Sabotage in Promotion Tournaments
Chen (2003)

- When workers can sabotage each others in a promotion tournament, not only is there inefficiency in effort, but also inefficiency in picking winner, in the sense that higher ability agents are promoted with lower probability.
- Intuition: High-ability workers, being one with higher promotion probability, will be subject to more attacks. The total attack might over-weigh the advantage in ability.

Model

- One principal, n agents. All risk-neutral.
- Effort vector of agent i:

$$(e_i; a_{i1}, a_{i2}, \ldots, a_{i,i-1}, a_{i,i+1}, \ldots, a_{in});$$

  where e_i is own-effort, and a_{ij} is i's attack against j.
- Output of agent i:

$$W_i = t_i e_i - g\Big(\sum_{j \neq i} s_j a_{ji}\Big) + \varepsilon_i;$$

  where t_i is i's ability in productive activity, and s_j is his ability in sabotage.
- ε_i is a r.v. with density function $f(\cdot)$ and distribution function $F(\cdot)$. Assume $f(\cdot)$ is single-peaked and symmetric around 0.

- $g(\cdot)$ is the function that transforms total attack into loss of output.
- $r_i \equiv t_i/s_i$: i's relative ability in productive activity.
- Incentives are provided by a promotion tournament: If $W_i > W_j$ for all $j \neq i$, then i is promoted and receives a utility u. Otherwise his utility is 0.
- Utility of agent i:

$$u_i(e, a) = p_i(W_i, \ldots, W_n)u - v(e_i + \sum_{j \neq i} a_{ij}); \qquad (1)$$

where $p_i(\cdot)$ is i's promotion probability.

- $u_i(e, a)$ can be rewritten as

$$
\begin{aligned}
&\text{Prob}\Big(W_i + \varepsilon_i \geq W_j \geq \varepsilon_i,\ \forall j\Big) u - v\big(e_i + \sum_{j \neq i} a_{ij}\big) \\
&= \Big[\int_{-\infty}^{\infty} f(\varepsilon_i) \Big(\Pi_{j \neq i} \int_{-\infty}^{\varepsilon_i - W_i} f(\varepsilon_j) d\varepsilon_j \Big) d\varepsilon_i \Big] u \\
&- v\big(e_i + \sum_{j \neq i} a_{ij}\big).
\end{aligned}
$$

- FOC:

$$t_i\Big[\sum_{j\neq i}\int_{-\infty}^{\infty} f(\varepsilon_i)f(\varepsilon_i - W_{ji})\big(\Pi_{k\neq i,j}\int_{-\infty}^{\varepsilon_i - W_{ki}} f(\varepsilon_k)d\varepsilon_k\big)d\varepsilon_i\Big]u$$
$$-\, v'\big(e_i + \sum_{k\neq i} a_{ik}\big) = 0,\;\; i = 1,\ldots,n. \tag{2}$$

$$s_i g'\big(\sum_{k\neq j} s_k a_{kj}\big)\Big[\sum_{j\neq i}\int_{-\infty}^{\infty} f(\varepsilon_i)f(\varepsilon_i - W_{ji})$$
$$\big(\Pi_{k\neq i,j}\int_{-\infty}^{\varepsilon_i - W_{ki}} f(\varepsilon_k)d\varepsilon_k\big)d\varepsilon_i\Big]u - v'\big(e_i + \sum_{k\neq i} a_{ik}\big) = 0,$$
$$\text{for } i,j = 1,\ldots,n,\;\; j\neq i. \tag{3}$$

- From (2) and (3) we can show that

$$g'\Big(\sum_{k\neq j} s_k a_{kj}\Big) = \frac{n-1}{\sum_{l=1}^{n} r_l^{-1} - (n-1) r_j^{-1}}. \tag{4}$$

As a result

$$r_i > r_j \ \text{ implies } \ \sum_{k\neq i} s_k a_{ki} > \sum_{k\neq j} s_k a_{kj}$$

- **Theorem**: A worker with higher comparative productive activity is subject to greater total attack.

Some Comparative Statics Results

- As a worker becomes more talented in negative activity, all his co-workers are subject to more attack:

$$g''\Big(\sum_{k\neq j} s_k a_{kj}\Big)\frac{\partial\big(\sum_{k\neq j} s_k a_{kj}\big)}{\partial s_j}$$
$$= -(n-1)\Big[\sum_{l=1}^{n} r_l^{-1} - (n-1)r_j^{-1}\Big]^{-2} t_i^{-1} < 0.$$

- As a worker becomes more talented in sabotage, he himself will be less attacked

$$g''\Big(\sum_{k\neq j} s_k a_{kj}\Big)\frac{\partial\big(\sum_{k\neq j} s_k a_{kj}\big)}{\partial s_j}$$
$$=(n-1)(n-2)\Big[\sum_{l=1}^{n} r_l^{-1}-(n-1)r_j^{-1}\Big]^{-2} t_i^{-1}\geq 0.$$

- As a worker becomes more productive, he will be attacked more and his co-workers less attacked: $\partial \sum_k s_k a_{kj}/\partial t_i < 0$, $\partial \sum_k s_k a_{kj}/\partial t_j > 0$.

- **Proposition 1**: The total attack a worker receives (i) decreases in her own negative ability and his opponent's productive ability; and (ii) increases in the negative ability of any of his co-worker and his own productive ability.
- A person with highest (either absolute or negative) productive ability is not necessarily one with the highest promotion chance.

- If a new worker with ability $r_{n+1} = t_{n+1}/s_{n+1}$ enters the firm. Then (4) becomes

$$g'\Big(\sum_{k=1}^{n+1} s_k a_{kj}\Big) = \frac{n}{\sum_{l=1}^{n+1} r^{-1} - nr_j^{-1}}.$$

Consequently,

$$\begin{aligned}&\frac{n}{\sum_{l=1}^{n+1} r_l^{-1} - n_j^{-1}} - \frac{n-1}{\sum_{l=1}^{n} r_l^{-1} - (n-1)r_j^{-1}} \\ &= \frac{\sum_{l=1}^{n} + r_l^{-1} - (n-1)r_{n+1}^{-1}}{\big(\sum_{l=1}^{n+1} r_l^{-1} - nr_j^{-1}\big)\big(\sum_{l=1}^{n} r_l^{-1} - (n-1)r_j^{-1}\big)},\end{aligned}$$

which is positive if r_{n+1} is large, and is negative if r_{n+1} is small.

- **Proposition 2**: Every worker is subject to more (less) attack when a new entrant relatively more (less) talented in sabotage joins the organization.

An Example of Non-monotonicity

- Suppose a_{ij} and e_i can only be 0 or 1.
- ε_i can only be 1 or -1, with equal probability.
- $r_1 = 5$, $r_2 = 4$, $r_3 = 1$. $v(0) = 0$, $v(1) = 1$, $v(2) = 3$, $v(3) = 8$.
- $g(o) = 0$, $g(1) = 2.5$, $g(2) = 4$.

Appendix B

Table B1. Utility Levels of Member 1

e_1	a_{12}	a_{13}	W_{21}	W_{31}	$v(e_1+a_1)$	$u_1(a_1, e_1, a^*_{-1}, e^*_{-1})$
1	1	0	0.5	−1	3	$\frac{1}{4}u-3$
1	1	1	0.5	−3.5	8	$\frac{1}{4}u-8$
1	0	1	3	−3.5	3	−3
1	0	0	3	−1	1	−1
0	1	0	5.5	4	1	−1
0	1	1	5.5	1.5	3	−3
0	0	1	8	1.5	1	−1
0	0	0	8	4	0	0

Table B2. Utility Levels of Member 2

e_2	a_{21}	a_{23}	W_{12}	W_{32}	$v(e_2+a_2)$	$u_2(e_2, a_2; e^*_{-2}, a^*_{-2})$
1	1	0	−0.5	−1.5	3	$\frac{5}{8}u-3$
1	1	1	−0.5	−4	8	$\frac{3}{4}u-8$
1	0	1	1	−4	3	$\frac{1}{4}u-3$
1	0	0	1	−1.5	1	$\frac{1}{4}u-1$
0	1	0	3.5	2.5	1	−1
0	1	1	3.5	0	3	−3
0	0	1	5	0	1	−1
0	0	0	5	2.5	0	0

Table B3. Utility Levels of Member 3

e_3	a_{31}	a_{32}	W_{13}	W_{23}	$v(e_3+a_3)$	$u_3(e_3, a_3; e^*_{-3}, a^*_{-3})$
0	1	0	1	1.5	1	$\frac{1}{8}u-1$
0	1	1	1	0	3	$\frac{3}{16}u-3$
0	0	1	2.5	0	1	−1
0	0	0	2.5	1.5	0	0
1	1	0	0	0.5	3	$\frac{3}{16}u-3$
1	1	1	0	−1	8	$\frac{7}{16}u-8$
1	0	1	1.5	−1	3	$\frac{1}{4}u-3$
1	0	0	1.5	0.5	1	$\frac{1}{8}u-1$

- $e_1^* = e_2^* = 1$, $e_3^* = 0$, $a_{21}^* = a_{31}^* = a_{12}^* = 1$, $a_{13}^* = a_{23}^* = a_{32}^* = 0$ is a NE.
- However, $W_{21} = 0.5 > 0$.

Some Remedies

- Pay equality: Reducing the value of u.
- Seniority promotion system: Partially severing the link between promotion and W_i.
- Group incentives: One's pay depends partially on group performance.
- Early designation of successor: Successor is named before promotion occurs.
- External recruitment: The higher position has positive probability of being filled by an outsider.