Price Discrimination: Case of Monopoly

- The firm charges different prices to different consumers, or different prices for different units purchased.
- Prevelent in reality.
- Examples:
  1. tickets (airline, movie, ...)
  2. bulk rate discount
  3. Disneyland pricing (two-part tariff)
  4. same good with different prices in different places
  5. hardbound vs paperback books
  6. bundling

- Conditions needed for successful price discrimination:

  1. Market power of the firm (here we are mainly concerned with monopoly).
  2. Information on different willingness-to-pays.
  3. Prevention of resale.

- Three types of price discrimination: 1st degree, 2nd degree, and 3rd degree.

First degree price discrimination:

- The monopolist charges every unit with a price which equals a consumer's maximum willingness to pay for that unit. Moreover, it does so for every (presumably different) consumer.
- Let $P(Q)$ be the demand curve of a consumer. Assume zero cost. The price charged for that consumer is $P^M(Q) = \int_0^Q p(x)dx$ if Q units are purchased.
- Also called perfect price discrimination.
- Requires strong information on the part of the monopolist.
- Must also be legally implementable.

- A discrete example:

  Let demand schedule of a consumer be

unit	willingness to pay
1	10
2	9
3	8
4	7
$\vdots$	$\vdots$

- One way to implement 1st degree price discrimination is bulk rate

  $P^M(1) = 10$

  $p^M(2) = 10 + 9 = 19$, unit price is thus $19/2 = 9.5$: 50% off.

  $P^M(3) = 10 + 9 + 8 = 27$, unit price is thus $27/3 = 9$: 10% off.

  $P^M(4) = 10 + 9 + 8 + 7 = 34$, unit price is thus $34/4 = 8.5$: 15% off.

$$\vdots$$

- What if there is cost of production for the monopolist, $C(Q)$? (Homework #1)
- Since all the consumer surplus is exploited, social welfare is actually maximized.

2nd degree price discrimination:

- Also called non-linear pricing.
- The monopolist still charges different prices for different units purchased, but it has only one price schedule.
- Suppose there are two types of consumers. Utility of consumer i is $U_i(x) + y$, $i = 1, 2$; where y is income and x is units of good consumed.
- Assume $u_2(x) > u_1(x)$ and $u_2'(x) > u_1'(x)$ for all x.
- Let $p(x)$ be the price schedule. Let consumer x_i demand i units of good, and thus spends $r_i = p(x_i)x_i$ on it.

- The monopolist actually offers (r_i, x_i) to consumer i. If consumer i buys x_i rather than x_j, then it must be

$$u_1(x_1) - r_1 \geq 0, \tag{1}$$

$$u_2(x_2) - r_2 \geq 0, (\text{IR}) \tag{2}$$

$$u_1(x_1) - r_1 \geq u_1(x_2) - r_2, \tag{3}$$

$$u_2(x_2) - r_2 \geq u_2(x_1) - r_1, (\text{IC}) \tag{4}$$

- We can rewrite the constraints as

$$r_1 \leq u_1(x_1), \tag{1'}$$

$$r_1 \leq u_1(x_1) - u_1(x_2) + r_2, \tag{2'}$$

$$r_2 \leq u_2(x_2), \tag{3'}$$

$$r_2 \leq u_2(x_2) - u_2(x_1) + r_1. \tag{4'}$$

- Since optimality requires r_1 and r_2 to be as large as possilbe, one of (1') and (2') must be binding, and one of (3') and (4') must be binding.

- Want to show (1) and (4) are binding:

  If $u_2(x_2) = r_2$, then by (4) we know that $r_1 \geq u_2(x_1) > u_1(x_1)$, a contradiction. Thus we must have $u_s(x_s) > r_2$. (4') is thus binding. If (3) holds as equality, then

  $r_1 = u_1(x_1) - u_1(x_2) + r_2 = u_1(x_1) - u_1(x_2) + u_2(x_2) - u_2(x_1) + r_1$, i.e., $u_1(x_2) - u_1(x_1) = u_2(x_2) - u_2(x_1)$. But since $u_2'(x) > u_1'(x)$, we have

  $$\int_{x_1}^{x_2} u_2'(t)dt > \int_{x_1}^{x_2} u_1'(t)dt,$$

  which means $u_2(x_2) - u_2(x_1) > u_1(x_2) - u_1(x_1)$, a contradiction. Thus (3) must hold as strict inequality.

- Lower-demand consumer (consumer 1) is fully exploited, while higher demand consumer has positive surplus.
- Profit of firm is thus (assuming constant unit cost)

$$
\begin{aligned}
& r_1 - cx_1 + r_2 - cx_2 \\
& = u_1(x_1) - cx_1 + u_2(x_2) - u_2(x_1) + u_1(x_1) - cx_2.
\end{aligned}
$$

- FOC:

$$
\begin{aligned}
& u_1'(x_1) - c - u_2'(x_1) + u_1'(x_1) = 0, \\
& u_2'(x_2) - c = 0.
\end{aligned}
$$

This implies $u_1(x_1) = c - u_1'(x_1) + u_2'(x_1) > c$. The consumption of consumer 1 is not efficient.

Third degree price discrimination

- Different consumers are charged different prices, but each unit is charged exactly the same unit price, no matter how many units are purchased.
- A two-consumer example: Assuming constant unit cost, and $p_i(x_i)$ is the demand function of consumer i. Then the problem of the monopolist is

$$\max_{x_1,x_2} p_1(x_1)x_1 + p_2(x_2)x_2 - cx_1 - cx_2$$

- FOC:

$$p_1(x_1) + p_1'(x_1)x_1 = c,$$

$$p_2(x_2) + p_2'(x_2)x_2 = c.$$

- We thus have

$$p_1(x_1)\left\{1 - \frac{1}{|\varepsilon_1|}\right\} = c,$$
$$p_2(x_2)\left\{1 - \frac{1}{|\varepsilon_2|}\right\} = c.$$

This implies

$$p_1 > p_2 \quad \text{iff} \quad |\varepsilon_2| > |\varepsilon_1|.$$

- Consumers who have lower demand elasticity are exploited more heavily in the sense that they are charged higher prices.

Two-part tariff

- The monopolist charges a fixed fee T, plus unit price p for every unit purchased.
- Consumers are indexed by α. $f(\alpha)$ is the number of consumer of type α. $\alpha \in [\underline{\alpha}, \overline{\alpha}]$. Consumer with higher value of α has higher valuation of the good.
- Let $s(p, \alpha)$ is consumer surplus of type α when the fixed fee is zero.
- Define α^* such that $s(p, \alpha^*) = T$. α^* called the marginal consumer.
- The number of consumers who purchase the good is $\int_{\alpha}^{\bar{\alpha}} f(\alpha) d\alpha \equiv N(p, T)$.

- Let $q(p,\alpha)$ be the demand function of type α consumer. Then total demand for the monopolist's good is $\int_{\alpha}^{\bar{\alpha}} q(p,\alpha)f(\alpha)d\alpha \equiv Q(p,T)$.
- Assume constant unit cost m. The profit of the monopolist is $TN(p,T) + (p-m)Q(p,T)$.
- FOC (Homework #2):

$$\frac{p-m}{p} = -\frac{1}{\varepsilon}\left\{1 - \frac{q^*}{\bar{q}}\right\},$$

where $q^* = q(p,\alpha^*)$, $\bar{q} = Q(p,T)/N(p,T)$,

and

$$\varepsilon = \int_{\alpha}^{\bar{\alpha}} \frac{q(p,\alpha)}{Q(p,T)} \frac{\partial q(p,\alpha)}{\partial p} f(\alpha)d\alpha$$

which is the demand elasticity of purchasing consumers.

- Recall that for a simple monopolist, $(p - m)/p = -1/\varepsilon$.
- If consumers are identical, then $q^* = \bar{q}$, and $p = m$. All profit comes from T.
- It is possible to have $p < m$. This occurs when consumers who have very high surplus buy little, and who have low surplus buy a lot.
- Impossible that $T < 0$.