## Reasons for Bundling:

1 Price discrimination.

2 Leverage Theory: Extending the monopoly power of a monopolist from one market to the other.

## McAfee, McMillan and Whinston (1989)

- Provide conditions under which bundling is a profitable strategy in discriminating consumers.
- Some preliminaries:

The rationale: To exploit the difference of consumers' willingness to pay.


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- By charging a bundled price of $\$ 100$, the firm can exploit all consumer surplus of all consumers.
- Question: Under what condition is bundling always better than unbundling?


## Model

- One monopolist, two goods (1 \& 2). Good i is produced with constant MC, $c_{i}$.
- A consumer buys good i if its price $P_{i}$, is lower than the utility it brings, $V_{i}$.
- Distribution of consumers' tastes: $F\left(V_{1}, V_{2}\right) ; f\left(V_{1}, V_{2}\right)$ : its density. $g_{i}\left(V_{1} \mid V_{2}\right)$ : conditional density. $G_{i}()$ and $H_{i}()$ are corresponding distribution functions.
- pure bundling: The firm offers bundled good only.
- mixed bundling: The consumers are allowed to buy both bundled good and separately.
- mixed bundling dominates pure bundling.

This is because for any pure bundling with price $P_{B}$, we can always create a mixed bundling with ( $P_{B}, P_{1}=P_{B}-C_{2}, P_{2}=P_{B}-C_{1}$ ), which makes at least equal profit.

- Proposition 1:

Let $\left(P_{1}^{*}, P_{2}^{*}\right)$ be the optimal non-bundling prices. Mixed bundling dominates unbundled sales if

$$
\begin{gather*}
\int_{0}^{P_{1}^{*}}\left\{\left[1-G_{2}\left(P_{2}^{*} \mid s\right)\right]-g_{2}\left(P_{2}^{*} \mid s\right)\left(P_{2}^{*}-c_{2}\right)\right\} h_{1}(s) d s \\
\quad+\left(P_{1}^{*}-c_{1}\right)\left[1-G_{2}\left(P_{2}^{*} \mid P_{1}^{*}\right)\right] h_{1}\left(P_{1}^{*}\right)>0 \tag{1}
\end{gather*}
$$

- $P_{1}^{*}+P_{2}^{*}=V_{1}+V_{2}$

Create a bundle with price $P_{1}^{*}+P_{2}^{*}$
Raise price of good 2 to be $P_{2}^{*}+\varepsilon$
Keep price of good 1 unchanged.


- Corollary 1.

If $V_{1}$ and $V_{2}$ are independently distributed, then bundling dominates unbundled sales.

- Proof of Corollary 1.

For the case of independently distributed reservation values, condition (1) reduces to (note that $h_{i}(P)=g_{i}(P \mid s)$ for all $(P, s)$ and $i=1,2$ ):

$$
\begin{align*}
& H_{1}\left(P_{1}^{*}\right)\left\{\left[1-H_{2}\left(P_{2}^{*}\right)\right]-h_{2}\left(P_{2}^{*}\right)\left(P_{2}^{*}-c_{2}\right)\right\} \\
&+\left(P_{1}^{*}-c_{1}\right) h_{1}\left(P_{1}^{*}\right)\left[1-H_{2}\left(P_{2}^{*}\right)\right]>0 \tag{2}
\end{align*}
$$

But, if $P_{2}^{*}$ is the optimal unbundled price for good 2, then the first term in (2) is equal to zero, so that (2) reduces to

$$
\begin{equation*}
\left(P_{1}^{*}-c_{1}\right) h_{1}\left(P_{1}^{*}\right)\left[1-H_{2}\left(P_{2}^{*}\right)\right]>0 \tag{3}
\end{equation*}
$$

Now, by the assumptions of no atoms and existence of a positive measure of valuations above cost, $\left(P_{1}^{*}-c_{1}\right)\left[1-H_{2}\left(P_{2}^{*}\right)\right]>0$. Also, under our continuity assumption it must be that $h_{1}\left(P_{1}^{*}\right)>0$ (again, from the non-bundling first-order condition). Thus, condition (3) holds, and a local gain from bundling is possible.

## Chen (1997)

- Considers the incentive to bundle for two firms which are duopolists in one market, and there is perfect competition in the other.
- Two firms: A, B
- Two goods: $\mathrm{X}, \mathrm{Y}$
- MC for $X$ is $c$, and $c_{Y}$ for $Y$
- A continuum of consumers with mass 1 .
- Consumers are identical, whose valuation for $X$ is $r$.
- Demands for $X$ and $Y$ are independent. Only one $X$ is needed.
- Valuation for Y is v , which is stochastic with density $g(v)$. $G(v)$ is corresponding distribution function.

$$
v \in[\underline{v}, \bar{v}], \underline{v}<c_{Y}<\bar{v} .
$$

## Two-stage game

- 1st stage: Firms decide whether to
(i) sell $X$ only; or
(ii) sell $\mathrm{X} \& \mathrm{Y}$ as bundle ( XY ); or
(iii) sell both $X$ and $X Y$ at the same time.
- 2nd stage: The firm decides the prices for its products. Thus the 2nd stage is a Bertrand competition.

There are 4 possible subgame in 2nd stage:
$(X, X),(X, X Y),(X Y, X)$, and (XY, $X Y)$. First exclude the possibility of mixed bundling. (i.e., (iii) above). This will be justified.

- Solution: (pure strategies only)
- Case one, (X, X):

Both offer $P_{X}=c$ and make zero profit.

- Case two, (XY, XY):

Both offer $P_{X Y}=c+c_{y}$ and make zero profit.

- Case three, (XY, X) or (X, XY):
(1) At any equilibrium, $P_{X}>c, P_{X Y}>c+c_{Y}$. That is, both firms make positive profits.
(2) In equilibrium, one firm offers $X$, and the other $X Y$.
- Reason for (1):

For a consumer with valuation $v$ for Y , whether he will buy X or XY depends on the relative size of
$v-P_{X}$ : utility for buying Y , and
$r+v-P_{X Y}$ : utility for buying XY.

- The \# of buyers for $X$ is thus

$$
q_{X}\left(P_{X Y}, P_{X}\right)=\left\{\begin{array}{l}
G\left(P_{X Y}-P_{X}\right) ; \text { if } P_{X Y}-P_{X}<c_{Y}, P_{X} \leq r \\
1 ; \text { if } P_{X Y}-P_{X} \geq c_{Y}, P_{X} \leq r \\
0 ; \text { if } P_{X}>r
\end{array}\right.
$$

- Demand for XY is thus $1-q_{X}$, if $P_{X} \leq r$.
- Profit for firms selling X and XY are, respectively,

$$
\begin{aligned}
& \pi_{X}=\left(P_{X}-c\right) q_{X}\left(P_{X}, P_{X Y}\right) \\
& \pi_{X Y}=\left(P_{X Y}-c-c_{Y}\right) q_{X Y}\left(P_{X}, P_{X Y}\right)
\end{aligned}
$$

- Suppose $P_{X}^{*}$ and $P_{X Y}^{*}$ are equilibrium prices. Then

$$
\begin{aligned}
& \pi_{X}\left(P_{X}^{*}, P_{X Y}^{*}\right) \geq \pi_{X}\left(c+\varepsilon, P_{X Y}^{*}\right)\left(\text { obviously, } P_{X}^{*} \geq c \text { and } P_{X Y} \geq c+c_{Y}\right) \\
& \geq \varepsilon G\left(P_{X Y}^{*}-(c+\varepsilon)\right) \geq \varepsilon(c Y-\varepsilon)>0
\end{aligned}
$$

if $\varepsilon$ is small enough.
This implies $P_{x}^{*}>c, \pi_{x}^{*}>0$.

- Given that $P_{X}^{*}>c$, we can set $\delta>0$ small enough so that $c+c_{Y}+\delta<P_{X}^{*}+c_{Y}$, and under that price the $\#$ of buyers for $X Y$ is at least $1-G\left(c_{Y}\right)>0$. That means it must be that $P_{X Y}^{*}>c+c_{Y}$ and $\pi_{X Y}^{*}>0$. QED
- (2) follows immediately from (1).
- Note that this argument critically depends on the assumption of Bertrand competition.
- FOC:

$$
\begin{aligned}
& P_{X}-c-\frac{G\left(P_{X Y}-P_{X}\right)}{g\left(P_{X Y}-P_{X}\right)}=0 \\
& P_{X Y}-c-c_{Y}-\frac{1-G\left(P_{X Y}-P\right)}{g\left(P_{X Y}-P_{X}\right)}=0 .
\end{aligned}
$$

- A unique pair $\left(P_{X}^{*}, P_{X Y}^{*}\right)$ solves FOC if

$$
\frac{d \frac{g(v)}{G(v)}}{d v} \leq 0, \text { and } \frac{d \frac{g(v)}{1-G(v)}}{d v} \geq 0
$$

- $P_{X}^{*}>P_{X Y}^{*}-c_{Y}$ : There is transfer of surplus form consumers to firms in the $X$ market. However, dead-weight loss occurs when consumers with $v<c_{Y}$ buy the bundle XY.
- When bundling is allowed, producers of $X$ are better off, and consumers worse off. Social welfare reduces.
- Even if the Y market is competitive, firms in X market can still benefit from bundling, because it provides a useful way to differentiate $X$ and thus gain market power.
- Example: $g(v)=1,0 \leq v \leq 1, c+2 / 3<\gamma,(3 \sqrt{5}-5) / 2 \leq c_{Y}<1$.

Then $P_{X}^{*}=c+\left(1+c_{Y}\right) / 3$ and $P_{X Y}^{*}=c+2\left(1+c_{Y}\right) / 3$.
$\Pi_{X}^{*}=\left[\left(1+c_{Y}\right) / 3\right]^{2}$ and $\Pi_{X Y}^{*}=\left[\left(2+c_{Y}\right) / 3\right]^{2}$.
Consumers with $\left(1+c_{Y}\right) / 3<v<c_{Y}$ buy $X Y$ and consume $Y$. There is inefficiency.

- When mixed bundling is allowed: When both firms offer ( $\mathrm{X}, \mathrm{XY}$ ), both earn zero profits. When one firm offers $X Y$ and the other $(X, X Y)$, the former makes zero profit and the latter $\Pi^{*}$, with $0 \geq \Pi^{*}<\Pi_{x}^{*}$.
- Mixed bundling is thus a weakly dominated strategy.
- Sharp contrast to McAfee et. al.

Intuition: Product differentiation role is undermined if mixed bundling is offered.

- Firm selling $X$ earns higher profit than firm selling XY.

The game is somewhat like a battle- of-sex game, in that every firm would like the opponent to bundle in order to soften competition. But will itself bundle if it expects the other firm not to.


- If bundling is allowed, producer of $X$ is better off, and consumers worse off. Social welfare is reduced.

- Contrary to conventional wisdom, mixed bundling is dominated by pure bundling.


## Literature review

- Schmalensee: monopoly competitive bundling is useless.
- Chen: oligopoly+ competitive; bundling useful.
- Whinston: monopoly + oligopoly; bundling useful.


## Topics to pursue

(1) Oligopoly + oligopoly prisoners' dilemma?
(2) Bundling goods are usually complements
e.g. Explorer + window

$$
\text { cable }+ \text { program }
$$

$$
\text { printer }+ \text { cartridge }
$$

$$
\text { phone }+ \text { internet }
$$

(i) What are the effects when we make special assumptions on characteristic of commodities?
(ii) In particular

- Perfect complement
- One good requires the other to use, but not the other way round.

