- 1 Price discrimination.
- 2 Leverage Theory: Extending the monopoly power of a monopolist from one market to the other.

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McAfee, McMillan and Whinston (1989)

- Provide conditions under which bundling is a profitable strategy in discriminating consumers.
- Some preliminaries:

The rationale: To exploit the difference of consumers' willingness to pay.



- By charging a bundled price of \$100, the firm can exploit all consumer surplus of all consumers.
- Question: Under what condition is bundling always better than unbundling?

- One monopolist, two goods (1 & 2). Good i is produced with constant MC, c_i.
- A consumer buys good i if its price P_i, is lower than the utility it brings, V_i.
- Distribution of consumers' tastes: F(V₁, V₂); f(V₁, V₂): its density. g_i(V₁|V₂): conditional density. G_i() and H_i() are corresponding distribution functions.

- pure bundling: The firm offers bundled good only.
- mixed bundling: The consumers are allowed to buy both bundled good and separately.
- mixed bundling dominates pure bundling.

This is because for any pure bundling with price P_B , we can always create a mixed bundling with $(P_B, P_1 = P_B - C_2, P_2 = P_B - C_1)$, which makes at least equal profit. Proposition 1:

Let (P_1^*, P_2^*) be the optimal non-bundling prices. Mixed bundling dominates unbundled sales if

$$\int_{0}^{P_{1}^{*}} \{ [1 - G_{2}(P_{2}^{*}|s)] - g_{2}(P_{2}^{*}|s)(P_{2}^{*} - c_{2}) \} h_{1}(s) ds + (P_{1}^{*} - c_{1})[1 - G_{2}(P_{2}^{*}|P_{1}^{*})] h_{1}(P_{1}^{*}) > 0.$$
(1)

 $\blacktriangleright P_1^* + P_2^* = V_1 + V_2$

Create a bundle with price $P_1^* + P_2^*$

Raise price of good 2 to be $P_2^* + \varepsilon$

Keep price of good 1 unchanged.



Corollary 1.

If V_1 and V_2 are independently distributed, then bundling dominates unbundled sales.

Proof of Corollary 1.

For the case of independently distributed reservation values, condition (1) reduces to (note that $h_i(P) = g_i(P|s)$ for all (P, s) and i = 1, 2):

$$H_{1}(P_{1}^{*})\{[1 - H_{2}(P_{2}^{*})] - h_{2}(P_{2}^{*})(P_{2}^{*} - c_{2})\} + (P_{1}^{*} - c_{1})h_{1}(P_{1}^{*})[1 - H_{2}(P_{2}^{*})] > 0.$$
(2)

But, if P_2^* is the optimal unbundled price for good 2, then the first term in (2) is equal to zero, so that (2) reduces to

$$(P_1^* - c_1)h_1(P_1^*)[1 - H_2(P_2^*)] > 0.$$
(3)

Now, by the assumptions of no atoms and existence of a positive measure of valuations above cost, $(P_1^* - c_1)[1 - H_2(P_2^*)] > 0$. Also, under our continuity assumption it must be that $h_1(P_1^*) > 0$ (again, from the non-bundling first-order condition). Thus, condition (3) holds, and a local gain from bundling is possible.

Chen (1997)

- Considers the incentive to bundle for two firms which are duopolists in one market, and there is perfect competition in the other.
 - Two firms: A, B
 - Two goods: X, Y
 - MC for X is c, and c_Y for Y
 - A continuum of consumers with mass 1.
 - Consumers are identical, whose valuation for X is r.
 - Demands for X and Y are independent. Only one X is needed.
 - Valuation for Y is v, which is stochastic with density g(v).

G(v) is corresponding distribution function.

$$v \in [\underline{v}, \overline{v}], \ \underline{v} < c_Y < \overline{v}.$$

1st stage: Firms decide whether to

(i) sell X only; or

(ii) sell X & Y as bundle (XY); or

(iii) sell both X and XY at the same time.

<u>2nd stage</u>: The firm decides the prices for its products. Thus the 2nd stage is a Bertrand competition.

There are 4 possible subgame in 2nd stage:

(X, X), (X, XY), (XY, X), and (XY, XY). First exclude the possibility of mixed bundling. (i.e., (iii) above). This will be justified.

- Solution: (pure strategies only)
- Case one, (X, X):

Both offer $P_X = c$ and make zero profit.

Case two, (XY, XY):

Both offer $P_{XY} = c + c_y$ and make zero profit.

Case three, (XY, X) or (X, XY):

(1) At any equilibrium, $P_X > c$, $P_{XY} > c + c_Y$. That is, both firms make positive profits.

(2) In equilibrium, one firm offers X, and the other XY.

Reason for (1):

For a consumer with valuation v for Y, whether he will buy X or XY depends on the relative size of

 $v - P_X$: utility for buying Y, and

 $r + v - P_{XY}$: utility for buying XY.

► The # of buyers for X is thus

$$q_X(P_{XY}, P_X) = \begin{cases} G(P_{XY} - P_X); \text{ if } P_{XY} - P_X < c_Y, P_X \le r \\\\ 1; \text{ if } P_{XY} - P_X \ge c_Y, P_X \le r \\\\ 0; \text{ if } P_X > r. \end{cases}$$

- Demand for XY is thus $1 q_X$, if $P_X \leq r$.
- Profit for firms selling X and XY are, respectively,

$$\pi_X = (P_X - c)q_X(P_X, P_{XY}),$$

$$\pi_{XY} = (P_{XY} - c - c_Y)q_{XY}(P_X, P_{XY}).$$

► Suppose P_X^* and P_{XY}^* are equilibrium prices. Then $\pi_X(P_X^*, P_{XY}^*) \ge \pi_X(c + \varepsilon, P_{XY}^*)$ (obviously, $P_X^* \ge c$ and $P_{XY} \ge c + c_Y$) $\ge \varepsilon G(P_{XY}^* - (c + \varepsilon)) \ge \varepsilon (c_Y - \varepsilon) > 0$

if ε is small enough.

This implies $P_X^* > c$, $\pi_X^* > 0$.

Given that P^{*}_X > c, we can set δ > 0 small enough so that
 c + c_Y + δ < P^{*}_X + c_Y, and under that price the # of buyers for XY is at least 1 − G(c_Y) > 0. That means it must be that P^{*}_{XY} > c + c_Y and
 π^{*}_{XY} > 0. QED

- ▶ (2) follows immediately from (1).
- Note that this argument critically depends on the assumption of Bertrand competition.

► FOC:

$$P_X - c - rac{G(P_{XY} - P_X)}{g(P_{XY} - P_X)} = 0,$$

 $P_{XY} - c - c_Y - rac{1 - G(P_{XY} - P)}{g(P_{XY} - P_X)} = 0.$

• A unique pair (P_X^*, P_{XY}^*) solves FOC if

$$\frac{d\frac{g(v)}{G(v)}}{dv} \leq 0, \text{ and } \frac{d\frac{g(v)}{1-G(v)}}{dv} \geq 0.$$

P^{*}_X > P^{*}_{XY} − c_Y: There is transfer of surplus form consumers to firms in the X market. However, dead-weight loss occurs when consumers with v < c_Y buy the bundle XY.

- When bundling is allowed, producers of X are better off, and consumers worse off. Social welfare reduces.
- Even if the Y market is competitive, firms in X market can still benefit from bundling, because it provides a useful way to differentiate X and thus gain market power.
- ► Example: g(v) = 1, $0 \le v \le 1$, $c + 2/3 < \gamma$, $(3\sqrt{5} 5)/2 \le c_Y < 1$. Then $P_X^* = c + (1 + c_Y)/3$ and $P_{XY}^* = c + 2(1 + c_Y)/3$. $\Pi_x^* = [(1 + c_Y)/3]^2$ and $\Pi_{XY}^* = [(2 + c_Y)/3]^2$.

Consumers with $(1 + c_Y)/3 < v < c_Y$ buy XY and consume Y. There is inefficiency.

- When mixed bundling is allowed: When both firms offer (X, XY), both earn zero profits. When one firm offers XY and the other (X, XY), the former makes zero profit and the latter Π*, with 0 ≥ Π* < Π^{*}_X.
- Mixed bundling is thus a weakly dominated strategy.
- Sharp contrast to McAfee et. al. Intuition: Product differentiation role is undermined if mixed bundling is offered.

Firm selling X earns higher profit than firm selling XY.

The game is somewhat like a battle- of-sex game, in that every firm would like the opponent to bundle in order to soften competition. But will itself bundle if it expects the other firm not to.

	Α	В
A	3,3	2,1
В	1,2	3,3

 If bundling is allowed, producer of X is better off, and consumers worse off. Social welfare is reduced.



 Contrary to conventional wisdom, mixed bundling is dominated by pure bundling.

- Schmalensee: monopoly competitive bundling is useless.
- Chen: oligopoly+ competitive; bundling useful.
- Whinston: monopoly + oligopoly; bundling useful.

Topics to pursue

(1) Oligopoly + oligopoly

prisoners' dilemma?

- (2) Bundling goods are usually complements
 - e.g. Explorer + window

 $\mathsf{cable} + \mathsf{program}$

 $\mathsf{printer} + \mathsf{cartridge}$

phone + internet

(i) What are the effects when we make special assumptions on characteristic

of commodities?

(ii) In particular

- Perfect complement
- One good requires the other to use, but not the other way round.