A Dynamic Model of Auctions with Buy-Out: Theory and Evidences

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Preliminarily

- Purpose of paper: To propose a dynamic model of English auction with buy-out.
- Buy-out price: A price (set by the seller) at which the buyer can obtain the object immediately (at any time during the auction) by paying that price.
- Two possible explanations for the existence of buy-out price:
  1. (1) Time preference
  2. (2) Risk-aversion
- We adopt the second approach

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Ships to: United States

Item location: USA, United States

Quantity: 4 available

History: Purchases

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Seller Info

Seller (rating): **wholesale0607** (unrated) Send IM

Yahoo! Auctions Member since May 16, 2006

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Payment Types Accepted

- Accepts Cashiers Checks and Money Orders

Shipping Info

- **Seller Pays Shipping**
- Seller Ships on Payment
- Seller will ship internationally

Auction Info

Current Bid:	**$127.50** Quick Bid
Buy Price:	**$275.00** Buy It Now
Starting Price:	$100.00
High Bidder:	**goalieisone** (unrated)
# of Bids:	7 (Bid History)
Bid Increment:	$2.50
Time Left:	1d 10h 27m 10s (**Countdown Ticker**)
Opened:	May 28 11:10 AM
Closes:	May 31 09:10 AM
Available Qty:	1
Location:	Philadelphia, PA
ID #:	99740645

Notes:

- Seller can close auction early.
- Auction may get automatically extended.
- This auction has a buy price of $275.00.

Before you bid, review the seller's ratings and comments.

Learn more about auction safety.

Place a Bid

Minimum Bid: $130.00

Maximum Bid: $ USD

- Bid up to this amount on my behalf
- Bid this exact amount

Preview Bid

Need Help? View the Bidders' Guide

Want to Buy it Now?

Buy Price: **$275.00**

Buy It Now

Questions? View Buy Price Help

- Intuition:

1. The seller can exploit the buyer's aversion toward uncertainty, and make more profit by setting a buy-out price.
2. The seller is risk-averse himself, and buy-out serves as an instrument to avoid price risk.

- Questions to ask:

1. What is the optimal buy-out price of the seller?
2. What is the optimal bidding strategy of buyer?
3. What is the equilibrium out-come?

- Results:

(1) Optimal buy-out price is in inverse (direct) relation to the seller's (buyer's) degree of risk aversion.

(2) Seller's expected utility is higher with buy-out.

(3) Buyer's expected utility is lower with buy-out.

(4) Transaction price is higher with buy-out.

(5) Transaction price is in reverse relationship with the time it takes to reach (contrary to usual ascending bid auction).

Model

- Two bidders ($i = 1, 2$), one seller.
- The value of the object to bidders i is v_i.
- v_1 and v_2 are independently and uniformly drawn from $[0, \bar{v}]$.
- The utility of buyer i:
  $(v_i - p)^\alpha / \alpha$; where p is the price paid, and $\alpha \in (0, 1]$ is degree of risk-aversion.
- The utility of seller is x^β / β, where $\beta \in (0, 1]$.

Equilibrium Buy-out Strategy

- Let v_b be the buy-out price set by the seller.
- One result of the standard English auction remains true: The bidder will stay active as long as the prevailing price is lower than his valuation of the object.
- But when to buy out?
- Let $p(v)$ be the but-out strategy of the buyer. That is, a buyer with valuation v is willing to buy out the objective (by paying v_b) when the prevailing price is $p(v)$.

- $p(v)$ is decreasing in v.
- Let $v(p)$ be the inverse of $p(v)$: $v(p) = p^{-1}(v)$.
- $v(p)$ is decreasing in p.
- Suppose at price p, both bidders are still active.
- This implies $v_i \in [p, \bar{v}]$ for all i.
- Hasn't been bought-out by any bidder yet, implying $v_i \notin [v(p), \bar{v}]$, $i = 1, 2$.
- Thus the posterior of v_i is $UNI[p, v(p)]$.
- If the buyer (with valuation v) buys out the object, his utility is $u(v, v_b) = (v - v_b)^\alpha / \alpha$.

- If he waits until $p + dp$ to buy out, then there are three consequences (Figure 1):

(1) His opponent buys out during $[p, p + dp]$. This occurs with probability $-dv(p)/(v - p)$, and his utility is 0.

(2) His opponent drops out during $[p, p + dp]$. This occurs with probability $dp/(v - p)$, and his utility is $(v - p)^\alpha/\alpha$.

(3) None of the above, which occurs with probability $1 - (\frac{-dv(p)}{v-p} + \frac{dp}{v-p})$, and his utility is $(v - v_p)^\alpha/\alpha$.

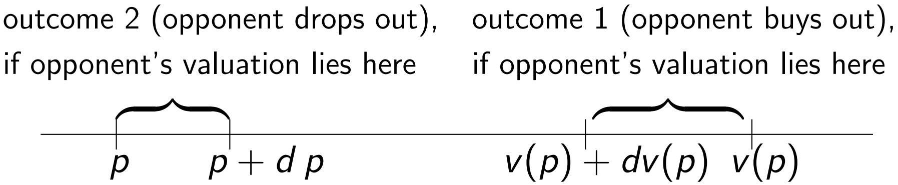


Figure 1: Possible outcomes of waiting.

- The total utility to buy out at $p + dp$ is thus

$$\frac{dp}{v-p}\frac{(v-p)^\alpha}{\alpha} + [1 + \frac{dv(p)}{v-p} - \frac{dp}{v-p}]\frac{(v-v_b)^\alpha}{\alpha}.$$

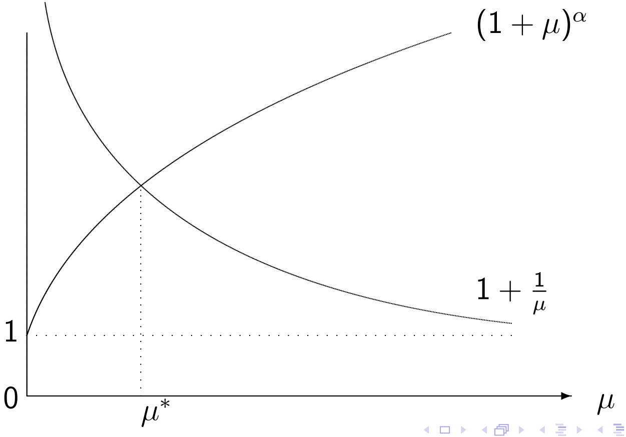
- Total change in utility by postponing buy-out from p to $p + dp$:

$$du = \frac{dp}{v-p}\frac{(v-p)^\alpha}{\alpha} + \frac{dv(p) - dp}{u-p}\frac{(v-v_b)^\alpha}{\alpha}.$$

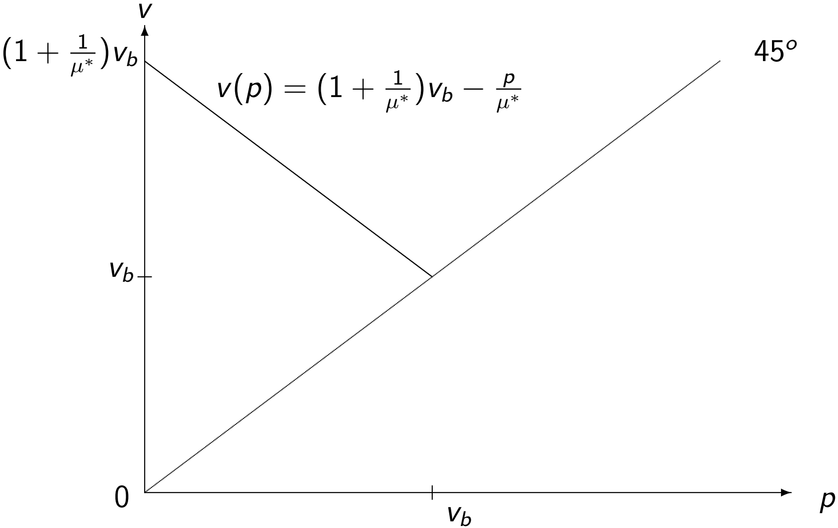
- $v(p)$ being optimal implies $\frac{du}{dp} = 0$:

$$(v-p)^\alpha - (v-v_b)^\alpha = -(v-v_b)^\alpha \frac{du}{dp}.$$

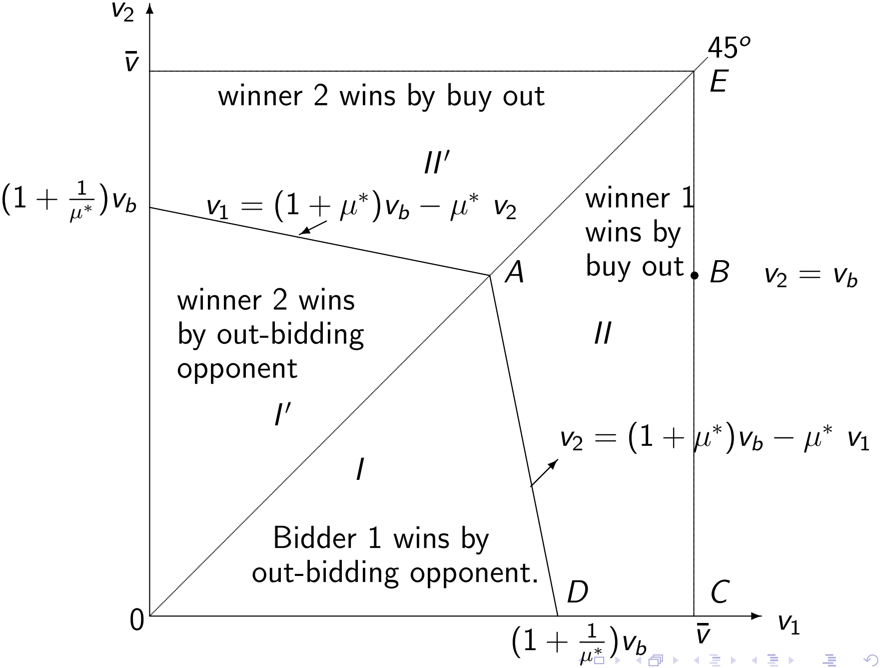
- Let $y = v - v_b$ and $x = v - p$, this becomes $(x+y)^\alpha - y^\alpha = y^\alpha \frac{du}{dp}$.
- Let $x = \mu y$, then this again becomes $(1+\frac{1}{\mu^*})^\alpha = 1 + \frac{1}{\mu^*}$, $\mu^* \in [1, \infty)$.



- We thus have $v(p) = (1 + \frac{1}{\mu^*})v_b - \frac{p}{\mu^*}$.
- $p(\mu) = (1 + \mu^*)v_b - \mu^* v$.



- We are interested in symmetric equilibrium, i.e., $p_1(v) = p_2(v) = p(v)$.
- In this case bidder 1 (2) wins if and only if $v_1 > v_2$ $(v_2 > v_1)$.
- Bidder i wins by competitive bidding if $v_i > v_j$ and $v_j < p(v_i)$.
- Bidder i wins by buy-out if $v_i > v_j$ and $v_j > p(v_i)$.



v_2
$\bar{v}$
45^o
E
winner 2 wins by buy out
II'
$(1+\frac{1}{\mu^*})v_b$
$v_1 = (1+\mu^*)v_b - \mu^*\ v_2$
winner 1 wins by buy out
A
B
$v_2 = v_b$
winner 2 wins by out-bidding opponent
II
I'
$v_2 = (1+\mu^*)v_b - \mu^*\ v_1$
I
Bidder 1 wins by out-bidding opponent.
D
C
0
$(1+\frac{1}{\mu^*})v_b$
$\bar{v}$
v_1

Optimal Buy-out Price

- Trade-off for seller in setting up buy-out price:
  Region OAD unchanged: sold with $p = v_2$
  Region AEB loses: $v_2 < v_b$
  Region ABCD gains: $v_2 > v_b$
- Expected utility of seller:

$$\pi(v_b) = \frac{v_b^\beta}{\beta}[1 - \frac{\beta(3+\beta)(1+u^*)}{(\beta+1)(\beta+2)u^*}(\frac{v_b}{\bar{v}})^2].$$

- FOC:

$$\frac{\partial \pi}{\partial v_b} = v_b^{\beta-1}[1 - \frac{(3+\beta)(1+u^*)}{(1+\beta)u^*}(\frac{v_b}{\bar{v}})^2] = 0$$

- $v_b^* = \sqrt{\frac{\mu^*(1+\beta)}{(1+\mu^*)(3+\beta)}}\bar{v}.$

(1) v_b^* is in inverse relation with α.

(2) v_b^* is in direct relation with β.

- Plugging v_b^* into $\pi(v_b^*)$ we get the utility of the seller is the function of β: $\pi(\beta)$.

- The expected utility of seller's without buy-out price
$$\pi^0(\beta) = \frac{2v^{-\beta}}{\beta(\beta+1)(\beta+2)}.$$

- $\pi(\beta) - \pi^0(\beta) \geq 0$; equality holds only if $\alpha = \beta = 1$.

- **Proposition 1**: If either buyer or seller is risk-averse, then
  (1) Expected price is higher with buy-out.
  (2) Expected utility of seller is higher with buy-out.
- **Proposition 2**: Buyer's expected utility is lower, unless he is close to risk-neutral and the seller is very risk-averse (in this case bidders with high valuations gain).

- Empirical Implications:

  (1) If we look at auctions of identical objects, but some with buy-out prices and some without, then the average transacted price is higher in the former.

  (2) For items that are sold, average transaction price is increasing in buy-out price.

  (3) Transaction price is in inverse relationship with the time it takes to be sold.
- Data: Digital cameras in Taiwan's Yahoo! auction site.
- Empirical Result: Confirmative

Table 1. Sample Distribution of Brands.

Brand Name	Number of Observations
BenQ	124
Canon	336
Casio	215
Fujifilm	407
Kodak	79
Konica	137
Kyocera	21
Nikon	315
Olympus	59
Panasonic	232
Pentax	177
Ricoh	28
Sanyo	52
Total	2,182

Table 2. Bidding Outcome

Total number of observations (2,171)				
Auction resulted in a sale (1,166)			Auction did not result in a sale (1,005)	
Auctions with buyout option (936)		Auctions without buyout option (230)	Auctions with buyout option (805)	Auctions without buyout option (200)
Transacted with buyout price (744)	Transacted with highest bid (192)			
Average transaction price: NT$9,674.874 Average buyout price: NT$9,674.874	Average transaction price: NT$6,293.33 Average buyout price: NT$7,859	Average transaction price: NT$6,594.9	Average buyout price: NT$10,963.02	

Table 3. Summary Statistics of Related Variables in Tobit Model

Variables	Definition	Mean	Std. Dev.	Min	Max
REP	Seller's reputation	461.054	963.958	-25	5806
NEW	A dummy variable with the value one if the item is new; zero otherwise.	.550	.498	0	1
BUYOUT	Buyout price	9.062	.581	4.605	11.086
BUYOUTD	A dummy variable with the value one if the auction has buyout option; zero otherwise.	.802	.399	0	1
MINIBID	Minimum bid	8.572	1.891	0	11.082
LENGTH	Length of auction in terms of the number of days	7.609	3.097	0	2.398
TRADE	A dummy variable with the value one if the auction results in a sale; zero otherwise.	.537	.499	0	1
PRICE	Transaction price	8.951	.695	0	11.082
	Number of Observations		2,171		

Table 4. Regression Results of Tobit Model

Independent Variable	Transaction Price Equation
Constant	8.1263***
	(.1072)
Buyout Dummy	.1006**
	(.0512)
Reputation	.0001***
	(.0000)
Length of Auction	-.0675***
	(.0092)
New Subject Dummy	.2828***
	(.0532)
Brand Dummy 1	.0562
	(.0838)
Brand Dummy 2	.6391***
	(.0575)
Brand Dummy 3	.5287***
	(.0623)
Brand Dummy 4	.2927***
	(.0612)

Table 4. Regression Results of Tobit Model (continued)

Independent Variable	Transaction Price Equation
Brand Dummy 5	.0975
	(.0699)
Brand Dummy 6	.6629***
	(.0592)
Brand Dummy 8	.5451***
	(.0570)
Brand Dummy 9	.4065***
	(.0558)
Brand Dummy 10	.6032***
	(.0597)
Brand Dummy 11	.4804***
	(.0643)
Brand Dummy 12	.3395***
	(.0629)
Brand Dummy 13	.6476***
	(.0641)
Number of Observations	2,171

Notes: * denotes significance at the 10% level,
** at the 5% level,*** at the 1% level.

Table 5. Summary Statistics of Related Variables in Sample Selection Model

Variables	Definition	Mean	Std. Dev.	Min	Max
REP	Seller's reputation	544.228	1054.446	-25	5806
NEW	A dummy variable with the value one if the item is new; zero otherwise.	.624	.485	0	1
BUYOUT	Buyout price	9.062	.581	4.605	11.086
BUYOUTD	A dummy variable with the value one if the auction has buyout option; zero otherwise.	1	0	0	1
MINIBID	Minimum bid	8.770	1.538	0	11.082
LENGTH	Length of auction in terms of the number of days	7.502	3.133	0	11
TRADE	A dummy variable with the value one if the auction results in a sale; zero otherwise.	.538	.499	0	1
PRICE	Transaction price	9.013	.613	4.605	11.082
	Number of Observations		1,741		

Table 6. Regression Results of Sample Section Model

Independent Variable	Transaction Price Equation	Selection Equation
Constant	.1398	3.5643***
	(.2257)	(.9813)
Buyout Price	.9734***	
	(.0250)	
Reputation	.0000*	.0003***
	(8.87e-06)	(.0000)
Length of Auction	-.0183**	-.1921***
	(.0073)	(.0155)
New Subject Dummy	.0623***	-.2331**
	(.0177)	(.0946)
Minimum Bid		-.2510*
		(.1289)
Brand Dummy 1	-.0113	.6642***
	(.0154)	(.0810)
Brand Dummy 2	.0934***	.2439*
	(.0153)	(.1356)
Brand Dummy 3	.0825***	.0310
	(.0149)	(.1524)

Table 6. Regression Results of Sample Section Model (continued)

Independent Variable	Transaction Price Equation	Selection Equation
Brand Dummy 4	.0457***	.1609
	(.0083)	(.1192)
Brand Dummy 5	.0397***	.1763
	(.0110)	(.0973)
Brand Dummy 6	.1081***	.2904
	(.0189)	(.1529)
Brand Dummy 8	.0941***	.1503
	(.0118)	(.1327)
Brand Dummy 9	.0794***	.6579***
	(.0127)	(.0482)
Brand Dummy 10	.0923***	.0893
	(.0145)	(.1628)
Brand Dummy 11	.0999***	.2250*
	(.0156)	(.1365)
Brand Dummy 12	.0876***	.4916***
	(.0172)	(.0462)
Brand Dummy 13	.0987***	.2163
	(.0197)	(.1473)
Number of Observations	1,713	1,713

Extension to n-Bidder Case

- In the 2-bidder case, when a bidder drops out, the auction ends.
- In a general n-bidder auction, when a bidder drops out, the remaining bidders will update their information.
- Information updating will lead to change in buy-out strategy.

- Theorem : In an n-bidder auction, the bidder's optimal but-out strategy is

$$[p_i(v) = (1 + \mu_i) - \mu_i v]_{i=2}^n; \qquad (1)$$

where $\mu_{i+1} > \mu_i$.

- That is, when the prevailing price is p and when these are still i active bidders in the auction, an ative bidder will buy out the item as soon as price reaches $p_i(v)$.

- The greater the number the bidders, the more willing a bidder is to buy-out (since $p_i(v) > p_{i+j}(u)$ for $j \geq 1$).
- When some bidders drop out, the remaining bidders respond with increasing the prevailing price at which they are willing to buy out.