Levin (2003)

- ▶ Studying the optimal contact between a principal and an agent when they interact repeatedly.
- ▶ Features:
    - (a) Output is observable to both, but is not contractible.
    - (b) The agent possesses certain private information, regarding either his own type or action.
    - (c) Performance evaluation is subjective.
- ▶ The model therefore incorporates both hidden information (adverse selection) and hidden action (moral harzard) consideration.
- ▶ Optimal contract takes a very simple form.

Model

- One principal (P), one agent (A). Both risk-neutral.
- They trade on dates $t = 0, 1, 2, \ldots$.
- At the beginning of date t, P proposes a contract to A.
- Content of contract: Base salary w_t, plus a contingent payment $b_t : \Phi \to R$; where Φ is the set of observable performance outcomes.
- Content of Φ depends on how much information is observable to both.
- After contract is offered, A decides whether to accept ($d_t = 1$) or to reject ($d_t = 0$).

- If rejected, they each receives reservation payoff $\underline{\pi}$ (for P) and $\underline{u}$ (for A).
  Note that their relation can restart at date $t+1$ even if $d_t = 0$.
- Upon acceptance, A observes a cost parameter θ_t, and chooses an effort $e_t \in [0, \bar{e}]$.
- $\theta_t \in [\underline{\theta}, \bar{\theta}]$, with distribution function $P(\cdot)$.
- Cost of effort: $v(e_t, \theta_t)$; with $v_1 > 0$ and $v(0, \theta_t) = 0 \; \forall \, \theta_t$.
- Output: $y_t \in Y \equiv [\underline{y}, \bar{y}]$, with distribution function $F(\cdot | e)$.

- Expected surplus of production:

$$S(e, \theta) = E_y[y|e] - v(e, \theta).$$

- A can observes all of θ_t, e_t, y_t.
- P can observes y_t, but might or might observe θ_t or e_t.
- If P can only observe e_t, then it is an adverse selection setup.
- If P can only observe θ_t, then it is a moral hazard setup.
- If P can observe both e_t and θ_t, it is a symmetric information setup.

- The model can thus incorporate both moral hazard and adverse selection consideration.
- At the end of date t, P is contractually obligation to pay w_t.
- If $b_t(\varphi_t) > 0$, it is P's decision whether to honor promise of contingent payment.
- If $b_t(\varphi_t) < 0$, it is A's decision whether to honor promise of contingent payment.
- W_t denotes payments actually made. That is, $W_t = w_t + b_t(\varphi_t)$ if contingent payment is honored, and is w_t if not.

- Payoff of A: $W_t - v(e_t, \theta_t)$.
  Payoff of P: $y_t - W_t$.

- Let $\underline{s} = \underline{u} + \underline{\pi}$. Assume

$$\max_e S(e, \theta) > \underline{s} > S(0, \theta) \quad \text{for all } \theta.$$

- δ: discount factor between periods.

- Starting from date t, the "continuation payoff" for P and A are, respectively,

$$\pi_t = (1 - \delta)E \sum_{\tau=t}^{\infty} \delta^{\tau-t}\Big\{ d_\tau(y_\tau - W_\tau) + (1 - d_\tau)\underline{\pi}\Big\},$$

$$u_t = (1 - \delta)E \sum_{\tau=t}^{\infty} \delta^{\tau-t}\Big\{ d_\tau\big(W_\tau - v(e_\tau, \theta_\tau)\big) + (1 - d_\tau)\underline{u}\Big\}.$$

Relational Contract

- Though φ_t is observable, $b_t(\varphi_t)$ is not legally enforceable.
- $b_t(\cdot)$ thus has to be self-enforcing.
- This kind of contract, in which terms are enforced by equilibrium, rather than law, is called relational contract or implicit contract.
- Given any "history" at t, $h^t = (w_0, d_0, \varphi_0, W_0, w_1, d_1, \varphi_1, W_1, \ldots, w_{t-1}, d_{t-1}, \varphi_{t-1}, W_{t-1})$, a relational contract specifies

  (i) the compensation P should pay;
  (ii) whether A should accepts a certain offer; and in case of acceptance;
  (iii) the effort level to be taken, given θ_t.

- A relational contract is self-enforcing if it is a SE.
- A self-enforcing contract is optimal if no other self-enforcing contract generates greater surplus.

The Basic Mathematics

- Consider a contract which calls for payments w, $b(\varphi)$ and effort $e(\theta)$ in the first period.

- Let $u(\varphi)$ and $\pi(\varphi)$ be the continuation payoffs if contract is accepted and contingent payment made in the first period.

- Without loss of generality, assume the contract is enforced by reversion to the worst equilibrium — the static equilibrium in which payoffs are $\underline{u}$ and $\underline{\pi}$ — if there is deviation.

- $W(\varphi) \equiv w + b(\varphi)$.

- $u \equiv (1 - \delta)E_{\theta,y}\big[W(\varphi) - v\big|e(\theta)\big] + \delta E_{\theta,y}\big[u(\varphi)\big|e(\theta)\big].$
  $\pi \equiv (1 - \delta)E_{\theta,y}\big[y - W(\varphi)\big|e(\theta)\big] + \delta E_{\theta,y}\big[\pi(\varphi)\big|e(\theta)\big].$
  u and π are the expected payoffs under this contract.

- $s \equiv u + \pi$: total surplus generated by this contract.
  That is,

$$s = (1 - \delta)E_{\theta,y}\big[y - v|e(\theta)\big] + \delta E_{\theta,y}\big[s(\varphi)\big|e(\theta)\big];$$

  where $s(\varphi) = u(\varphi) + \pi(\varphi)$.

- The contract is self-enforcing iff

  (i) $u \geq \underline{u}$ and $\pi \geq \underline{\pi}$.

  (ii) For all θ,

  $$e(\theta) \in \arg \max_{e} E_y \left[W(\varphi) + \frac{\delta}{1-\delta} u(\varphi) \big| e \right] - v(e, \theta).$$

  (iii) For all φ, both parties are willing to make the contingent payment:

  $$b(\varphi) + \frac{\delta}{1-\delta} u(\varphi) \geq \frac{\delta}{1-\delta} \underline{u},$$

  $$-b(\varphi) + \frac{\delta}{1-\delta} \pi(\varphi) \geq \frac{\delta}{1-\delta} \underline{\pi}.$$

  (iv) For all φ, $u(\varphi)$ and $\pi(\varphi)$ must correspond to a self-enforcing contract that initiates in the next period.

- Theorem 1: If there is a self-enforcing contract that generates surplus $s \geq \underline{s}$, then there exists a self-enforcing contract that gives as expected payoff <u>any</u> (π, u) such that $\pi + u \leq s$, $\pi > \underline{\pi}$ and $u > \underline{u}$.
- Intuition: Changes the value of w_t. This redistributes surplus without changing incentives.

Stationary Contract

- Focus on a type of equilibrium in which the strategy of either P or A does not change over periods.
- A contract is stationary if on the equilibrium path,
  (i) $W_t = w + b(\varphi_t)$ for some w,
  (ii) $e_t = e(\theta_t)$,
      where $b : \Phi \to R$ and $e : [\underline{\theta}, \bar{\theta}] \to E$.
- In a stationary contract, the wage policy and effort decision do not change over periods.
- Theorem 2: If an optimal contract exists, there are stationary contracts which yield same payoffs to A and P.

► Certain facts before proof:

    1. The set of possible payoffs for a self-enforcing contract is

$$\Big\{(u,\pi)|u \geq \underline{u}, \pi \geq \underline{\pi}, \text{ and } u + \pi \leq s^*\Big\};$$

    where s^* is the surplus generated by the optimal contract.

    2. For any self-enforcing contract, its continuation payoffs after any history φ must satisfy $u(\varphi) \geq \underline{u}$ and $\pi(\varphi) \geq \underline{\pi}$

    3. For any optimal self-enforcing contract, it must be that $u(\varphi) + \pi(\varphi) = s^*$ for any history that occurs positive probability.

► Proof: the aim is to contract a stationary equilibrium which attains the same payoff for both A and P.

Let $u^* \in [\underline{u}, s^* - \underline{\pi}^*]$ be given.

Define

$$b^*(\varphi) = b(\varphi) + \frac{\delta}{1-\delta}u(\varphi) - \frac{\delta}{1-\delta}u^*, \text{ and}$$

$$w^* = u^* - E_{\theta,y}[b^*(\varphi) - v|e(\theta)].$$

Then let $W^*(\varphi) = w^* + b^*(\varphi)$. $\pi^* \equiv s^* - u^*$.

This is a stationary contract, as the determination of continuation payoffs are same in every period.

This contract obviously satisfies (i). Also, by construction of the new contract,

$$b^*(\varphi) + \frac{\delta}{1 - \delta} u^* = b(\varphi) + \frac{\delta}{1 - \delta} u(\varphi).$$

As a result, A faces the same incentive structure as the original contract, which in turn implies that the new contract satisfies (ii) and (iii). Finally, (iv) must also be satisfied because the new contract repeats in each period, and continuation contract thus must be self-enforcing. Under the contract, A's expected payoff is u^* and P's π^*.

- An important principle in the theory of repeated game is that in order to deter a player from deviating, it always suffices to punish him with his <u>lowest</u> possible equilibrium payoffs (optimal penal code).

- Applied to our stationary cases, this means in order to deter P or A from deviating, it suffices that

$$\frac{\delta}{1-\delta}(\pi - \underline{\pi}) \geq \sup_{\varphi} b(\varphi), \text{ and}$$

$$\frac{\delta}{1-\delta}(u - \underline{u}) \geq -\inf_{\varphi} b(\varphi).$$

► Theorem 3: An effort level $e(\theta)$ that generates expected surplus s can be implemented with a stationary contract iff there exists a payment scheme $W : \Phi \to R$ satisfying

$$e(\theta) \in \arg\max_e E_y\big[W(\varphi)|e\big] - v(e,\theta) \text{ for all } \theta, \quad (IC) \text{ and}$$

$$\frac{\delta}{1-\delta}(s - \underline{s}) \geq \sup_\varphi W(\varphi) - \inf_\varphi W(\varphi). \qquad (DE)$$

► The variation in contingent payments (RHS of (DE)) is limited by the surplus relationship.

▶ Proof:

Necessity: Consider a self-enforcing contract $W(\varphi) = w + b(\varphi)$ which implements $e(\theta)$. For this contract to be self-enforcing, it must be that, for all φ,

$$\frac{\delta}{1-\delta}(u - \underline{u}) \geq -b(\varphi), \text{ and}$$

$$\frac{\delta}{1-\delta}(\pi - \underline{\pi}) \geq b(\varphi)$$

Naturally, the above hold when the RHSs are replaced by $-\inf_\varphi b(\varphi)$ and $\sup_\varphi b(\varphi)$. Summing up we have

$$\frac{\delta}{1-\delta}(s - \underline{s}) \geq \sup_\varphi b(\varphi) - \inf_\varphi b(\varphi).$$

Since $b(\varphi) = W(\varphi) - w$, we get (DE).

Sufficiency: Suppose there is a $W(\varphi)$ and an $e(\theta)$ which satisfy (DE) and (IC). Let

$$b(\varphi) \equiv W(\varphi) - \inf_{\varphi} W(\varphi) \text{ and}$$

$$w \equiv \underline{u} - E_{\theta,y}\big[b(\varphi) - v(e,\theta)|e(\theta)\big].$$

Note that $b(\varphi)$ is just $W(\varphi)$ minus a constant, and w is (of course) a constant. The incentive structure faced by A is exactly the same under $(b(\varphi), w)$ and $W(\varphi)$. Therefore, $e(\theta)$ must satisfy (IC).

Also, under the constructed contract, A's expected payoff is $\underline{u}$ and that of P is $\pi \equiv s - \underline{u}$. Easy to see that $\pi > \underline{\pi}$, so both P and A are willing to enter into the relationship. Finally,

$$\frac{\delta}{1-\delta}(\underline{u} - \underline{u}) = 0 \geq -b(\varphi) = -\big(W(\varphi) - \inf_{\varphi} W(\varphi)\big), \text{ and}$$

$$\frac{\delta}{1-\delta}(\pi - \underline{\pi}) = \frac{\delta}{1-\delta}(s - \underline{u} - \underline{\pi}) \geq \sup_{\varphi} W(\varphi) - \inf_{\varphi} W(\varphi)$$
$$= \sup_{\varphi} W(\varphi) + b(\varphi) - W(\varphi) \geq b(\varphi).$$

This implies neither A nor P is willing to deviate from the contract $(b(\varphi), w, e(\theta))$.

Optimal Incentive Provision: Case of Adverse Selection

- ▶ Assume P can observe e_t, but not θ_t.
- ▶ Assume $v_{ee} > 0$, $v_\theta > 0$, $v_{e\theta} > 0$, $v_{\theta ee} > 0$, $v_{\theta e\theta} > 0$.
- ▶ Assume $S(e, \theta)$ is differentiable and concave in e with interior maximizer $e^{FB}(\theta)$.
- ▶ Assume $P(\cdot)$ is concave, with density function $p(\cdot)$.
- ▶ Theorem 4: An effort level $e(\theta)$ that generates surplus s can be implemented by a stationary contract iff $e(\theta)$ is increasing and

$$\frac{\delta}{1 - \delta}(s - \bar{s}) \geq v\big(e(\underline{\theta}), \underline{\theta}\big) + \int_{\underline{\theta}}^{\bar{\theta}} v_\theta\big(e(\theta), \theta\big) d\theta.$$

- ▶ Form of the optimal contract:
    1. If δ is small or outside option high, self-enforcing constraint can never be satisfied, and no relationship can realize.
    2. If, at the other extreme, none of IC or DE is binding, $\mu = 0$ and first-best can be implemented: $e(\theta) = e^{FB}(\theta)$.
    3. In the intermedian case when $\mu > 0$, boundary condition implies $v(\underline{\theta}) > 0$ and, therefore by complementary slackness, $\dot{e}(\underline{\theta}) = 0$: There is pooling on the bottom.
       (a) If self-enforcing constraints are very restrictive, all types are pooled.
       (b) If not very so, only partially pooled.

▶ Theorem 5: Assuming it is worthwhile for A and P to enter a relationship, there are 3 possible forms for $e(\theta)$:

(i) Pooling $e(\theta)$ is constant for all θ.

(ii) Partial pooling: There exists $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ so that $e(\theta)$ is constant on $[\underline{\theta}, \hat{\theta}]$ and strictly increasing on $(\hat{\theta}, \bar{\theta}]$.

(iii) First best outcome: $e(\theta) = e^{FB}(\theta)$ for all θ.

Moreover, in case (i) and (ii), $e(\theta) < e^{FB}(\theta)$ for all θ.

Moral Hazard

- Assume θ_t is observable but not e_t.
- The stationary contract then takes the form $W(\theta, y) = w + b(\theta, y)$.
- Assuming MLRP for $F(y|e)$ and that $F(y|e = v^{-1}(x, \theta))$ is convex in x for all θ. This ensures first-order condition approach is valid.

▶ Theorem 6: An optimal contract implements some effort $e(\theta) \leq e^{FB}(\theta)$. For each θ, either $e(\theta) = e^{FB}(\theta)$ or $e(\theta) < e^{FB}(\theta)$. Moreover,

$$W(\theta, y) = \begin{cases} \underline{W} & \text{if } y < \hat{y}(\theta), \\ \overline{W} & \text{if } y \geq \hat{y}(\theta); \end{cases}$$

where $\overline{W} = \underline{W} + \frac{\delta}{1-\delta}(s - \bar{s})$, $\hat{y}(\theta)$ is the point at which the likelihood ratio $f_e(y)/f(y)$ turns from being negative to positive.

Subjective Performance Evaluation

- ▶ Consider a special case of subjective performance measures in which A privately knows e_t and P privately observes y_t.
- ▶ Assume away θ_t, so that now cost function $v(\cdot) = v(e)$.
- ▶ After observing y_t, P sends a "message" $m_t \in M$. W_t will be in the form $w_t + b_t(m_t)$.
- ▶ Stationary contract is no longer effective.
- ▶ Reason: To provide incentive, A's payoff must depend on m_t. However, P is willing to report different value for m_t and m_t' upon observing different y_t and y_t' only if his payoff are same following m_t and m_t' (i.e., $b(m_t) = b(m_t')$ if contract is stationary). That means production cannot be optimal after every history.

- Consider only the <u>full review</u> contracts: Given any history up to t and compensation after at t, any two different outputs $y_t \neq y_t'$ must generate distinct reports, $m_t \neq m_t'$.
- Reason: There won't be private information that P possesses: He fully reveals it in report. This greatly simplifies math stuffs.

▶ Theorem 7: If an optimal full review contract exists, it can be achieved by the following "termination" contract:

(i) In every period, A's effort level is some $e \leq e^{FB}$.

(ii) There exists some $\hat{y}$ such that

$$W_t = \begin{cases} w, & \text{if } y_t < \hat{y}, \text{ and relation terminates,} \\ w + b, & \text{if } y_t \geq \hat{y}, \text{ and relation continues;} \end{cases}$$

where $w = \bar{u} + v(e) + k$ and $b = \frac{\delta}{1-\delta}(s - \underline{s} - k)$ for some $k \in [0, s - \underline{s}]$.

▶ Under this contract, the expected surplus is

$$s = \underline{s} + \frac{(1-\delta)E(y - v - \underline{s}|e)}{1 - \delta[1 - F(\hat{y}|e)]}.$$

▶ Since P has incentive to report bad outcome to save bonus payment, there must be cost for him to do that. Termination of contract is this cost.