A Dynamic Model of Auctions with Buy-Out: Theory and Evidence*

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Abstract

The paper considers a two-bidder ascending price auction with buy-out (or buy price). We characterize the symmetric optimal bidding strategy of the bidder and the optimal buy-out price of the seller, and show that the more risk-averse a buyer, the earlier he is willing to buy out the object. Moreover, the seller's optimal buy-out price is decreasing in his own degree of risk-aversion, and increasing in that of the buyer. Buy-out options are shown to benefit the seller at the cost of the bidders. The expected transaction price is higher in auctions with the buy-out option, and is increasing in buy-put price. Finally, contrary to the usual ascending price auctions, the longer it takes for an item to be sold, the lower is its transaction price. All the theoretical predictions are confirmed in the data we collect from Taiwan's Yahoo! auction site.

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1 Introduction

An interesting feature of the recent on-line bidding auction format which is absent from the traditional English auction is the existence of the buy-out option.[1] There are two main explanations of how a buy-out benefits the seller. The first is the seller's ability to exploit bidders' time preference (see, e.g., Mathews, 2004). Under this explanation, the bidder is impatient, and is willing to pay a higher price to obtain an objective immediately, rather than through a time-consuming bidding process. The seller can then set up a buy-out price to satisfy this need and thereby make more profit. The second explanation is that if the buyers are risk-averse, then they will be willing to buy the object with a high, but fixed, price rather than obtaining the object through the bidding process, which has the risk of either losing to other bidders or, even if they win, paying an uncertain price.[2]

Both explanations imply that if we compare the auctions without buy-out prices and those of identical objects but with buy-out prices, the average transaction price will be higher for the latter. Both reasonings are incomplete, because the same reasoning can also be applied to the seller. That is, not only buyers, but also the sellers can be impatient or risk-averse. In either case the sellers will be willing to set a lower buy-out price so that the objects can be sold earlier (if they are impatient) or at a fixed price (if they are risk averse). Thus the two mentioned explanations have implicitly assumed that it is the buyers, rather than the sellers, who are impatient or risk averse. When the seller is also

[1] See Lucking-Reiley (2000) and Bajari and Hortaçsu (2004) for general discussions on internet auctions.

[2] See, e.g., Budish and Takeyama (2001), Reynolds and Wooders (2004) and Mathews and Katzman (2006).

risk-averse, the function served by buy-out for the seller and its consequence needs further investigation.

In this paper we focus on how both the buyer's and the seller's attitudes toward risk affect the bidder's incentives to buy out, and the seller's optimal buy-out price in response to the bidders' strategies. We develop a dynamic model of English auction with two bidders who, at every prevailing price, need to decide whether to continue with bidding or to buy out. Either the buyer or the seller (or both) can be risk-averse. We solve for the optimal buy-out and bidding strategy of the bidders. This optimal strategy is in turn used to solve for the optimal buy-out price of the seller. Under the optimal strategy, the higher a bidder's valuation of the object, the earlier is he willing to buy out the object. Also, the optimal buy-out price is an increasing (decreasing) function of the bidders' (seller's) degree of risk-aversion. We also show that whether the auction will end with one bidder out-bidding his opponent or a buy-out depends on the configuration of the bidders' valuations of the object.

In our model, the buy-out price serves two purposes for the seller. First, if the buyers are risk-averse, it can be used as an instrument to exploit the buyer's aversion to risk, by forcing them to pay a premium in order to aviod the risks in the bidding process. Second, if the seller is risk-averse, it also serves to decrease price risk for the seller himself. This implies that even if the buyers are risk-neutral, the seller still has incentives to offer the buy-out option, not to make more profit, but to avoid the more risky outcome of the bidding process. Indeed, our result indicates that the only case in which the seller does not gain from a buy-out option is when both the buyers and sellers are risk-neutral. This is in contrast to Budish and Takeyama (2001), who show that the buy-out option is of value if and only if the buyers are risk-averse.

Mathews and Katzman (2006) also propose a theory with buy-out price. In their model, a bidder decides whether to buy out the item at the beginning of the auction, or not to buy out and enter the bidding process, and the choice is irreversible. In particular, it is impossible for a bidder to join the competitive bidding in the beginning, and to buy out the item half-way in the bidding process. Thus their model is the eBay-type temporary buy-out model. Reynolds and Wooders (2004) compare two formats of auctions with buy-out price, and find that when the bidders are risk-neutral, the eBay (temporary) and Yahoo! (permanent) types of auction with buy-out price have the same expected revenue for the seller. However, when the bidders are risk-averse, the Yahoo! version raises more revenue. Their model assumes constant absolute risk-averse bidders and general distribution of the bidder's valuation. Hidvegi et al. (2006) solve for the equilibrium for the n-bidder English auction with buy-outs, under general assumptions on the utility function and the distribution of bidder's valuations. All papers assume risk-neutral seller.

Our paper thus makes two theoretical contributions to the literature. First, it characterize the symmetric equilibrium of auctions with buy-out when both the buyers and the sellers are risk-averse. In particular, it provides a rationale for buy-out option even when the buyers are risk-neutral. Buy-out is shown to benefit the seller, but in general hurt the buyers. Second, we derive the optimal buy-out price for the seller, together with its relation to the degree of risk-aversion of both the bidder and the seller. Most importantly, our derivation is intuitive, showing clearly the cost and benefit of a buy-out option for the seller, and how he can design the buy-out price to balance its cost and benefit.

Besides the theoretical results, we also derive three empirical implications for auctions with buy-out: (i) The expected transaction price of an item will be higher in an auction with buy-out than an auction of the same item but without buy-out; (ii) fot auctions with buy-out, the expected transaction price is in direct relationship with the buy-out price; and (iii) the expected transaction price is in inverse relation with the time taken for an item to be sold. To verify these implications, we use data collected from Taiwan's Yahoo! on-line auctions and test these predictions. All three predictions are confirmed by the data. The third contribution of this paper is thus to identify certain new empirical regularities for auctions with buy-out, and to collect data to confirm these regularities. Our empirical results also support certain previously tested results. For example, good reputation of the seller increases both the probability an item is sold and its transaction price (see, e.g., Livingston, 2005).[3]

2 The Model

A risk-averse seller conducts an English auction to sell an object. Two bidders ($i = 1, 2$) are participating in the auction. The value of the object to bidder i, v_i, is his own private information, but is known to be independently and uniformly drawn from $[0, \bar{v}]$.

A bidder can either buy the object by out-bidding the other bidder, or by buying the objective with the buy-out price, v_b, set by the seller in the beginning of the auction. The

[3] Other empirical works of auctions with buy-out include Dodonova and Khoroshilov (2004), Kirkegaard and Overgaard (2004), and Wang et al. (2004).

utility function of a bidder with valuation v is

$$u(v, p) = (v - p)^{\alpha}/\alpha \tag{1}$$

if he buys the objective with price p, and is 0 if he does not buy it; where $0 < \alpha \leq 1$. The value of α denotes the bidder's reverse degree of relative risk-aversion. The smaller the value of α, the more risk-averse is the buyer. The utility function of the seller is assumed to be $\pi(p) = p^{\beta}/\beta$; where $\beta \in (0, 1]$ is the seller's reverse degree of relative risk-aversion. Similarly, the smaller the value of β, the more risk-averse is the seller.[4]

2.1 Equilibrium Buy-Out Strategy

In this subsection, we derive the optimal buy-out strategy of the bidder under a given buy-out price. Thus throughout this subsection, we assume that buy-out price is fixed at v_b.

Even with a buy-out price, a basic result of the standard English auction remains true: It is a dominant strategy for a bidder to stay active in the auction as long as the prevailing price is lower than his valuation of the object. The complication comes from the fact that, at every prevailing price, now he has the additional option to pre-empt his opponent by buying the objective immediately at the buy-out price v_b.

Given v_b, let $p(v)$ be the buy-out strategy of the bidder whose valuation of the objective is v. That is, a bidder who values the object at v is willing to buy out the object (by

[4] Our assumption on utility function thus implies constant relative risk-aversion. A recent paper (Chiappori, 2006) using the Italian Survey of Household Income and Wealth data set suggests that individuals do exhibit constant relative risk-aversion.

paying v_b) when the prevailing price reaches $p(v)$. Since the greater the value of v, the more willing is the bidder to obtain the object immediately by paying v_b, we know that $p(v)$ is a decreasing function. It turns out to be easier to work with the inverse function of $p(v)$. Let $v(p)$ be the inverse function of $p(v)$. It relates the prevailing price p with bidder's valuation v, who at p is just willing to buy out the objective. That is, a bidder with valuation $v(p)$ is just willing to obtain the object by paying the buy-out price, when the prevailing price reaches p. Similarly, if a bidder with valuation v is willing to buy out the object, when the prevailing price is p, then a bidder with valuation $v' > v$ will be even more willing to do so at that moment. This implies that $v(p)$ is a decreasing function.

Suppose that both bidders are still active at the moment when the prevailing price is p. This implies that the valuations of both bidders are greater than p, which in turn implies that the possible valuations of any bidder must be distributed on $[p, \bar{v}]$. Moreover, by definition of $v(p)$, any bidder with valuation $v > v(p)$ would have bought out the object before the price has risen to p. The fact that this object has not been bought out at price p implies that the bidder's possible valuations cannot lie in $(v(p), \bar{v}]$. As a result, both bidders' valuations must lie in $[p, v(p)]$. In other words, if both bidders are still active when the prevailing price is p, then (by Bayes rule) any bidder's possible valuations of the object must be distributed uniformly on $[p, v(p)]$.

Consider the decision of a bidder (whose valuation is v) at the moment when the prevailing price is $p < v$. If he buys the object immediately with buy-out price v_b, his utility will be $u(v, v_b) = (v - v_b)^\alpha/\alpha$. If instead he holds out and waits until price is $p + dp$ to buy out the object, then he will face three possible outcomes. First, his opponent buys out the object while he waits. Second, his opponent drops out between p and $p + dp$. Third, neither of the above happens so that he eventually buys out the object when the prevailing price is $p + dp$. Whether the bidder should buy the object immediately (by paying v_b), or waits until $p + dp$, depends on the difference of the utility between an immediate buy-out and the combined expected utility under the three possible outcomes of waiting until $p + dp$.

Figure 1 depicts the possible intervals at which outcomes 1 and 2 occur. When the valuation of the bidder's opponent lies in $[v(p) + dv(p), v(p)]$,[5] then his opponent will buy out the object while he waits. This is the first outcome we mentioned above, which occurs with probability $\frac{-dv(p)}{v-p}$, and his utility is 0. Similarly, if his opponent's valuation lies in $[p, p + dp]$, then his opponent will drop out while he waits, and he will win the bidding with price p. This is the second outcome mentioned, which occurs with probability $\frac{dp}{v-p}$, and his utility is $(v - p)^\alpha/\alpha$. Under the third outcome, which occurs with probability $1 - \left(-\frac{dv(p)}{v-p} + \frac{dp}{v-p}\right)$, his utility is $(v - v_b)^\alpha/\alpha$.

Figure 1: Possible outcomes of waiting.

[5] Since $dv(p) = v'(p)dp$ and $v'(p) < 0$, $dv(p) < 0$.

The total expected utility of waiting until $p + dp$ to buy out is thus

$$\frac{dp}{v-p}\frac{(v-p)^\alpha}{\alpha} + \left[1 + \frac{dv(p)}{v-p} - \frac{d\,p}{v-p}\right]\frac{(v-v_b)^\alpha}{\alpha}. \tag{2}$$

The total change in utility of waiting until $p + d\,p$ to buy out, instead of buying out now, is

$$du = \frac{dp}{v-p}\frac{(v-p)^\alpha}{\alpha} + \frac{dv(p) - dp}{v-p}\frac{(v-v_b)^\alpha}{\alpha}. \tag{3}$$

For the function $v(p)$ to be the optimal buy-out strategy, it must be the case that $\frac{du}{dp} = 0$, i.e., the first-order condition must hold at any p. This implies that

$$(v-p)^\alpha - (v-v_b)^\alpha = -(v-v_b)^\alpha\,\frac{dv}{dp}. \tag{4}$$

Let $y = v - v_b$ and $x = v_b - p$, then $\frac{dv}{dp} = -\frac{dy}{dx}$, and equation (4) becomes

$$(x+y)^\alpha - y^\alpha = y^\alpha\frac{dy}{dx}. \tag{5}$$

It is difficult to directly solve for equation (5), but the boundary condition and the fact that (5) is homogeneous of degree α on both sides supply a clue. Since $v(v_b) = v_b$,[6] the solution of (5) must pass through $(x, y) = (0, 0)$. We then conjecture that the solution of (5) is linear. Let $x = \mu y$, then (5) becomes

$$(1+\mu)^\alpha = 1 + \frac{1}{\mu}. \tag{6}$$

Denote μ^* as the solution of (6). Figure 2 then depicts how μ^* is determined. It can

[6] If the current price is v_b, and buy out price is v_b, then it must be optimal to buy out immediately.

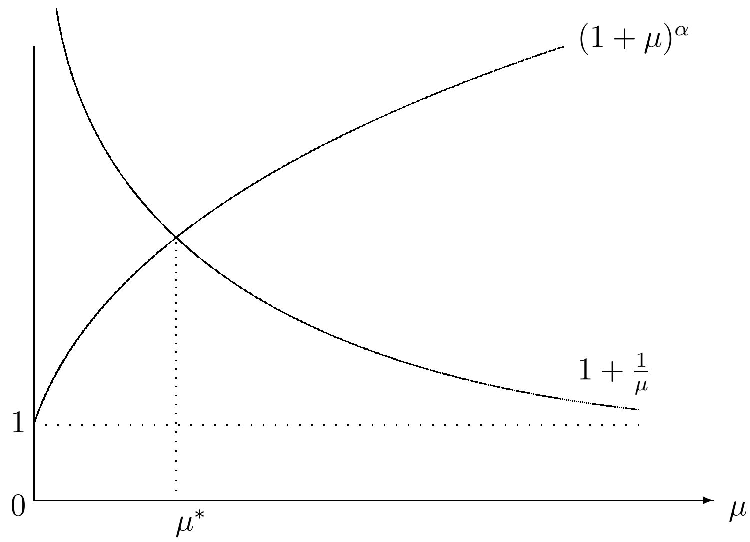

Figure 2: Determination of μ^*.

be shown easily that $\mu^* \geq 1$ and that μ^* is decreasing in α. In particular, $\mu^* = 1$ when the buyers are risk neutral ($\alpha = 1$). We thus have $x = \mu^* \, y$. Substituting for $y = v - v_b$ and $x = v_b - p$ we eventually have

$$v(p) = (1 + \frac{1}{\mu^*})v_b - \frac{p}{\mu^*}. \tag{7}$$

Solving for the inverse of the function $v(p)$ we have

$$p(v) = (1 + \mu^*)v_b - \mu^* \, v. \tag{8}$$

The function $p(v)$ is exactly the optimal buy-out strategy of the bidder. It shows that a bidder, whose valuation of the object is v, will be willing to buy out the object (by paying v_b) when the prevailing price reaches $(1 + \mu^*)v_b - \mu^* v$. Given the optimal buy-out policy, the optimal strategy of the bidder with valuation v is then easy to describe: Stay active as long as the prevailing price is lower than $p(v)$, and buy out the object when price reaches $p(v)$. Note that since a bidder will consider buying out only if $v > v_b$, we know $v - p(v) = (1 + \mu^*)(v - v_b) > 0$. That is, if a bidder will buy out the object, then he will do so before the price reaches his valuation. This also implies that the transaction price cannot be higher than v_b. In other words, by setting v_b as the buy-out price, the seller essentially sets v_b as the upper-bound for the possible transaction prices.

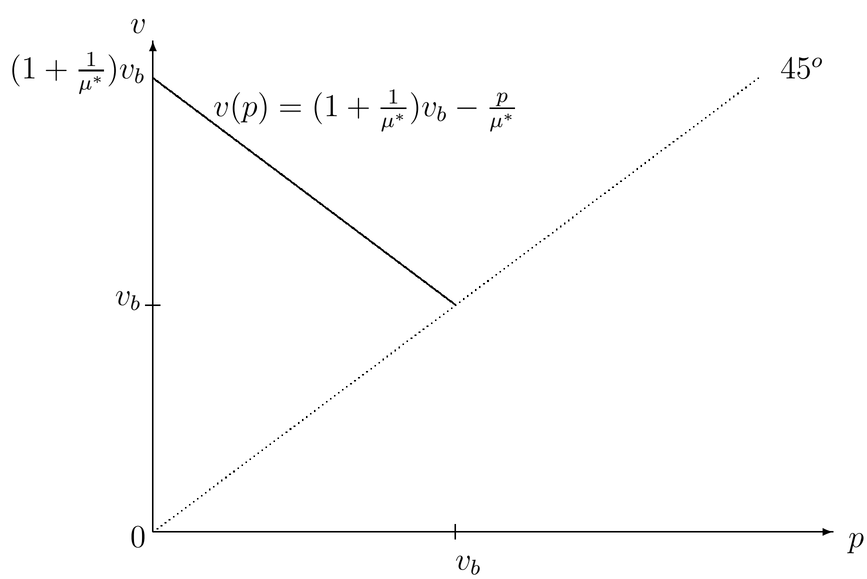

Figure 3: Buyer's optimal buy-out policy.

The graph of $v(p)$ is depicted in Figure 3. It visualizes the relation between a bidder's valuation and the prevailing price at which he wants to buy out. The higher a bidder's valuation for the objective, the lower is the prevailing price at which he is willing to buy it out. In particular, if his valuation $v \geq (1 + \frac{1}{\mu^*})v_b$, then his valuation is so high that he is willing to buy out the objective right at the beginning of the auction (i.e., when $p = 0$). Moreover, since μ^* is decreasing in α, it implies that the more risk-averse a buyer, the earlier is he willing to buy out the object. This result is fairly intuitive. The more risk-averse a bidder, the less willing is he to face the uncertain outcome of the bidding process. Thus he is more willing to buy it out early. (Note that to buy-out early is costly, as the expected gain from the bidding process is still high.) The solution $v(p)$ also makes it possible to characterize the outcomes of the auction as a function of v_1 and v_2. Note that by the symmetric nature of the equilibrium, a bidder will win if and only if his valuation of the object is greater than his opponent's. The question is only whether he will win by out-bidding his opponent or by direct buy-out. Since the line $v(p) = (1 + \frac{1}{\mu^*})v_b - \frac{p}{\mu^*}$ characterizes the relation between a bidder's valuation and the prevailing price at which he is willing to buy out, bidder i will win by bidding if and only if $v_i > v_j$ and $v_i < (1 + \frac{1}{\mu^*})v_b - \frac{v_j}{\mu^*}$. On the other hand, i will win by buy-out if and only if $v_i > v_j$ and $v_i > (1 + \frac{1}{\mu^*})v_b - \frac{v_j}{\mu^*}$. We can thus characterize the outcomes of the auction as a function of the bidders' valuations of the item in Figure 4. In the figure, regions I and I' depict the case in which the winner wins by out-bidding his opponent. In regions II and II', the winner obtains the object by buy-out.

A technical problem occurs when the valuation of every bidder is greater than $(1 + \frac{1}{\mu^*})v_b$. In that case both will want to buy out at beginning of the auction; i.e., when the prevailing price is 0. A reasonable assumption to make is to assume that every bidder wins with probability 1/2. Therefore, there will be a mass of bidders (specifically, those

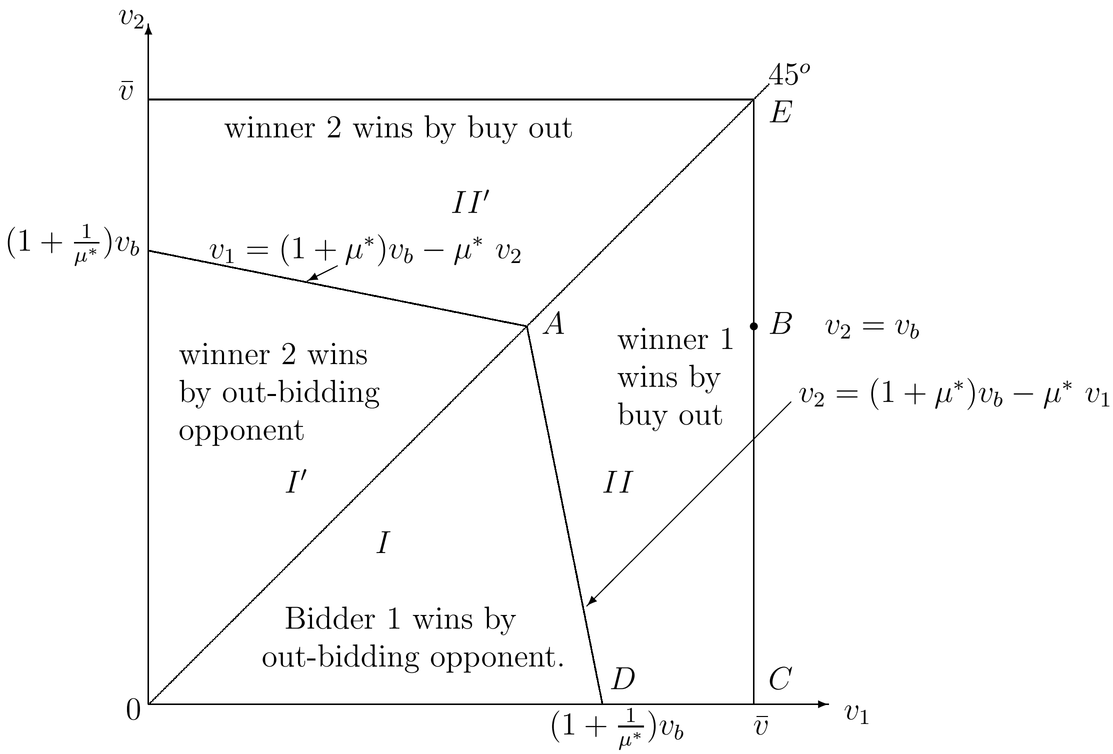

Figure 4: Outcomes of bidding as a function of bidders' valuations.

whose valuations of the item are higher than $(1+\frac{1}{\mu^*})v_b$) who will buy out the object when the bidding price is still 0. This will create a discontinuity in the expected utility for bidders whose valuations are just below $(1+\frac{1}{\mu^*})v_b$. The reason is that according to the optimal strategy derived, a bidder whose valuation is slightly lower than $(1+\frac{1}{\mu^*})v_b$ will wait until the price is slightly higher than 0 to buy out. In that case he will lose for sure if his opponent's valuation is greater than his. If, however, rather than wait until price is slightly above 0, he buys out the item immediately (when prevailing price is 0), then his chance to win, when his opponent's valuation is greater than his, will surge from 0 to 1/2.

Thus, the assumption that a bidder's winning chance is 1/2, when both propose to buy out when price is 0, creates a jump in expected utility for those bidders whose valuations are sufficiently close to $(1 + \frac{1}{\mu^*})v_b$ if they buy immediately rather than follow the strategy $p(v)$. As a result, they will deviate instead of following the strategy $p(v)$. This prevents $p(v)$ from being the optimum for bidders whose valuations are close to $(1 + \frac{1}{\mu^*})v_b$.

To overcome this technical problem, we will make the following strong assumption: Whenever two bidders propose to buy out at the beginning of the auction, the bidder with higher valuation will win. Although a strong assumption, it has certain justification. In an on-line auction, an item is put up for auction at a much longer time span than the traditional English auction.[7] Moreover, a bidder needs not be present during the whole auction process. A bidder can enter any time to bid as long as the item is still open. That means a bidder might miss the chance even if he is willing to buy out when the prevailing price is 0, as he might be absent. A bidder with higher valuation, being one having greater surplus from buying the item, is then more alert to stay on-line searching for the item. Therefore, he is more likely to be present when auction of the item in question starts, and thus has greater chance to buy out. Our assumption essentially says that this advantage for the higher valuation bidder is absolute, in that he wins with probability 1. If this assumption is made, then the discontinuity in expected payoff mentioned above will cease to exist, as a bidder wins if and only if his valuation is greater, even if both bidders propose to buy out at price 0. Consequently, $v(p)$ is indeed the optimal buy-out strategy.[8]

[7] In Taiwan's Yahoo! auction site, it can be from 1 to 10 days.

[8] We also analytically solve for the case in which the winning chance is 1/2 for each bidder when

The strategic effect of a buy-out option on the seller's revenue can be seen very clearly in Figure 4. Take the case when buyer 1 eventually wins (i.e., the region OEC). Without a buy-out option, bidder 1 will win by paying bidder 2's valuation, v_2. With a buy-out, there are three types of outcomes to consider. First, the outcomes in region OAD are the same as the case without buy-out: Bidder 1 wins by paying bidder 2's valuation v_2. Second, in region ABCD, bidder 1 wins by paying the buy-out price v_b. Note that in this region $v_2 < v_b$. What would have been sold with price v_2 is now sold with a higher price v_b. The seller thus gains by setting up a buy-out price in this region. Third, in region ABE, bidder 1 pays the buy-out price, v_b, to win the item, but now $v_b < v_2$. This is the region in which the seller actually loses with the buy-out option. The optimal buy-out price of the seller must thus balance the latter two types of outcomes; that is, to maximize the expected revenue from region ABCD net of the expected loss from region ABE.

Figure 4 also shows clearly that a buy-out option reduces the risk that both the buyers and seller face in the bidding process. Again, consider the case in which bidder 1 eventually wins, i.e., region OEC. In region OAD, the outcomes of bidding (and thus the uncertainty faced by both) are the same regardless of whether there is a buy-out option, since in both cases bidder 1 wins by paying bidder 2's valuation v_2. In region AECD, if there is no buy-out option, bidder 1 will win by paying bidder 2's valuation v_2, which is

both propose to buy out in the beginning. But since there exists no close-form solution, the comparative statics derivation and price comparison become extremely complicated and burdensome. We thus use simulation to check for the properties that we derive in Section 2 of the paper. All the results go through. These simulation results can be downloaded from the following website:
www.sinica.edu.tw/~kongpin/auctionsimulation.nb. The file must be viewed with a Mathematica software. Please contact the corresponding author for a pdf printout file. The file however, is truncated, as the simulation results are too wide to be contained in a letter size paper.

uncertain. However, with a buy-out option, bidder 1 will win by paying a fixed price v_b. Obviously, the price risk faced by both the buyers and the seller is reduced by the buy-out option.

2.2 Optimal Buy-Out Price

Given the outcomes of the bidding process depicted in Figure 4, it is straightforward to compute the expected utility of the seller under any buy-out price v_b:

$$\pi(v_b) = \frac{2}{\bar{v}^2}\Bigg\{\int_0^{v_b}\int_0^{v_1}\frac{v_2^\beta}{\beta}dv_2dv_1 + \int_{v_b}^{(1+\frac{1}{\mu^*})v_b}\int_0^{(1+\mu^*)v_b-\mu^*v_1}\frac{v_2^\beta}{\beta}dv_2dv_1 + \frac{v_b^\beta}{\beta}\left[\frac{\bar{v}^2}{2}-\frac{v_b}{2}\left(1+\frac{1}{\mu^*}\right)v_b\right]\Bigg\}; \tag{9}$$

where the first two terms in the braces are profits from region I, and the third term is that from region II. $\pi(v_b)$ can be shown to be equal to

$$\frac{v_b^\beta}{\beta}\left[1-\frac{\beta(3+\beta)(1+\mu^*)}{(\beta+1)(\beta+2)\mu^*}\cdot\left(\frac{v_b}{\bar{v}}\right)^2\right]. \tag{10}$$

The seller chooses the values of v_b to maximize $\pi(v_b)$. The first-order condition for v_b is

$$\frac{\partial\pi}{\partial v_b} = v_b^{\beta-1}\left[1-\frac{(3+\beta)(1+\mu^*)}{(\beta+1)\mu^*}\left(\frac{v_b}{\bar{v}}\right)^2\right] = 0. \tag{11}$$

This implies that $v_b^* = \sqrt{\frac{\mu^*(1+\beta)}{(1+\mu^*)(3+\beta)}}\bar{v}$. By plugging v_b^* into (10), we can compute the expected utility of the seller under the optimal buy-out price to be

$$\pi(\beta) \equiv \frac{2}{\beta(2+\beta)}\left(\sqrt{\frac{\mu^*(1+\beta)}{(1+\mu^*)(3+\beta)}}\,\bar{v}\right)^\beta. \tag{12}$$

On the other hand, the expected utility of the seller without buy-out option is

$$\pi^0(\beta) \equiv \frac{2}{\bar{v}^2} \int_0^{\bar{v}} \int_0^{v_1} \frac{v_2^\beta}{\beta} dv_2 dv_1 = \frac{2\bar{v}^\beta}{\beta(\beta+1)(\beta+2)}.$$

The difference in expected utility is thus

$$\pi(\beta) - \pi^0(\beta) = \frac{2\bar{v}^\beta}{\beta(\beta+1)(\beta+2)} \left[\left(\frac{\mu^*}{1+\mu^*} \right)^{\frac{\beta}{2}} (1+\beta)^{\frac{\beta}{2}+1} (3+\beta)^{-\frac{\beta}{2}} - 1 \right]. \qquad (13)$$

Let $\Phi(\mu^*, \beta)$ be the term in the brackets of (13). It is easy to see that Φ is increasing in μ^*. Moreover, $\Phi(1, \beta) = (\frac{1}{2})^\beta (1+\beta)^{\frac{\beta}{2}+1} (3+\beta)^{-\frac{\beta}{2}} - 1$, which is increasing in β initially, then decreasing in β. Note that $\Phi(1, 0) = \Phi(1, 1) = 0$, implying that $\Phi(1, \beta) \geq 0$ for all $\beta \in (0, 1]$. By the fact that Φ is an increasing function of μ^*, we know that $\Phi(\mu^*, \beta) \geq 0$ for all $\beta \in (0, 1]$ and $\mu^* \geq 1$. That is, the expected utility of the seller is always greater with the buy-out option. Moreover, $\Phi(\mu^*, \beta) = 0$ only if $\mu^* = 1$ (i.e., $\alpha = 1$) and $\beta = 1$, meaning that the expected utility of the seller is strictly higher with buy-out unless both agents are risk-neutral. We thus have the following proposition.

Proposition 1. *If both the buyers and the seller are risk-neutral, then the seller's expected revenues are the same in auctions with and without buy-out. If either the seller or the buyer is risk-averse, the seller's expected utility is strictly higher with buy-out.*

Next we compare the expected utility of the bidder between cases with and without buy-outs. We will show that although the seller gains from buy-out, unless the bidder has a very high valuation of an item and the seller is not very risk-averse; otherwise the bidder is worse off with the buy-out option. This is summarized in the next proposition.

Proposition 2. *Consider a bidder with valuation v. He has higher expected utility with buy-out if and only if $v/\bar{v} > v_c(\alpha, \beta) \equiv \sqrt{(\frac{\mu}{1+\mu})(\frac{1+\beta}{3+\beta})}/(1-(\alpha+1)^{\frac{-1}{\alpha}})$, which is greater than $(1+\frac{1}{\mu^*})v_b$.*

Proof: See Appendix B.

Proposition 2 shows that unless a bidder has very high valuation, he is worse off with the buy-out option. Note that if α is close enough to 1 and β is close enough 0 (i.e., if the seller is sufficiently risk-averse and the bidder is close to risk-neutral), then $v_c(\alpha, \beta) > 1$, implying that the inequality in Propostion 2 cannot hold. That is, regardless of their valuations, the bidders must be worse off under buy-out. Although the buy-out option might seem to help the bidders by offering them an option to buy the item with a more predictable transaction price, it actually serves as the seller's instrument to increase the competition between the bidders. With the buy-out option, the bidders not only have to compete in the bidding process, but have to compete in buy-out. The seller thus extracts more rent (Proposition 1) at the bidders' expense in the auction. Recall (see Figure 4) that buy-out price actually helps the bidders with high valuations, because it enables them to buy the items at the buy-out price rather than risk bidding into very high price. Consequently, a bidder who has very high valuation will have a higher expected utility in the case with buy-out (Proposition 2). Note that $v_c(\alpha, \beta)$ is increasing in β, implying that the more risk-averse the seller, the more likely the bidder will gain from buy-out.

There are other properties of the optimal buy-out price which deserve to be discussed. First, since v_b^* is an increasing function of μ^*, which is in turn decreasing in α, we know that the optimal buy-out price is increasing in the degree of the buyer's degree of risk-aversion. This is an intuitive result, since one purpose of setting up a buy-out price is to make more profit by exploiting the aversion of bidders to the uncertainty of whether he will win or, if he wins, the uncertainty of price he needs to pay. What is surprising is that even if the bidders are risk-neutral, there is still incentive for the seller to set a non-trivial buy-out price (i.e., a buy-out price lower than $\bar{v}$). This can be seen clearly from the fact that when $\alpha = 1$, $\mu^* = 1$, $v_b^* = \sqrt{\frac{1+\beta}{2(3+\beta)}}\bar{v} < \bar{v}$ and equation (13) is strictly positive.[9] This is in contrast to the conventional wisdom that the reason for the buy-out price is to satisfy the bidder's desire to avoid risks.[10] Again, the intuition for this is actually quite clear. In the case when both buyer and seller are risk-averse, the buy-out price serves two purposes for the seller. First, it can be used to exploit the bidder's aversion to risk and increase the seller's revenue. Second, it can also be used as a way to avoid risk for the seller. Therefore, even if the buyers are risk-neutral, the seller still has incentives to evoke the buy-out option, not to increase revenue, but to reduce his own risk.

Also note that v_b^* is increasing in β, meaning that the optimal buy-out price is decreasing in the seller's degree of risk-aversion. The reason behind this is transparent. The more risk-averse the seller, the more he abhors the uncertainty brought about by the result of the competitive bidding between the buyers. He is then more willing to set up a lower, but fixed and certain, buy-out price to avoid price risk.

[9] If the seller sets $v_b > \bar{v}$, then the buy-out price is redundant.

[10] For example, in Budish and Takeyama (2001), a buy-out price benefits the buyer only if the buyers are risk neutral. Since the seller can always set up an impossibly high buy-out price to make the auction equivalent to one without buy-out price, this implies that only if the buyers are risk-neutral will the buy-out price have any function.

2.3 Empirical Implications

In this subsection, we derive three empirical implications from our theoretical model. First, we compare the expected transaction price of an item between auctions with and without buy-out. The expected transaction price in the case without buy-out can be easily computed to be $\bar{v}/3$. The average transaction price, when buy-out price is v_b, is

$$\begin{aligned}
&\frac{2}{\bar{v}^2}[\int_0^{v_b}\int_0^{v_1} v_2 dv_2 dv_1 + \int_{v_b}^{(1+\frac{1}{\mu^*})v_b}\int_0^{(1+\mu^*)v_b-\mu^* v_1} v_2 dv_2 dv_1 + \frac{v_b}{2\mu^*}(\mu^*\bar{v}^2 - (1+\mu^*)v_b^2)] \\
&= 2[\frac{v_b}{2} - \frac{(1+\mu^*)v_b^3}{3\mu^*\bar{v}^2}]. \qquad (14)
\end{aligned}$$

The difference in expected transaction prices under optimal buy-out price, v_b^*, is thus

$$\frac{1}{3}[\bar{v} + \left(\frac{2(1+\beta)}{3+\beta} - 3\right) v_b^*]. \qquad (15)$$

As a result, the expected transaction price is greater with buy-out option if and only if

$$\mu^* > \frac{(3+\beta)^3}{6\beta(6+\beta)+22}. \qquad (16)$$

Note that $\mu^* \in [1, \infty)$ and $(3+\beta)^3/[6\beta(6+\beta)+22] \in (1, 27/22]$. That means the expected transaction price without buy-out can be greater only if μ^* falls in the narrow interval $[1, 27/22]$. (Even in this case the average transaction price with buy-out still has good chance to be greater). Consequently, for reasonable assumptions on the values of α and β, we will expect the transaction price to be greater for auctions having buy-out options than for ones without.[11] That is, unless in the extreme case when α is very close to 1 and

[11] For example, assuming that both α and β are uniformly distributed on $(0, 1]$, then we can show that the probability that $\mu^* \geq 27/22$ (i.e., $\mu^* > (3+\beta)^3/[6\beta(6+\beta)+22]$ for *all* possible values of β) is 0.744.

β very close to 0 (i.e., the buyer is almost risk-neutral and the seller is very risk-averse), otherwise the transaction price is greater when there is buy-out option.[12] Since in the on-line auctions whether to set up a buy-out price is the option of the seller, if we look at auctions with identical objects, there will be ones that go with buy-out prices and those go without. The empirical implication for this fact and our discussions above is that, the average transaction price for items sold under buy-out (but not necessarily sold with buy-out price) will be greater than those without.

Second, by plugging the optimal value of v_b into (14), we can easily show that the transaction price is an increasing function of β. That means as the seller becomes more risk-averse, both the transaction price and the optimal buy-out price will be lower. This result has a strong empirical implication. If we assume that different sellers have different degrees of risk-aversion, but they face the same pool of potential buyers, then different sellers will have different buy-out prices only if they differ in degree of risk-aversion. However, since both optimal buy-out price and average transaction price are decreasing in seller's degree of risk-aversion, if we sample only those items which are sold without a reservation price (so that all items are sold), then the average transaction price will be in direct relationship with the buy-out price.

Third, for those items that come with buy-out options, we can also compare the average transaction price for items which are eventually sold under buy-out (regions II

[12] Note that when $\beta = 1$, the average transaction price is exactly the expected utility of the seller, $\pi(1)$. Although Proposition 1 shows that $\pi(1) \geq \pi^0(1)$, this does not prove that the transaction price is always higher with buy-out. It only shows that average transaction price is higher with buy-out *when the seller is risk-neutral*. In order to make the appropriate comparison, we have to do it for the general case when β is not necessarily 1.

and II') and those that are sold under competitive bidding (regions I and I'). It is obvious from the figure that the average price is higher in the former case. This fact has a strong empirical implication. Note that in an on-line auction, the length of time an item is put up for sale is fixed in advanced by the seller. The only possibility that the object is sold by a time shorter than this period is if someone buys it out. But since we already know that the average transaction pirce is higher for items that are sold under buy-out, there should be an *inverse* relationship between the transaction price of an item and the time it takes to be sold. This is a fact that is in contrast to a usual ascending price auction.[13]

There are thus three predictions from our model. First, the average transaction price should be higher for auctions with buy-out than without. Second, the average transaction price is in positive relation with buy-out price. Third, the average transaction price of an item is in inverse relation with the time it takes to be sold. In section 3, we will empirically test the three predictions.

3 Empirical Study

In this section, we perform two empirical tests to examine the three implications of our theoretical model. Our first test investigates whether the average transaction price in auctions with buy-out options is higher than that without the option. The second test investigates whether in auctions with buy-out options, the average transaction price is

[13] Another possible explanation is that, for popular items, the sellers are more confident that they can be sold more quickly with higher prices. As a result, they choose to list the items for a shorter number of days. We thanks Hideo Owan for suggesting this to us.

increasing in buy-out price. They need to be tested separately mainly because they have different sample sizes. In both we also test our third implication, namely whether the transaction price of an item is in inverse relationship to the time it takes to be sold. All implications are confirmed using Taiwan's Yahoo! on-line auction data of digital cameras. We also subject these results to a number of robustness checks. First, we eliminate the explanatory variables except one at a time, and go through the same estimation procedure. Second, we drop all the dummy variables and run the same estimation. Third, we consider the impact of different types of digital camera by specifying each type of the top three brands digital camera (Nikon, Canon and Fujifilm),using types, rather than brands, as the dummy variables (after dropping one as the base type) and re-estimate the two models. Finally, we run our tests with only a single type of camera, Fijifilm F4, which has the largest number of observations in the whole sample (141). None of these alternative specifications changes our results qualitatively.

3.1 Data Description and Variable Specification

We collect data from Taiwan's Yahoo! auction site during the period from April 1, 2005 to July 1, 2005.[14] Our data contains 2182 observations (items) for the auction of digital cameras. There are a total of 13 brands. Table 1 lists the sample distribution of these brands. Among these observations, we find that there are 11 observations whose buy-out prices (averaged at NT$ 111,253.9)[15] are well above the average of the rest of the

[14] Taiwan's Yahoo! is currently the largest on-line auction site in Taiwan in terms of sales revenue.
[15] 1 U.S. dollar roughly equals 30 NT$ during this study period.

observations (NT$ 10,070.3) to be credible buy-out prices.[16] We thus delete these outliners and have a total of 2171 observations.[17]

Table 2 displays the bidding outcomes of our sample. Among the 2171 items, 1166 result in a sale, and 1005 items remain unsold. Furthermore, among the 1166 (1005) items that are eventually sold (remain unsold), 936 (805) come with buy-out options, and 230 (200) do not. Thus, more than 80% of the items in our data are listed with the buy-out option. For the 936 items (with the buy-out option) that result in a sale, 744 are sold at buy-out prices, and the average transaction price equals to NT$ 9,674.874 (which is also the average buy-out price); the other 192 items are sold to the highest bidders. The average transaction price in the latter case is NT$ 6,293.33. (The average buy-out price for these items is NT$ 7,859.) For the 230 items that are sold but without buy-out options, the average transaction price is NT$ 6,594.9. Finally, the average buy-out price for the 805 items that remain unsold is NT$ 10,963.02.

Our theoretical model assumes zero reservation price, thus the auction will always result in a sale. But in reality there exists a non-zero probability that the auction fails to reach a deal if the amount of the highest bid is less than the reservation price. In such a case, we do not observe a transaction price. Therefore, a sample selection model

[16] These outliers include Canon (6), Pentax (2), Casio (1), Fujifilm (1) and Nikon (1) with the number of items shown in parentheses.

[17] Three out of these 11 outliers resulted in sale. We also included these three observation in our estimation procedure. We found no signicant change in our empirical results.

is adopted for estimation, and is specified as follows.[18]

$$P_j^* = x_j\beta + \mu_j, \tag{17}$$

$$W_j^* = z_j\alpha + v_j, \tag{18}$$

$$P_j = \begin{cases} P_j^* \ \text{ if } \ W_j^* > M_j, \ j = 1, 2, ..., n. \\ M_j \ \text{ if } \ W_j^* \leq M_j, \end{cases} \tag{19}$$

We call (17) the regression equation, and (18) the selection equation. In the specifications, P_j^* is the value of the transaction price for item j, and W_j^* is the bidding amount. Since some items are not sold, we use P_j to denote the transaction price (P_j^*) if an item is sold, and the minimum bid for that item, M_j, if it is not. That is, minimum bid is used as a proxy for the unobserved reservation price, even though in some cases the fact that a winner's bid is greater than the minimum bid does not necessarily means that these a sale. (μ_j, v_j) are assumed to be iid normal with zero mean and variances σ^2 and 1, respectively. Assume their correlation is ρ. Also, P_j^* and W_j^* are assumed to be a linear function of observed variables x and z, respectively. The empirical model of sample selection contains a correlation term ρ. As long as ρ is not zero, OLS is biased. We thus use full information maximum likelihood to estimate the sample selection model. As a result, the estimates are consistent and asymptotically efficient.

In our regressions, the dependent variables are whether the auction results in a sale or not (TRADE), and the transaction price (PRICE) if the item is sold. The former takes a

[18] See Livingston (2005) for the reasons for using the sample selection model rather than the censoring model such as the Tobit model. In his paper, however, the purpose is to study the effects of a seller's reputation on bidders' participation decisions and on the decision of how much to bid.

value of one if the auction ends up with a deal, and is zero otherwise. The latter is equal to the buy-out price if it is sold with buy-out; otherwise it is the amount of winning bid.[19] For the explanatory variables, we include a number of controls (depending on which of the two tests we run) such as BUYOUT (buy-out price), BUYOUTD (a dummy variable which equals one if the auction has buy-out option; and is zero otherwise), LENGTH (the length of auction in terms of the number of days),[20] REP (seller's reputation, which is the accumulated ratings given by past buyers), NEW (a dummy variable with the value one if the auction subject is new; and is zero otherwise), and MINIBID (minimum price to start bidding, set by the seller). Note that in the regressions, the values of BUYOUT, PRICE, and MINIBID are all in Logrithm.

The key variables in our test are buy-out price (BUYOUT) and the buy-out dummy (BUYOUTD). The buy-out dummy is used to test whether the average transaction price with buy-out option is higher than that without buy-out if an auction results in sale. If our theory is correct, it should have a positive coefficient. On the other hand, the buy-out price is used to verify whether the average transaction price is increasing in buy-out price if we only look at the auctions with buy-out option.

In a usual ascending price auction, we expect that the longer the auction lasts, the higher will be the transaction price. However, this may not be the case for on-line auctions with buy-outs. This is because in an on-line auction the time period that an item is put up for sale is fixed in advanced. If no bidder buys out the object, then the seller will

[19] The transaction price excludes shipping charge.

[20] Note that Length is zero, if the auction is ended within one day.

wait until the end of the period to determine who the winner is, and at what price the object is transacted. This implies that if it takes a shorter time for the object to be sold, it must be because some bidder buys it out. Given that our theoretical model has shown that the average transaction price is higher for auctions that result in buy-out, we would expect there to be an inverse relationship between the transaction price and the time it takes until it is sold. That means the coefficient for LENGTH should be negative.[21]

Previous work on the impact of a seller's reputation on the transaction price of an auction indicates that the return to reputation is either insignificant or small if it exists.[22] One recent exception is Livingston (2005), who argued that previous studies underestimate the returns to reputation because they assumed that the relationship between the transaction price and the seller's reputation is linear or log-linear, and they might fail to control for sample selection bias. Using the Probit model and sample selection model, he showed that the probabilities that the auction receives a bid, that it results in a sale, and the amount of the winning bid all increase substantially with the first few positive reports of the seller. Given this result, in order to control for the effect of reputation on transaction price, we also put a reputation variable in our regression.

There is a possibility that the common omitted variables of the selection equation and regression equation might cause the error terms to be correlated, with each other. In such case, we need to find an intrumental variable which affects whether the auction ends

[21] The numbers in Table 2 also indicate this fact very clearly. Among the 936 items that are listed with buy-out options and are eventually sold, the average transaction price for the 144 items sold with buy-out is 9,674.874, which is substantionally higher than the 192 items that are eventually sold to the highest bidders, 6,293.33.

[22] See Bajari and Hortaçsu (2004).

up with a trade but not the level of transaction price. One of the possible candidates of this intrumental variable is the reservation price. However, the reservation price is unobserved. We thus use minimum bid as a proxy for this purpose. Minimum bid is the lowest possible bid set by the seller for the buyer to start the bidding process. Higher minimum bid may lower the chance that an auction results in a sale by discouraging the buyer from placing a bid. As is explained in Livingston (2005), minimum bid should only affect the buyer's participation decision, but not the decision of how much to bid. In term of our theory in Section 2, there may be some observed and unobserved factors that affect the participation decision, but not the fact that a buyer is willing to buy the object as long as price is lower than his valuation.

3.2 Transaction Price and Buy-out

Our theoretical model predicts that the average transaction price of auctions with buy-out option (regardless of whether items are sold through buy-out or bidding) is higher than that of those without. A simple look at the data indicates that the average transaction price with buy-out option (NT$8,981.2)[23] is indeed substantially greater than that without buy-out (NT$ 6,594.9). We first use a simple t-test to test the difference between their mean values. The null hypothesis is that average transaction price with buy-out option is greater than that without buy-out. The value of t-statistics is 5.364, and the null hypothesis cannot be rejected at the 1% significance level. Given this, we then use the sample selection model that controls for the relevant variables to run a full test.

[23] NT$ $9,674.9 \times \frac{744}{936}$+NT$ $6,293.3 \times \frac{192}{936}$ =NT$ 8,981.2.

The data we use contains the 2171 items no matter whether they result in a sale or not. Also, we use the following explanatory variables to control for the influence of other factors on the transaction price: BUYOUTD, LENGTH, REP, and NEW. As is explained, we do not include minimum bid in the equation because minimum bid affects only the probability that the auction results in a sale, but not the transaction price.[24] Besides, we add twelve dummy variables (one for each brand) to control for the effects of brand names on the winning bids. Note that Kyocera digital camera is chosen as the base brand because it has the smallest number of items. Summary statistics of the related variables are displayed in Table 3.

The estimation results of the sample selection model by the full information maximum likelihood estimation procedure are presented in Table 4.[25] The coefficient on the buyout dummy is positive and significant at the one percent level, confirming our prediction that the average transaction price for auctions with buy-out options is greater than those without.

The sign of the coefficient of other explanatory variables are also consistent with our theory. For example, the coefficient for the variable LENGTH is negative and significant, indicating that the transaction price of an object is in inverse relationship with the time it takes to be sold. This is a result that is in stark contrast to the usual ascending price auctions. As is explained, the force that drives this result is that it takes a shorter time for an item to be sold only when some bidder buys it out, and our theoretical calculation

[24] For more on this see Section 3.3.

[25] We use STATA Release 8 to estimate the empirical models in this paper.

shows that the average transaction price for items that are eventually sold with buy-out is greater than those that are eventually sold under the winning bid.

Consistent with Livingston (2005), our result also shows that a good reputation of the seller has statistically and economically significant effects on the level of the transaction price.

3.3 Transaction Price and Buy-out Price

The second empirical implication of our theoretical model is that the value of the winning bid, given that a deal occurs, will be higher, the higher the buy-out price. Since we are investigating the relation between buy-out price and transaction price, our sample will consist of the 1741 items that are listed with buy-out options, no matter whether the auction ends in a deal or not. Thus, we include the 936 items with buy-out option that result in sale, plus the 805 items with buy-out option but do not result in a sale. Note that these are 430 auctions without buyout options in our sample. Among them, 230 result in a sale, while 200 did not. The reason why these sellers do not set a buy-out price might be that they are risk neutral and they think buyers are risk neutral as well.

The vector of independent variables x of the sample selection model includes BUYOUT, LENGTH, REP, and NEW. The vector of observed variables in selection equation, z, contains LENGTH, REP, NEW and MINIBID. It is worth noting that the explanation variable BUYOUT appears only in the regression equation but not the selection equation. The reason is that the buyout price specified by a seller should affect the level of transaction price but not the probability that the auction results in a sale.[26]

The dependent variable in the regression equation is the transaction price if the auction results in a sale. Otherwise, it is the minimum bid.[27] The dependent variable in the selection equation is a dummy variable which takes the value of one if the auction results in a sale, and zero otherwise. It is worth noting that, in contrast to the buy-out option, the variable MINIBID affects the probability that the auction results in a sale but not the winning bid. Higher minimum bid lowers the probability that an auction ends up with a sale by decreasing the incentives of the bidders to place bids, but not the transaction price. Thus, minimum bid is used to serve as an exclusion restriction in the sample selection equation, but not in the regression equation. As in Section 3.2, we also control for the impact of different brands on both the probability that the auction results in a sale and the value of transaction price by adding twelve dummy variables to the selection and regression equations. The base brand is Kyocera since it has the smallest number of items. Table 5 presents the summary statistics of the related variables.

Table 6 shows the results of our estimation. The standard errors in parentheses have been adjusted for clustering at the brand level of digital camera. The Wald test rejects the null hypothesis that ρ is equal to zero at the one percent level of significance. Note that the coefficient on the buy-out price is positive and significant at the one percent level,

[26] An item will be sold if and only if one of the bidders' valuations is greater than the reservation price. Since the price at which a bidder is willing to buy out an item must be always smaller than his valuation, buy-out option only affects the price at which the item is eventually sold, but not its probability of sale.

[27] As pointed out by Livingston (2005), setting the transaction price equal to the minimum bid when the auction did not result in a sale has no effect on the likelihood function, and merely represents the event that the bidding amount is less than the minimum bid. See also Amemiya (1985).

indicating that if we consider the auctions with buy-out option, the average transaction price is increasing in buy-out price no matter whether the auction results in a sale or not.[28] This result confirms the prediction of our theoretical model. Furthermore, the coefficient on the minimum bid is negative and significant at the one percent level. Therefore, a higher minimum bid, which is used as an exclusion restriction in the sample selection model, reduces the probability that the auction ends up with a deal. Finally, the signs of all other explanatory variables are consistent with that of our first test in Section 3.2.

In summary, our two tests confirm our theoretical predictions that an item listed with buy-out options are on average sold at a higher price; transaction price is increasing in buy-out price and decreasing in the time an item takes to be sold.

4 Conclusion

In this paper we propose a dynamic model of auction with buy-out option, in which both the seller and bidders are risk-averse. We completely characterize the optimal bidding strategy of the bidder and the optimal buy-out price of the seller. The seller is shown to benefit from the buy-out option from two sources. He can either use it to exploit the bidder's aversion to price risk in the bidding process, or to reduce price risk in the bidding process for himself. In contrast to the literature, the buy-out option benefits the seller even if the bidders are risk-neutral. Since buy-out is also used as an additional instrument

[28] Endogeneity in the length of auction might arise when the seller's valuation is high and he specifies a short auction period. Thus, we may observe a negative relationship between transaction price and length of auction. However, the seller would then have to set a lower buy-out price. In such case, the nagative relationship mentioned above might not be so significant.

to intensify the competition between the bidders, unless a bidder has a high valuation of the item, he is worse off when there is buy-out.

Our model predicts three testable implications. First, the transaction price of an identical object will be higher when there is buy-out. Second, the transaction price of an item is in direct relation to its buy-out price. Third, the longer it takes for an item to be sold, the lower will be its transaction price, which is opposite to what is expected from a usual ascending price auction. All predictions are confirmed by the data we collected from Taiwan's Yahoo! on-line auction site.

An interesting option in auctions that is omitted from our model is the reserve price. In contrast to the buy-out price, which essentially sets an upper-bound on the possible transaction prices, the reserve price sets a lower bound. Moreover, like the buy-out price, the reserve price can also serve as a strategic instrument for the seller. That is, the seller can strategically set a reserve price to increase his expected revenue. When a reserve price consideration is incorporated, our model becomes substantially complicated, and requires major modification. But it also points to a promising direction for future research.

Appendix A: Uniqueness of Solution

In this appendix we show that the solution we find in Section 2.1 is the unique solution satisfying the initial condition $v(v_b) = v_b$. That is, $x = \mu^* y$ is the only solution of (4) that passes through $(0,0)$. Rewrite (4) as $(\frac{x}{y}+1)^\alpha - 1 = \frac{dy}{dx}$. Then by the definition of μ^* we know that for all points on line $x = \mu^* y$, $\frac{dy}{dx} = \frac{1}{\mu^*}$, i.e., direction of change points to the origin. Figure A1 depicts the phase diagram of solution of (4).

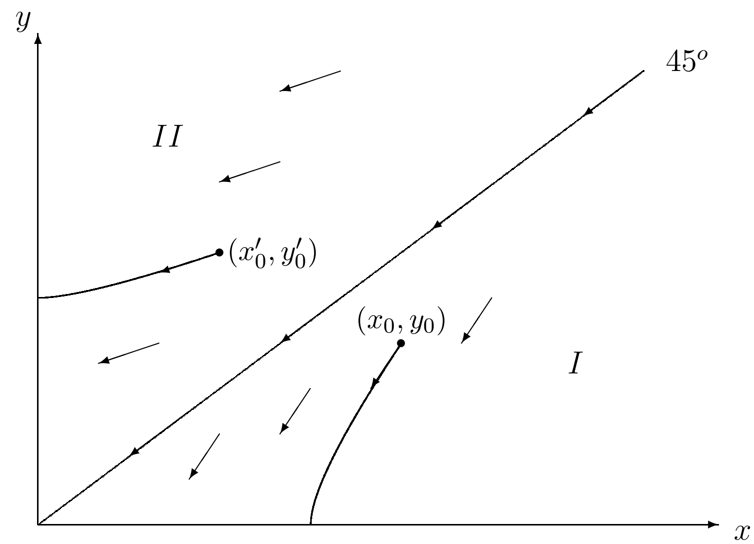

Figure A1

Consider a solution of (4) that passes through a point (x_0, y_0) in region I. Since $\frac{x_0}{y_0} > \mu^*$, $(\frac{x_0}{y_0} + 1)^\alpha - 1 > (\mu^* + 1)^\alpha - 1 = \frac{1}{\mu^*}$ by definition of μ^*. But a solution passing (x_0, y_0) must satisfy (4), namely, $(\frac{x_0}{y_0} + 1)^\alpha - 1 = \frac{dy}{dx}|_{(x_0,y_0)}$. That means $\frac{dy}{dx}|_{(x_0,y_0)} > \frac{1}{\mu^*}$. By the same argument we know that $\frac{dy}{dx}|_{(x_0,y_0)} < \frac{1}{\mu^*}$ for any solution passing (x_1, y_1) in region II. These are shown in Figure A1. The result implies that any solution of (4) not on the line $x = \mu^* y$ will not pass through $(0, 0)$. It is then obvious that, among solutions of (4), only the solution $x = \mu^* y$ can pass through $(0, 0)$, i.e., only $v(v_b) = (1 + \frac{1}{\mu^*})v_b - \frac{p}{\mu^*}$ satisfies the boundary condition $v(v_b) = v_b$.

Appendix B: Proof of Proposition 2

The expected utility of a buyer with valuation v, when there is no buy-out option, is $\int_0^v \frac{1}{\bar{v}}\frac{(v-x)^\alpha}{\alpha}\, dx = v^{\alpha+1}/\alpha(\alpha+1)$. In the case with buy-out, the way to calculate buyer's expected utility depends on the value of v. For $v < v_b^*$, a buyer's expected utility is identical to the case without buy-out. If $v_b^* \leq v < (1+\frac{1}{\mu^*})v_b^*$, the buyer's expected utility is

$$\int_0^{(1+\mu)v_b-\mu v} \frac{1}{\bar{v}}\frac{(v-x)^\alpha}{\alpha}\, dx + \int_{(1+\mu)v_b-\mu v}^{v} \frac{1}{\bar{v}}\frac{(v-v_b)^\alpha}{\alpha}\, dx$$
$$+ \frac{1}{\bar{v}}\frac{v^{\alpha+1}}{\alpha(\alpha+1)} - \frac{1}{\bar{v}}\frac{(1+\mu)^{\alpha+1}(v-v_b)^{\alpha+1}}{\alpha(\alpha+1)} + \frac{1}{\bar{v}}\frac{(v-v_b)^{\alpha+1}}{\alpha}(1+\mu). \tag{20}$$

The difference in utility between the cases with and without buy-out is

$$\begin{aligned}
&\frac{1}{\bar{v}}\frac{(1+\mu)^{\alpha+1}(v-v_b)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{1}{\bar{v}}\frac{(v-v_b)^{\alpha+1}}{\alpha}(1+\mu) \\
&= \frac{1}{\alpha\bar{v}}(v-v_b)^{\alpha+1}(1+\mu)[\frac{(1+\mu)^\alpha}{\alpha+1} - 1] \\
&= \frac{1}{\alpha\bar{v}}(v-v_b)^{\alpha+1}(1+\mu[\frac{\frac{1}{\mu}-\alpha}{\alpha+1}]),
\end{aligned} \tag{21}$$

where the last equality comes from the fact that $(1+\mu)^\alpha = 1+\frac{1}{\mu}$. Since $v > v_b^*$, the sign of (21) depends on that of $\frac{1}{\mu}-\alpha$. From the fact that $(1+\mu)^\alpha = 1+\frac{1}{\mu}$, we have $\alpha = \log(1+\frac{1}{\mu})/\log(1+\mu)$. Thus

$$\begin{aligned}
\frac{1}{\mu} - \alpha &= \frac{1}{\mu} - \frac{\log(1+\frac{1}{\mu})}{\log(1+\mu)} \\
&= \frac{1}{\mu\log(1+\mu)}\left[\log(1+\mu) - \mu\log(1+\frac{1}{\mu})\right] \\
&= \frac{1}{\mu\log(1+\mu)}\left[(1-\mu)\log(1+\mu) + \mu\log(\mu)\right].
\end{aligned} \tag{22}$$

First note that $g(\mu) \equiv (1-\mu)\log(1+\mu) + \mu\log(\mu) = 0$ when $\mu = 1$. Moreover,

$$\begin{aligned} g'(\mu) &= \frac{2}{1+\mu} + \log\ \mu - \log(1+\mu) \\ &> \frac{1}{\mu} + \log\ \mu - \log(1+\mu); \end{aligned} \tag{23}$$

which is positive by the concavity of the log function. This implies that, $\frac{1}{\mu} - \alpha > 0$ and, thus, the expected utility of the buyer is lower when there is buy-out.

In the case when $v \geq (1+\frac{1}{\mu})v_b$, the expected utility with buy-out is

$$\int_0^v \frac{1}{\bar{v}} \frac{(v-v_b)^\alpha}{\alpha}\ dx = \frac{1}{\bar{v}} \frac{(v-v_b)^\alpha}{\alpha} v. \tag{24}$$

We thus have

$$\frac{1}{\bar{v}} \frac{v^{\alpha+1}}{\alpha(\alpha+1)} \Big/ \frac{1}{\bar{v}} \frac{(v-v_b)^\alpha}{\alpha} v = \frac{v^\alpha}{(\alpha+1)(v-v_b)^\alpha}. \tag{25}$$

Plugging $v_b = \sqrt{\frac{\mu(1+\beta)}{(1+\mu)(3+\beta)}}$ into (25), setting (25) to be 1, and solving for v we have

$$\frac{v}{\bar{v}} = \frac{\sqrt{(\frac{\mu}{1+\mu})(\frac{1+\beta}{3+\beta})}}{1-(1+\alpha)^{\frac{-1}{\alpha}}} \equiv v_c(\alpha, \beta). \tag{26}$$

Since (25) is decreasing in v, we know that (25) is smaller than 1 (i.e., a bidder has higher expected utility with buy-out) if and only if $v \geq v_c(\alpha, \beta)$. QED

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Table 1. Sample Distribution of Digital Cameras.

Brand Name	Number of Observations
BenQ	124
Canon	336
Casio	215
Fujifilm	407
Kodak	79
Konica	137
Kyocera	21
Nikon	315
Olympus	59
Panasonic	232
Pentax	177
Ricoh	28
Sanyo	52
Total	2,182

Table 2. Bidding Outcome

<table>
<tr><td colspan="5">Total number of observations (2,171)</td></tr>
<tr><td colspan="3">Auctions resulting in a sale (1,166)</td><td colspan="2">Auctions not resulting in a sale (1,005)</td></tr>
<tr><td colspan="2">Auctions with buyout option (936)</td><td>Auctions without buyout option (230)</td><td>Auctions with buyout option (805)</td><td>Auctions without buyout option (200)</td></tr>
<tr><td>Items sold with buy-out (744)</td><td>Items sold to winning bids (192)</td><td></td><td></td><td></td></tr>
<tr><td>Average transaction price: NT$9,674.874
Average buyout price: NT$9,674.874</td><td>Average transaction price: NT$6,293.33
Average buyout price: NT$7,859</td><td>Average transaction price: NT$6,594.9</td><td>Average buyout price: NT$10,963.02</td><td></td></tr>
</table>

Table 3. Summary Statistics of Related Variables in the first Test

Variables	Definition	Mean	Std. Dev.	Min	Max
REP	Seller's reputation	461.054	963.958	-25	5806
NEW	A dummy variable with the value one if the item is new; zero otherwise.	.550	.498	0	1
BUYOUTD	A dummy variable with the value one if the auction has buyout option; zero otherwise.	.802	.399	0	1
MINIBID	Minimum bid	8896.049	6509.193	1	65000
LENGTH	Length of auction in terms of the number of days	7.609	3.097	0	11
TRADE	A dummy variable with the value one if the auction results in a sale; zero otherwise.	.537	.499	0	1
PRICE	Transaction price	9350.648	6368.257	1	65000
	Number of Observations		2,171		

Notes :1.Length is zero if the auction is ended within one day.
2.The maximum number of length is 11 because the auction is extended one more day due to the occurrance of last minute bid.

Table 4. Regression Results of the first test

Independent Variable	Transaction Price Equation	Selection Equation
Constant	8.1263***	4.0463***
	(.1072)	(.5433)
Buyout Dummy	.1006**	
	(.0512)	
Reputation	.0001***	.0003***
	(.0000)	(.0000)
Length of Auction	-.0675***	-.1457***
	(.0092)	(.0125)
New Subject Dummy	.2828***	-.1908***
	(.0532)	(.0874)
Minimum Bid		-.4007***
		(.0796)
Brand Dummy 1	.0562	.7696***
	(.0838)	(.0413)
Brand Dummy 2	.6391***	.6326***
	(.0575)	(.0948)
Brand Dummy 3	.5287***	.4935***
	(.0623)	(.0987)
Brand Dummy 4	.2927***	.4869
	(.0612)	(.0782)
Brand Dummy 5	.0975	.4696***
	(.0699)	(.0739)
Brand Dummy 6	.6629***	.7748***
	(.0592)	(.1076)
Brand Dummy 7	.5451***	.5992***
	(.0570)	(.0901)
Brand Dummy 8	.4065***	.6768***
	(.0558)	(.0555)
Brand Dummy 9	.6032***	.5005***
	(.0597)	(.1074)
Brand Dummy 10	.4804***	.5434***
	(.0643)	(.0954)
Brand Dummy 11	.3395***	.6651***
	(.0629)	(.0457)
Brand Dummy 12	.6476***	.5181***
	(.0641)	(.0988)
Number of Observations	2,171	2,171

Notes: standard errors appear in parenthess
parentheses. * denote significance at the 10% level, ** at the 5% level, *** at the 1% level.

Table 5. Summary Statistics of Related Variables in the second test

Variables	Definition	Mean	Std. Dev.	Min	Max
REP	Seller's reputation	544.228	1054.446	-25	5806
NEW	A dummy variable with the value one if the item is new; zero otherwise.	.624	.485	0	1
BUYOUT	Buyout price	10070.34	6472.57	100	65250
MINIBID	Minimum bid	9429.14	6606.35	1	65000
LENGTH	Length of auction in terms of the number of days	7.502	3.133	0	11
TRADE	A dummy variable with the value one if the auction results in a sale; zero otherwise.	.538	.499	0	1
PRICE	Transaction price	9726.26	6441.09	100	65000
	Number of Observations		1,741		

Notes: 1.Length is zero if the auction is ended within one day.
2.The maximum number of length is 11 because the auction is extended one more day due to the occurrance of last minute bid.

Table 6. Regression Results of the second test

Independent Variable	Transaction Price Equation	Selection Equation
Constant	.1398	3.5643***
	(.2257)	(.9813)
Buyout Price	.9734***	
	(.0250)	
Reputation	.0000*	.0003***
	(8.87e-06)	(.0000)
Length of Auction	-.0183**	-.1921***
	(.0073)	(.0155)
New Subject Dummy	.0623***	-.2331**
	(.0177)	(.0946)
Minimum Bid		-.2510*
		(.1289)
Brand Dummy 1	-.0113	.6642***
	(.0154)	(.0810)
Brand Dummy 2	.0934***	.2439*
	(.0153)	(.1356)
Brand Dummy 3	.0825***	.0310
	(.0149)	(.1524)
Brand Dummy 4	.0457***	.1609
	(.0083)	(.1192)
Brand Dummy 5	.0397***	.1763*
	(.0110)	(.0973)
Brand Dummy 6	.1081***	.2904*
	(.0189)	(.1529)
Brand Dummy 7	.0941***	.1503
	(.0118)	(.1327)
Brand Dummy 8	.0794***	.6579***
	(.0127)	(.0482)
Brand Dummy 9	.0923***	.0893
	(.0145)	(.1628)
Brand Dummy 10	.0999***	.2250*
	(.0156)	(.1365)
Brand Dummy 11	.0876***	.4916***
	(.0172)	(.0462)
Brand Dummy 12	.0987***	.2163
	(.0197)	(.1473)
Number of Observations	1,741	1,741

Notes: standard errors appear in parenthess
parentheses. * denote significance at the 10% level, ** at the 5% level, *** at the 1% level.