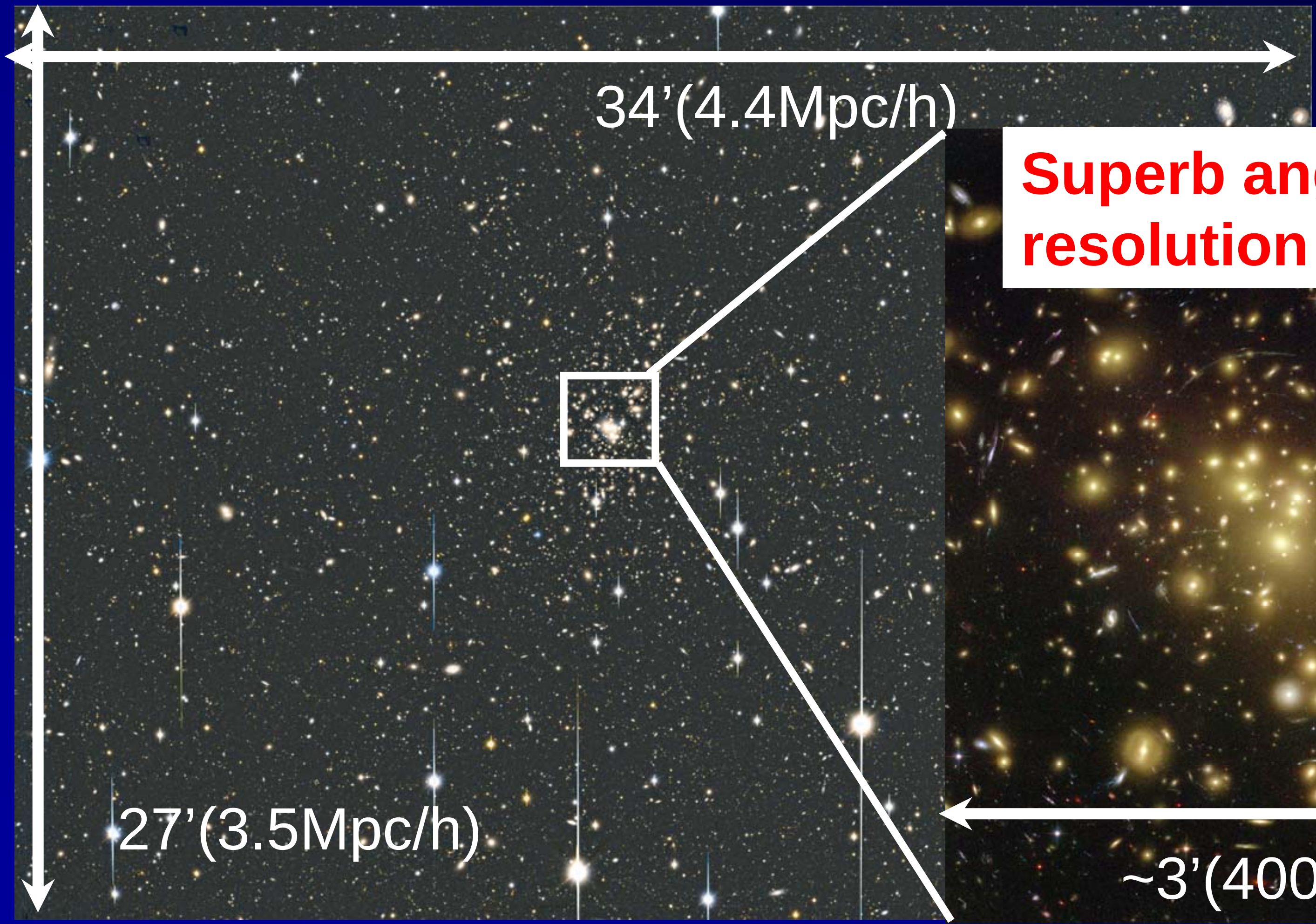
$$F \approx C_F^{-1}\left\langle \varsigma - C^q \varsigma_q \right\rangle \qquad G \approx D_F^{-1}\left\langle \delta - D^q \delta_q \right\rangle$$

Subaru Observations of A1689

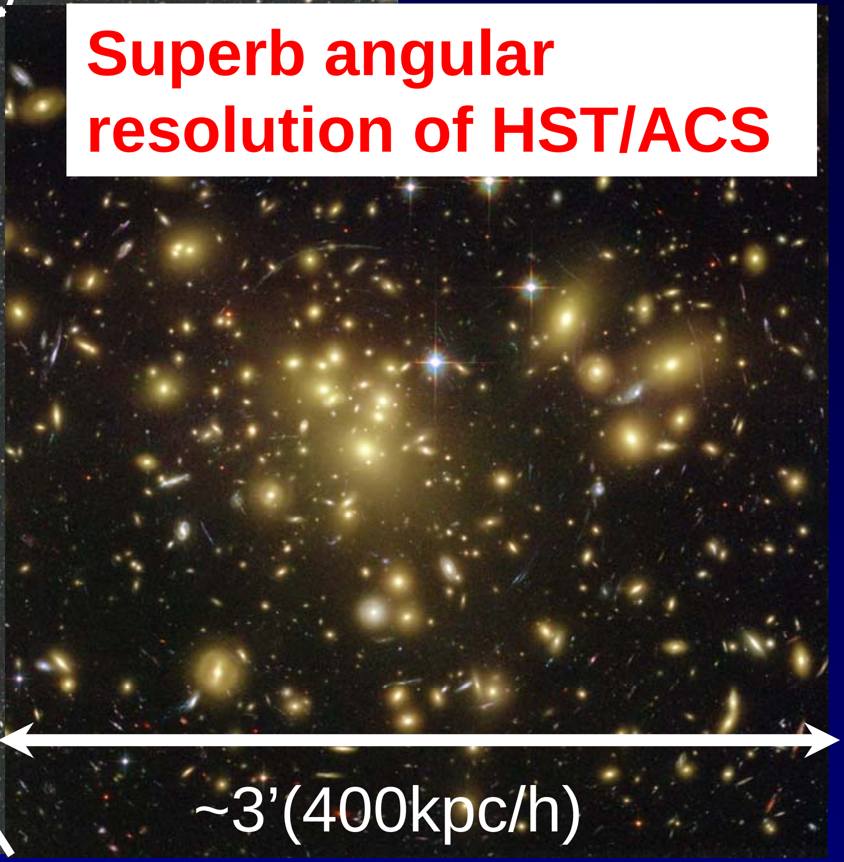
Wide-field imaging of Subaru/Suprime-Cam

Broadhurst, Takada, Umetsu+ 2005

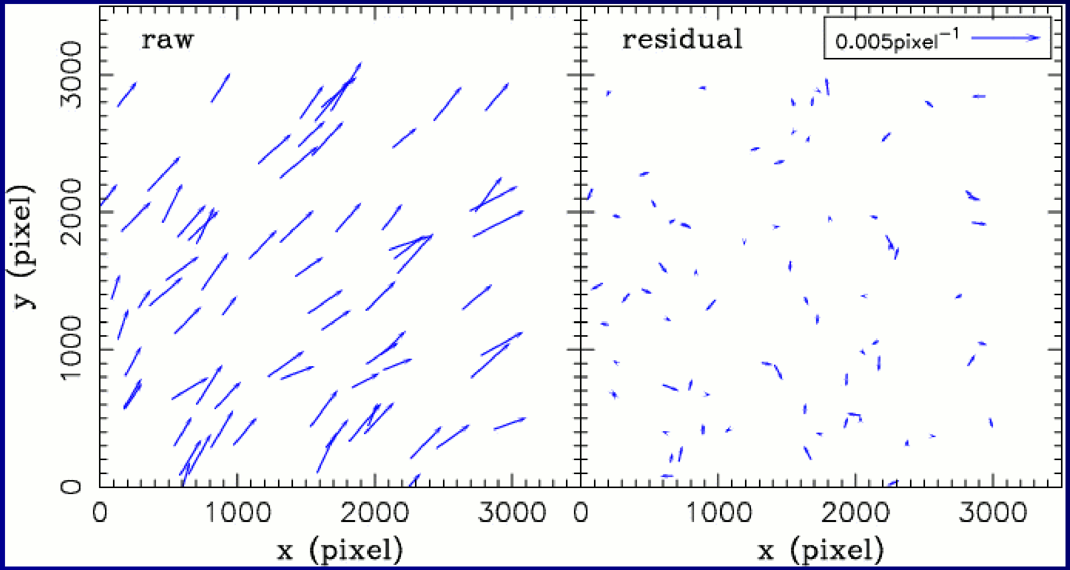
Umetsu & Broadhust 2007

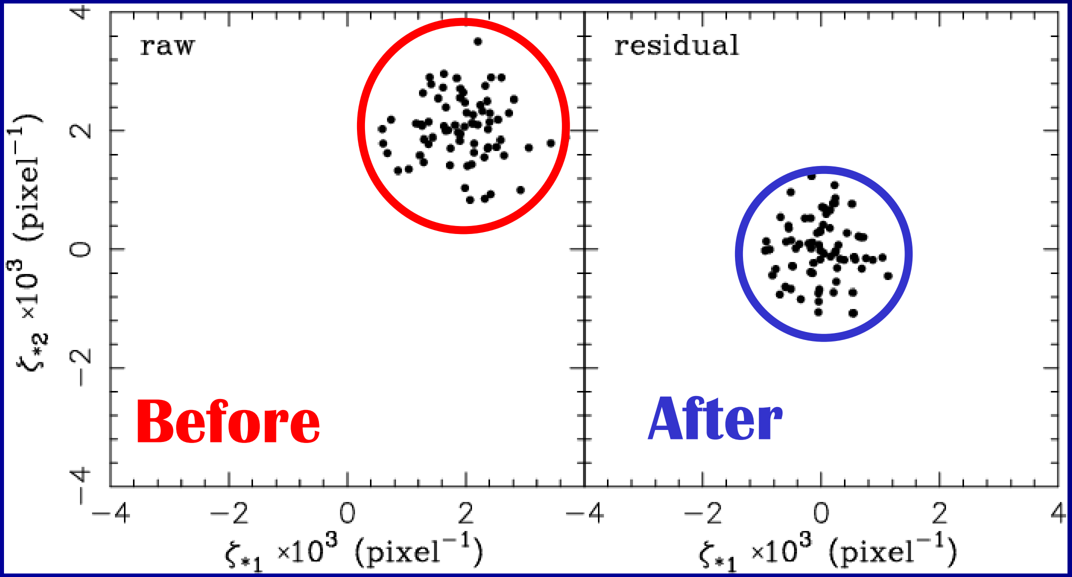


Superb angular resolution of HST/ACS



Spin-1 PSF Anisotropy Correction: Application to Subaru A1689 data





Spin-1 PSF anisotropy from stellar shape moments

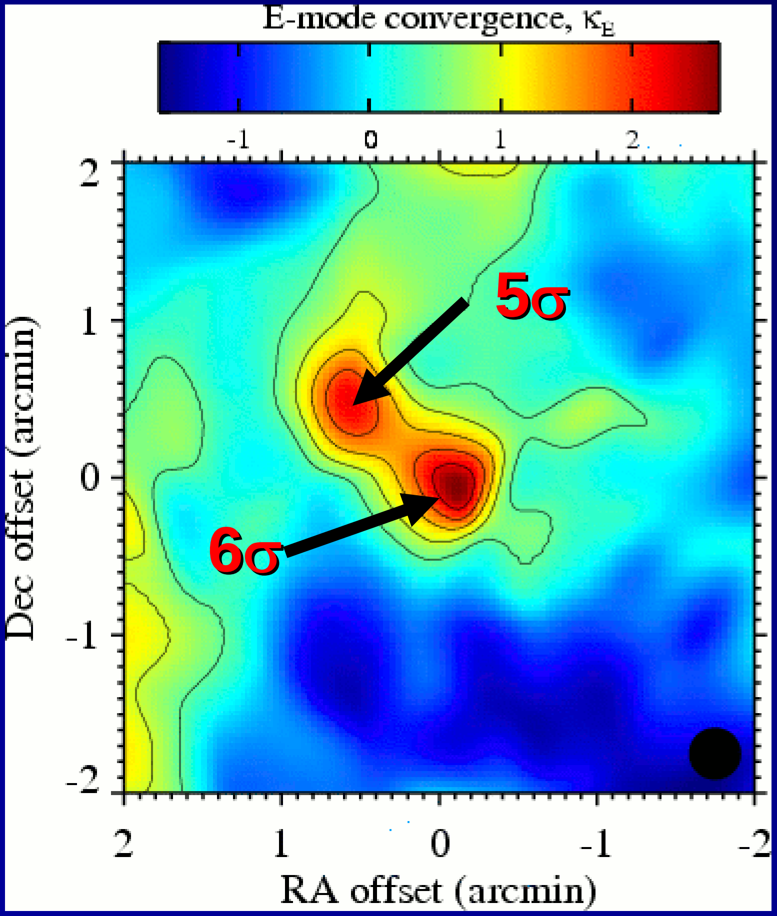
$$\varsigma_\alpha^{\ *} = (C^q)^*{}_{\alpha\beta}(\varsigma_q^{\ *})_\beta$$

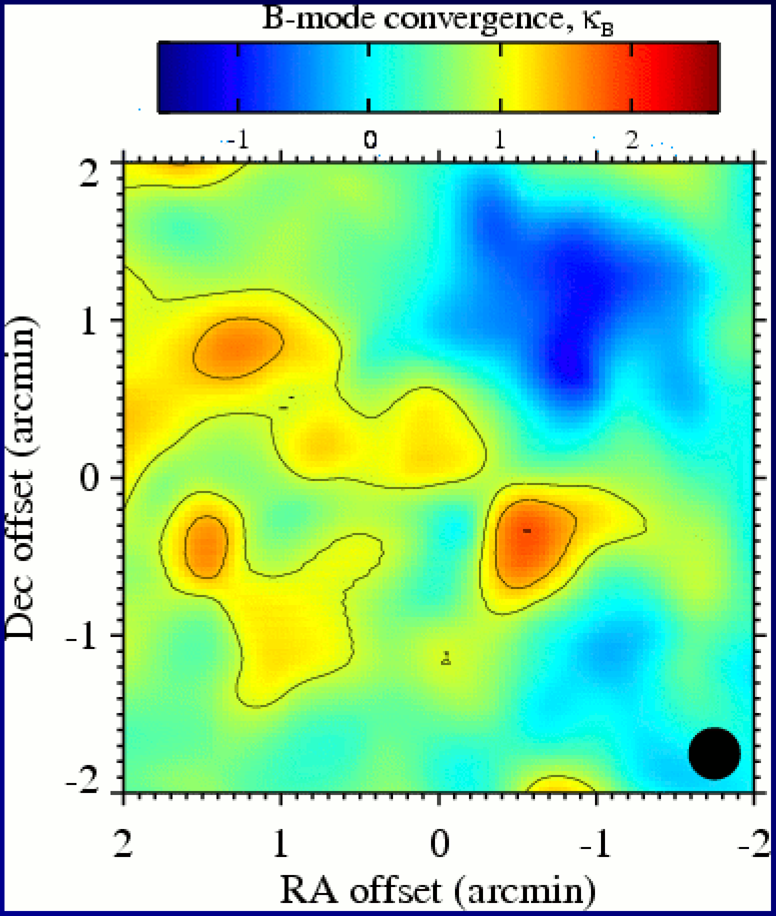
Okura, Umetsu, Futamase 2007b

Mass Map of A1689 from Spin-1 Flexion

Mass reconstruction in the 4’x4’ core region of A1689 (z=0.18)

E-mode (lensing) **B-mode (noise)**





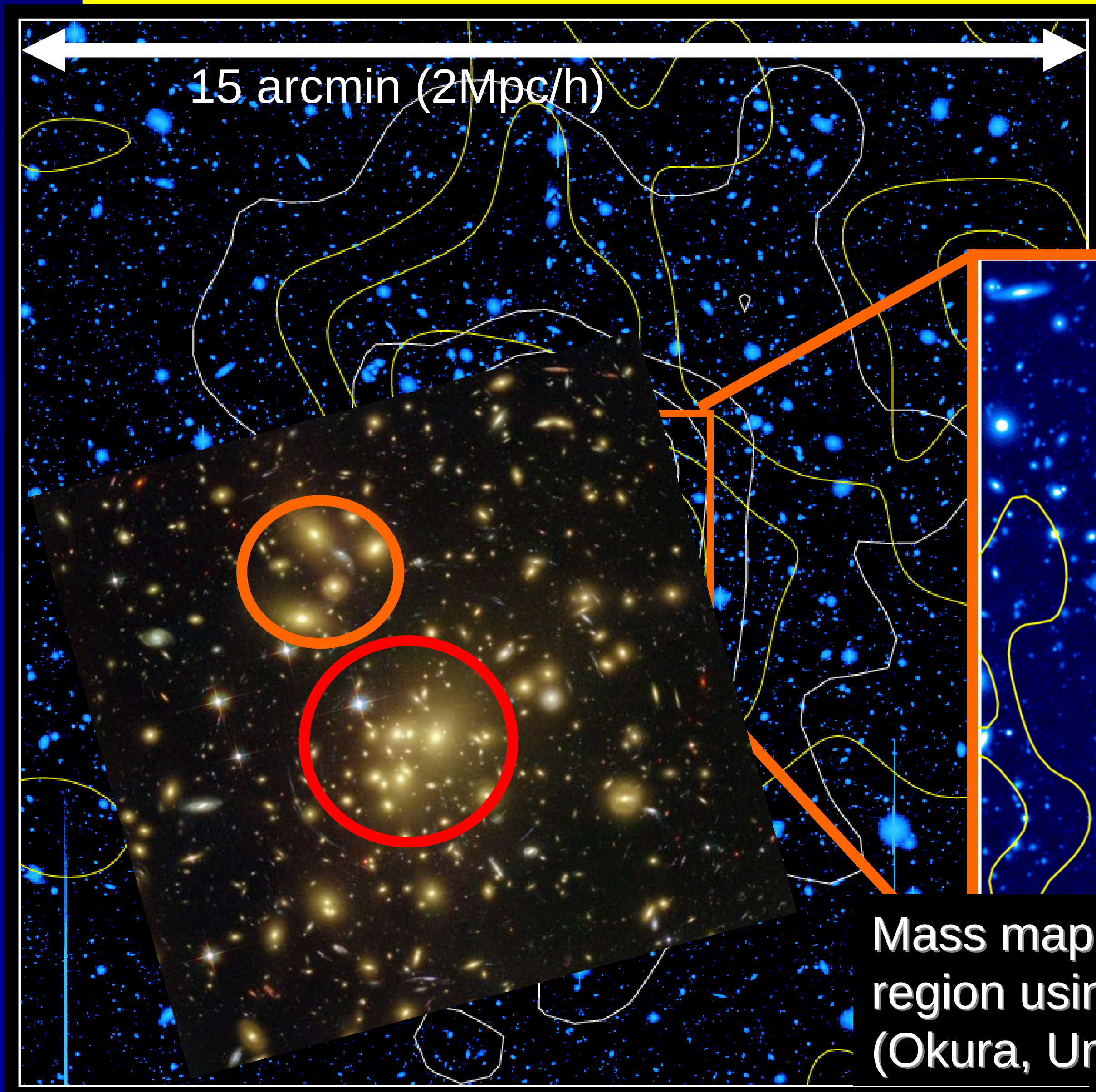
n_g=8 arcmin^-2

0.’3FWHM Gaussian

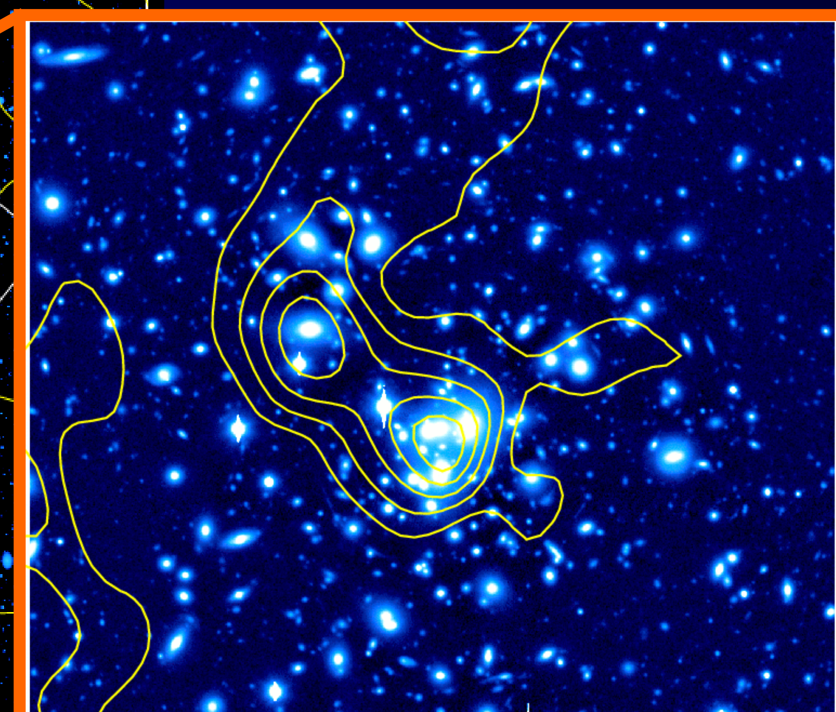
530kpc/h

Okura, Umetsu, Futamase 2007b

Mass and Light in A1689 (Subaru)



Mass + **Light** contours from Shear+Magbias data (Umetsu & Broadhurst 07)



Mass map from Fleixon in a 4'x4' region using ng=8 gal/arcmin^2 !!! (Okura, Umetsu, & Futamase 2007b)

Results

- First successful cluster mass reconstruction only by Flexion incorporating **full higher-order PSF anisotropy corrections**
- Significant detections of cD/group-scale substructures at 5-6 sigma levels
- Main density peak of $\kappa \sim 2.5^{+0.45}_{-0.45}$ is consistent with the ACS/Subaru mass model (Broadhurst, Takada, Umetsu et al. 05).

Conclusions and Future Work

- Weak lensing fiexion has been proposed in the past years, and is just becoming a usable practical tool for observational cosmology – not just a theoretical tool.
- A comparison with high-resolution HST/ACS data shows that indeed higher-order PSF anisotropy corrections are necessary for a high-precision measurement of the higher-order WL signal.
- Although the current HOLICs-based flexion analysis pipeline works well to locate mass substructures, an accurate mass measurement will require systematic tests and feasible "calibrations" as people have done for quadrupole WL (e.g., the STEP project).

FIN