Tying, Foreclosure, and Exclusion

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In recent years, the "leverage theory" of tied good sales has faced heavy and influential criticism. In an important sense, though, the models used by its critics are actually incapable of addressing the leverage theory's central concerns. Here I reconsider the leverage hypothesis and argue that tying can indeed serve as a mechanism for leveraging market power. The mechanism through which this leverage occurs, its profitability, and its welfare implications are discussed in detail. (JEL 610)

A firm engages in tying when it makes the sale (or price) of one of its products conditional upon the purchaser also buying some other product from it. Tying has a long history of scrutiny under the antitrust laws of the United States, and throughout this history it has been harshly treated by the courts.[1] A primary basis for this condemnation has been the courts' belief in what has come to be known as the "leverage theory" of tying: that is, that tying provides a mechanism whereby a firm with monopoly power in one market can use the leverage provided by this power to foreclose sales in, and thereby monopolize, a second market.

In recent years the leverage theory has come under heavy attack from a number of authors whose arguments are traceable to the University of Chicago oral tradition associated with Aaron Director (see, for example, Director and Edward Levi, 1956; Ward S. Bowman, 1957; Richard A. Posner, 1976; and Robert H. Bork, 1978). A typical rendition of their criticism goes along the following lines: Suppose that a firm is a monopolist of some good A that a consumer values at level v_A and that costs c_A to produce. The consumer also consumes some other competitively supplied product B that she values at level v_B and that can be produced at a unit cost of c_B. Now, the monopolist *could* require the consumer to purchase good B from him if she wants good A, but what will he gain? The consumer will only purchase such a bundle if its price is no larger than $v_A + c_B$, and so the monopolist can do no better than earning $(v_A - c_A)$, the level he earns selling good A independently. In short, there is only one monopoly profit that can be extracted.

Similar arguments are given for the case of complementary products. Richard Posner (1976), for example, comments as follows:

> [A fatal] weakness of the leverage theory is its inability to explain *why* a firm with a monopoly of one product would want to monopolize complementary products as well. It may seem obvious..., but since the products are by

[1]Tying doctrine was originally developed in patent cases (*Motion Pictures Patents Co. v. Universal Film Manufacturing Co.*, 243 U.S. 502 (1917)). Since then a long line of case law has developed under both Section 1 of the Sherman Act and Section 3 of the Clayton Act. (See, for example, *International Salt v. U.S.*, 332 U.S. 392 (1947) and *Northern Pacific Railway Co. v. U.S.*, 356 U.S. 1 (1958).) Similar ideas have also been developed under Section 2 of the Sherman Act. (See, for example, *U.S. v. Griffith*, 334 U.S. 100 (1948) and *U.S. v. United Shoe Machinery Corp.*, 110 F. Supp. 295 (D. Mass. 1953).) Two cases involving less harsh treatment are *Times-Picayune Publishing Co. v. U.S.*, 345 U.S. 594 (1953) and *U.S. v. Jerrold Electronics Corp.*, 365 U.S. 567 (1961).

hypothesis used in conjunction with one another..., it is not obvious at all. If the price of the tied product is higher than the purchaser would have to pay on the open market, the difference will represent an increase in the price of the final product or service to him, and he will demand less of it, and will therefore buy less of the tying product. To illustrate, let a purchaser of data processing be willing to pay up to $1 per unit of computation, requiring the use of 1 second of machine time and 10 punch cards, each of which costs 10 cents to produce. The computer monopolist can rent the computer for 90 cents a second and allow the user to buy cards on the open market for 1 cent, or, if tying is permitted, he can require the user to buy cards from him at 10 cents a card —but in that case he must reduce his machine rental charge to nothing, so what has he gained?   [p. 173]

Thus, the critics contend, if a monopolist does employ tying, his motivation cannot be leverage. In its place, they point to a number of socially beneficial, or at worst ambiguous, alternative explanations for tying: for example, price discrimination (Bowman, 1957), achieving economies of joint sales, protection of goodwill, risk sharing, and cheating on a cartel price. Almost inadvertently, the more formal economics literature on tying (Meyer L. Burstein, 1960; Roger D. Blair and David L. Kaserman, 1978; Richard Schmalensee, 1982) has reinforced this view as a result of its exclusive focus on price discrimination motivations for the practice. Thus, Posner (1976) goes on to note that "the replacement of leverage by price discrimination in the theory of tie-ins has been part of the economic literature for almost twenty years."[2] These criticisms have, in

fact, had a tremendous impact in both legal and economic circles.[3]

In an important sense, however, the existing literature does not really address the central concern inherent in the leverage theory, namely, that tying may be an effective (and profitable) means for a monopolist to affect the market structure of the tied good market (i.e., "monopolize" it) by making continued operation unprofitable for tied good rivals. The reason lies in the literature's pervasive (and sometimes implicit) assumption that the tied good market has a competitive, constant returns-to-scale structure. With this assumption, the use of leverage to affect the market structure of the tied good market is actually impossible. Thus, in contrast to a concern over the effects of tying on market structure, the existing literature's focus is on a demand-side notion of "leverage": the idea that, taking the prices charged by tied good competitors as given, a firm might be able to extract greater profits from consumers by tying.[4]

In this paper, I reexamine the leverage hypothesis. In particular, I examine several simple models that depart from the competitive, constant returns-to-scale structure assumed in the existing literature. In contrast,

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[2] Bork (1978) sums up his discussion of tying more emphatically: "[The leverage] theory of tying arrangements is merely another example of the discredited transfer of power theory, and perhaps no other variety of that theory has been so thoroughly and repeatedly demolished in the legal and economic literature."

[3] In a recent antitrust textbook, for example, Blair and Kaserman (1985) comment that "according to this view, somehow the seller expands or levers his monopoly power from one market to another. This, of course, is not possible. A seller cannot get two monopoly profits from one monopoly.... Thus, the leverage theory of tying is unsatisfactory." The 1985 Department of Justice Vertical Restraints Guidelines state that "Tying arrangements often serve procompetitive or competitively neutral purposes.... [They] generally do not have a significant anticompetitive potential." For a recent rebuttal to this view in the legal literature, however, see Louis Kaplow (1985).

[4] Indeed, this is exactly the sense in which the existing literature can be said to focus on price discrimination aspects of the practice; it analyzes whether tying is a profitable strategy *given* the prices of tied good competitors (which can be thought of as creating an induced demand structure for the monopolist). In contrast, here my focus is on the ability of tying to *change* those prices, in particular, by making continued operation unprofitable for competitors.

here I assume that scale economies exist in the production process for the tied good, and as a result, the structure of that market is oligopolistic.

In these models I address three basic questions. First, can tying succeed in altering the market structure of the tied good market, and if so, how? Second, is it a profitable strategy? Third, what are the welfare consequences? As we shall see, tying can lead to a monopolization of the tied good market. Most interestingly, the mechanism through which this exclusion occurs is foreclosure; by tying, the monopolist reduces the sales of its tied good market competitor, thereby lowering his profits below the level that would justify continued operation.

Tying is frequently a profitable strategy for the monopolist in these models, and it is often so precisely because of its potential for altering the market structure of the tied good market. The particular circumstances in which tying is a desirable strategy for the monopolist, however, depend in part on whether he is able to make a precommitment to tie. In many circumstances this is indeed possible. One of the primary ways in which this can be accomplished is through product design and the setting of production processes, both of which may involve significant sunk costs. By bundling components of its system together or by making interfaces between the separately sold components incompatible with their rivals' components, firms can precommit to their marketing strategy. IBM, for example, was accused of incorporating increased amounts of storage into its central processing units in order to prevent sales by plug compatible memory manufacturers and also of trying to achieve interface incompatibility for the same purpose (Franklin M. Fisher, John J. McGowan, and Joen E. Greenwood, 1983, pp. 332–33). Kodak was accused of designing its new film and camera in a format incompatible with rival manufacturers' products (*Berkey Photo v. Eastman Kodak Co.*, 603 F. 2d 263 (2d Cir., 1979)).

On the other hand, in a significant number of tying cases little more than an easily changed marketing decision seems to be involved. For example, in *Times-Picayune Publishing Co. v. U.S.* (345 U.S. 594 (1953)), the publisher of the only morning newspaper in New Orleans only sold an advertisement in his morning paper with an advertisement in that day's evening newspaper (which faced competition from another evening newspaper). In *U.S. v. Griffith* (334 U.S. 100 (1948)), a movie theater chain refused to show films in its theaters in towns in which it possessed a monopoly if the distributor did not give it that film in towns where it faced competition. In *United Shoe Machinery Corp. v. U.S.* (110 F. Supp. 295 (D. Mass. 1953)), United Shoe bundled repair service with its shoe machinery leases.

Finally, when tying does lead to exclusion of rivals, the welfare effects both for consumers and for aggregate efficiency are in general ambiguous. The loss for consumers arises because, when tied market rivals exit, prices may rise and the level of variety available in the market necessarily falls. Indeed, in the models studied here, tying that leads to the exit of the monopolist's tied market rival frequently leads to increases in all prices, making consumers uniformly worse off. More generally, though, as is common in models of price discrimination, some consumers may be made better off by the introduction of tying. The effect on aggregate welfare, on the other hand, is uncertain because of both the ambiguous effects of price discrimination and the usual inefficiencies in the number of firms entering an industry in the presence of scale economies and oligopolistic pricing (A. Michael Spence, 1976; N. Gregory Mankiw and Whinston, 1984).

Though most tying cases involve products that are complements (particularly those where precommitment is involved), for expositional purposes I begin below by considering the case of independent products. In Section I, I first analyze the simple case where all consumers have an identical valuation of the monopolized product, so that the monopolist, if he chooses to price his goods independently, can fully extract all of the surplus from his monopolized good. I

show that, *absent precommitment*, tying is not a useful strategy for the monopolist; any equilibrium outcome will be equivalent to one where only independent pricing is allowed. Despite this fact, however, a *precommitment* to tying can be a profitable strategy for the monopolist because of its potential for excluding his tied market rival. This exclusionary effect arises because of what I call "strategic foreclosure": tying represents a commitment to foreclose sales in the tied good market, which can drive its rival's profits below the point where remaining in the market is profitable. This strategic incentive to foreclose sales in the tied good market occurs because once the monopolist has committed to offering only tied sales, it can only reap its profit from its monopolized product by making a significant number of sales of the tied good. Thus, in this model, tying necessarily lowers the profits of the monopolist's tied good rival. I then discuss the implications of such a commitment to tying for the monopolist's profits, for consumers, and for aggregate efficiency, and present a simple example to illustrate these points.

In Section II, I investigate how the presence of heterogeneous preferences among consumers for the monopolized good affects these results. Two basic findings emerge. First, with heterogeneous preferences for the tying good, tying no longer necessarily results in strategic foreclosure and the lowering of the monopolist's tied good rival's profits (though it still does in many circumstances). If, for example, a significant number of consumers in the tied market have low valuations of the tying good, tying (not surprisingly) will not be a successful exclusionary device. In addition, a more subtle effect may prevent a commitment to tying from lowering the tied good rival's profits. This occurs when tying substantially decreases the responsiveness of the monopolist's demand to price changes relative to the level previously prevailing in the tied good market.

Second, with heterogeneous valuations, tying can now also be a profitable strategy in the absence of precommitment. There are two senses in which this is true. First, in

a purely static sense, the monopolist may find tying to be a profitable strategy given its rival's price. This motivation for tying is analogous to that in the monopolistic bundling literature (for example, W. J. Adams and J. L. Yellen, 1976; R. Preston McAfee, John McMillan, and Whinston, 1989), but here it can have important competitive effects: tied product rivals can find their sales foreclosed and continued operation unprofitable. Second, even when tying is not profitable in this static sense, it may be in a dynamic sense when the exclusion of rivals through predation is possible. In such cases, tying can be a profitable strategy for the monopolist precisely because it forecloses the sales of the monopolist's tied market rival.

In Section III, I turn to the case of complementary products used in fixed proportions. I first consider a model of fixed proportions that is essentially an extension of the simple example quoted above from Posner (1976) to the case where the tied good market involves scale economies and oligopolistic behavior. Despite these differences, Posner's central contention continues to hold: a monopolist of one component never finds it worthwhile to tie in order to reduce the level of competition in the market for the other component. The reason lies in the fact that when the monopolized product is essential for all uses of the two products, the monopolist can always benefit from more competition in the non-monopolized market through sales of its monopolized product. Nevertheless, I then show that in two natural extensions of this model where the monopolized product is no longer essential for all uses of the non-monopolized components, tying once again emerges as a profitable exclusionary strategy. In one case, the presence of an inferior, competitively supplied alternative to the monopolized component leads to results that parallel those for independent products. In the other case, the existence of a second use for the nonmonopolized product (such as a replacement part market) can give the monopolist an incentive to tie in order to reduce competition in this other market.

Finally, I conclude in Section IV with a brief discussion of the implications of these findings.

I. Independent Products

I begin by considering an extremely simple model with independent products. There are two markets, which I label A and B. Market A is monopolized by firm 1 (say, because of a patent). Market B, on the other hand, is potentially served by two firms, firm 1 and firm 2. The products of firms 1 and 2 in market B are differentiated. Production in market B involves fixed costs of K_i plus an expenditure of c_{Bi} per unit for firm i. Unit costs for good A are c_A. For expositional simplicity, I ignore the possibility that there are fixed costs for product A.

Consumers, who are indexed by $d \in (0,1)$ with total measure 1, each desire at most one unit of good A and one unit of good B. All consumers have a reservation value of $\gamma > c_A$ for good A, while a consumer of type d has a valuation of $v_{Bi}(d)$ for a unit of firm i's product B. Resale of products by consumers is assumed to be prohibitively costly. In the absence of tying by firm 1, consumers simply respond to individual product prices (P_A, P_{B1}, P_{B2}). Firm i's sales of product Bi are then given by some function $x^i(P_{B1}, P_{B2}) \le 1$, which I assume to be everywhere differentiable and satisfy (subscripts denote partial derivatives) $x^i_j(P_{B1}, P_{B2}) \ge 0$ if $j \ne i$ and ≤ 0 if $j = i$, with strict inequalities if $x^i(\cdot, \cdot) \in (0,1)$. That is, products $B1$ and $B2$ compete with each other for consumer purchases.

When bundling is not permitted (which I will refer to below as an "independent pricing game"), it is easy to see that firm 1 will always set $P_A = \gamma$. It is also useful for what follows to define each firm i's best response correspondence in market B by $P^*_{Bi}(P_{Bj})$, which solves

$$\max_{P_{Bi}} (P_{Bi} - c_{Bi}) x^i (P_{B1}, P_{B2}).$$

I assume that this correspondence is single-valued, continuous, and differentiable with $P^{*'}_{Bi}(P_{Bj}) \in (0,1)$ (so products $B1$ and $B2$ are strategic complements in the sense of Jeremy I. Bulow, John D. Geanakoplos, and Paul D. Klemperer, 1985).

In the next two subsections I analyze the use of tying both for cases where firm 1 can precommit to tie and where it cannot. For the case without precommitment, I analyze a simple two-stage game. In stage one, each firm simultaneously decides whether to be active in market B. If firm i decides to be active, it incurs the cost K_i. In stage two, the firms pick prices (simultaneously if both are active). If firm 1 is active in market B, it can offer three different items for sale: good A at a price of P_A, good $B1$ at a price of P_{B1}, and a bundle consisting of one unit of good A and one unit of good $B1$ at a price of $\bar{P}$. If firm 2 is active, on the other hand, it can only offer good $B2$ at price P_{B2}. Throughout I assume that firm 1 is unable to monitor customer purchases; this assumption rules out the use of requirements contracts (where a consumer agrees as a condition of buying good A not to buy good $B2$) and also implies that a bundle will be purchased only if $\bar{P} \le P_A + P_{B1}$.

To analyze the case where precommitment is possible, I extend this game to three stages. In the (new) first stage of the game, firm 1 commits to which subset of three possible products—good A, good $B1$, and a bundle—it will be able to produce. For example, firm 1 can commit itself to a position where it will only be able to produce a bundle. The second and third stages are then identical to the no commitment game, but with firm 1 only able to offer for sale those items that it is able to produce.[5] Thus, as discussed in the introduction, by setting its design and production process, firm 1 is able to commit to a tying strategy.

Finally, at various points below I make comparisons between the outcomes of these two games and those of a game where firm 1 only offers goods A and $B1$ indepen-

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[5]Note that as long as firm 1 can produce both goods A and $B1$ separately it can still offer a bundle for sale.

dently (more precisely, a game that is the same as the no precommitment game but where bundling is prohibited). I refer to this game as the "independent pricing game." Firm 1 is said to tie whenever its pricing is not identical (or, more generally, economically equivalent) to that arising in this independent pricing game.

A. Tying Without Precommitment

Consider first the no commitment game. If firm 1 is active in market B, then in the second stage of this game it selects three (nonnegative) prices: $(P_A, P_{B1}, \bar{P})$. As the following proposition makes clear, however, tying is not a useful strategy in this game.

PROPOSITION 1: *Any subgame perfect equilibrium outcome of the no commitment game is economically equivalent to a subgame perfect equilibrium outcome in the independent pricing game.*

PROOF:

The proposition is established by arguing that in the subgames of the no commitment game in which firm 1 is active in market B, any Nash equilibrium in prices is equivalent to a Nash equilibrium in the corresponding subgame of the independent pricing game. Then, given the equivalence of the equilibria in the pricing subgames, firms' decisions about whether to be active in market B must also be equivalent in the two games.

Consider the subgame where both firms are active in market B. The equivalence of equilibria is demonstrated by arguing that for any set of prices $((P_A^0, P_{B1}^0, \bar{P}^0); P_{B2}^0)$ that constitute a Nash equilibrium in the no commitment game there is a set of independent prices $(\hat{P}_A, \hat{P}_{B1})$ such that sales and profits are the same for *both* firms under prices $((\hat{P}_A, \hat{P}_{B1}); P_{B2})$ as under prices $((P_A^0, P_{B1}^0, \bar{P}^0); P_{B2})$ when $P_{B2} = P_{B2}^0$ and are the same for firm 2 for *any* P_{B2}. This implies that $((\hat{P}_A, \hat{P}_{B1}); P_{B2}^0)$ is a Nash equilibrium in the independent pricing game (note that firm 1 now has fewer possible deviations).

The equivalence clearly holds if firm 1's equilibrium strategy has $\bar{P}^0 > P_A^0 + P_{B1}^0$, so suppose that $\bar{P}^0 \leq P_A^0 + P_{B1}^0$. There are two

cases to consider. First, suppose that $P_A^0 > \gamma$. If this is firm 1's best response, then it must be that all consumers are buying firm 1's bundle since otherwise firm 1 could do better by setting $P_A = \gamma$ while leaving all of its other prices unchanged: this price would make profitable sales of product A to those consumers not buying the bundle, while having no effect on firm 1's sales of either good $B1$ or the bundle (since consumers are indifferent about buying good A at this price).[6] In addition, since all consumers are purchasing the bundle (and therefore none are purchasing either A or $B1$ alone) it cannot be that $\bar{P}^0 < \gamma$ since, if it were, firm 1 could do better by offering only the bundle at a price of γ. But, if so, then setting $(\hat{P}_A = \gamma, \hat{P}_{B1} = \bar{P}^0 - \gamma)$ yields identical sales and profits to both firms given P_{B2}^0 and identical profits to firm 2 for all P_{B2}. Second, suppose instead that $\gamma \geq P_A^0$. Note first that we must have $\bar{P}^0 \geq P_A^0$ in such an equilibrium: otherwise all consumers would be buying firm 1's bundle (all consumers would be willing to buy good A individually and they can get good A cheaper by buying the bundle) and firm 1 would increase its profits by offering only the bundle at a price of γ. But if $\gamma \geq P_A^0$ and $\bar{P}^0 \geq P_A^0$, then each consumer buys either good A alone or the bundle from firm 1. In this case, prices of $(\hat{P}_A = P_A^0, \hat{P}_{B1} = \bar{P}^0 - P_A^0)$ yield identical sales and profits for both firms for all P_{B2}.

A similar argument establishes the equivalence for the subgame where only firm 1 is active. □

The basic idea behind Proposition 1 is fairly straightforward. First, it is always worthwhile for firm 1 to make sure that all consumers purchase product A either alone or in the bundle. Given that all consumers are consuming good A, however, if firm 1 engages in tying, then consumers choose between buying only good A or the bundle from firm 1. They do so by imputing

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[6] I assume that all consumers will buy good A when $P_A = \gamma$. This assumption can be avoided through the use of limiting arguments, but it is made in order to ease the exposition.

an effective price of $(\bar{P}_1 - P_A)$ $((\bar{P}_1 - \gamma)$ if $P_A > \gamma)$ to the product $B1$ portion of the bundle, so that tying is effectively equivalent to an independent pricing strategy.

B. Commitment and Strategic Foreclosure

The negative result of Proposition 1 changes dramatically if firm 1 is able to precommit to tying through its choice of which goods it will be able to produce. In the three-stage game that I have described above, firm 1 can choose to produce seven different sets of goods: both goods individually, both goods individually and also a bundle, the bundle only, the bundle and product A, the bundle and product $B1$, A only, and $B1$ only. The argument in Proposition 1 implies that the first two options both yield outcomes equivalent to those in the independent pricing game and so they are strictly better for firm 1 than the last two (which yield lower profits to firm 1 in any subgame where it is active and at least as large profits to firm 2 when it is active). In fact, the following two lemmas indicate that firm 1's choice is essentially between producing independent goods and producing only the bundle.

LEMMA 1: *Any subgame perfect equilibrium outcome in the subgame of the commitment game where firm 1 can produce only the bundle and product A is equivalent to a subgame perfect equilibrium outcome of the independent pricing game.*

PROOF:
  The argument closely parallels that used to prove Proposition 1 and is omitted here.
  □

LEMMA 2: *Any subgame perfect equilibrium outcome in the subgame of the commitment game where firm 1 can only produce the bundle and product B1 is equivalent to a subgame perfect equilibrium outcome that arises in the subgame of the commitment game where firm 1 can only produce a bundle.*

PROOF:
  In Appendix A.   □

Given these results, firm 1 can restrict its attention to either producing goods A and $B1$ separately, which yields an outcome equivalent to that in the independent pricing game, or to committing to producing only a bundle. I now turn to an investigation of the competitive effects of this tying strategy. As the following result makes clear, such a commitment may make it unattractive for firm 2 to be active in the market.

PROPOSITION 2: *In the subgame of the commitment game where both firms are active and firm 1 has committed itself to producing only the bundle, firm 2 earns less than it does in the independent pricing game.*

PROOF:
  In Appendix A.   □

One might at first think that bundling in this context would have no effect at all: if firm 1 were charging independent prices of $P_A = \gamma$ and P_{B1}, a switch to bundling at a total price of $\gamma + P_{B1}$ would not change the demand for good $B1$ at all. The intuition for Proposition 2, however, centers on the way in which firm 1's pricing incentives change when it bundles. In an independent pricing game, firm 1's best response $P_{B1}^*(P_{B2})$ satisfies,

$$(1) \quad [P_{B1}^*(P_{B2}) - c_{B1}]x_1^1(P_{B1}^*(P_{B2}), P_{B2})$$
$$+ x^1(P_{B1}^*(P_{B2}), P_{B2}) = 0.$$

By contrast, when firm 1 bundles and sets price $\bar{P}$, the demand for its bundle is given by $x^1(\bar{P} - \gamma, P_{B2})$ and its best response to firm 2's price P_{B2} by $\bar{P}^*(P_{B2})$ such that

$$(2) \quad [\bar{P}^*(P_{B2}) - c_A - c_{B1}]$$
$$\times x_1^1(\bar{P}^*(P_{B2}) - \gamma, P_{B2})$$
$$+ x^1(\bar{P}^*(P_{B2}) - \gamma, P_{B2}) = 0.$$

Note first that if $\gamma = c_A$, then $\bar{P}^*(P_{B2}) = P_{B1}^*(P_{B2}) + \gamma$. However, if $\gamma > c_A$, then at $\bar{P} = P_{B1}^*(P_{B2}) + \gamma$ the left-hand side of (2) is strictly negative. Thus, it must be that $\bar{P}^*(P_{B2}) < P_{B1}^*(P_{B2}) + \gamma$: firm 1's optimal ef-
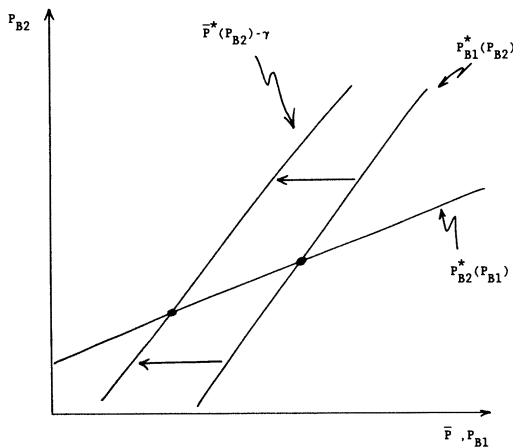
FIGURE 1

fective price for good $B1$ is lower under bundling than under independent good pricing. The reason is straightforward: when firm 1 is bundling, in order to make profitable sales of its monopolized product, good A, it must also make sales of good $B1$. This leads it to cut price in an effort to take sales away from firm 2, an effect I call "strategic foreclosure."[7] The effect on the equilibrium can be seen in Figure 1, where the equilibrium effective price for $B1$ and actual price for $B2$ both fall as a result of firm 1's bundling, thereby lowering firm 2's profits. Thus, by committing to tie by producing only a bundle, firm 1 may make continued operation unprofitable for its tied good rival. This point emerges particularly clearly in the following simple example, which is a limiting case of the above model.

Example 1. Suppose that all consumers view products $B1$ and $B2$ as perfect substitutes with value v, that $(c_{B1} - c_{B2}) > K_2 > 0$ and, to focus attention on firm 2's activity decision, that $K_1 = 0$ (this could be a situation of entry deterrence where only firm 1 has already sunk its market B set-up costs). Then the subgame perfect equilibrium outcome of the independent pricing game has firm 2 being active in market B, making all sales in that market, and earning profits of $(c_{B1} - c_{B2}) - K_2 > 0$.[8] By contrast, if $(c_{B2} - c_{B1}) + (\gamma - c_A) > 0$ and firm 1 commits to bundling, firm 2 earns zero if it is active, and so the unique equilibrium outcome involves firm 2 being inactive and firm 1 extracting all of the consumers' surplus.

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Note that if both firms are active, firm 1's profits are also lower in the bundling regime than under independent pricing. This is true because bundling not only loses some profitable sales of good A, but also causes firm 2 to lower its price.[9] Thus, in this model, firm 1 would never commit to tying unless this would succeed in driving firm 2 out of the market.[10]

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[7]Jean Tirole has pointed out a nice analogy to situations in which firms can invest in cost reduction. Here, by bundling, firm 1 can incur an "investment cost" of $(\gamma - c_A)$ (the lost good A sales) but thereby lowers its effective marginal cost in market B by $(\gamma - c_A)$. This lowering of marginal cost makes firm 2 more agressive in market B. As noted in Drew Fundenberg and Tirole (1984) (see also Tirole, 1988), with price competition (strategic complements) and entry deterrence/exit inducement, firms overinvest in cost reduction relative to what they would do absent this strategic effect (a "top dog" strategy), a comparison analogous to my commitment versus no commitment games.

[8]I am ignoring subgame perfect equilibria in which a firm prices below cost and makes no sales. These equilibria involve weakly dominated strategies and can be formally eliminated here through the use of R. Selten's (1975) notion of trembling-hand perfect equilibria (formally one examines discrete approximations to the game considered in the text where prices must be named in some discrete unit of account).

[9]Note that these lower profits can potentially force firm 1 to exit should it commit to tying and have positive fixed costs in market B (if it believes that firm 2 will be active). If product A is very profitable, however, this effect is unlikely to occur.

[10]One special feature of this model, however, is that firm 2 can "concede" to firm 1 only by fully withdrawing from the market. In other models in which concession can be partial, this need not be true. For example, if market competition is of the form described in David M. Kreps and Jose A. Scheinkman (1983) (product production followed by output constrained price competition), then firm 2 will respond to firm 1's more aggressive behavior by reducing its production level, which can make tying profitable even in the absence of complete exclusion.

When tying would drive firm 2 out of the market, firm 1 may or may not find it profitable to do so. The advantage of tying in such an instance is the gain from converting market B from duopoly into a monopoly. The potential loss, however, comes from the fact that firm 1 will be a monopolist who can only offer a bundle. Thus, the presence of a large number of consumers who strongly dislike product $B1$ may make a commitment to bundling unprofitable, even when it leads to exclusion.

At the same time, the welfare consequences of allowing tying in this circumstance are unclear both for consumers and for aggregate efficiency. First, consumers can lose both because of the price effects stemming from the exclusion and also because there is less variety available in market B. The price effect, however, can potentially go either way. The reason is that the same incentive to lower the effective price of good $B1$ that drives firm 2 from the market is also present when firm 1 becomes a monopolist in market B. In general, though, one should expect that if the gains from monopoly in market B are large, the standard price movement should be upward, making consumers uniformly worse off. The effect on aggregate efficiency is still less certain. This is due to two different common welfare ambiguities. First, the biases associated with the free entry process (Spence, 1976; Salop, 1979; Mankiw and Whinston, 1986) imply that exclusion of firms does not necessarily reduce aggregate welfare. Second, it is known from the monopolistic bundling literature (Adams and Yellen, 1976) that bundling in a monopoly setting has ambiguous welfare consequences.

The following example illustrates these points more concretely and also helps to set up the discussion in Section II.

Example 2. Suppose that a consumer of type d has a valuation for good Bi of $v_{Bi} = w - \alpha_i d$, and that d is uniformly distributed on $[0,1]$. Assuming that we have all consumers purchasing from some firm and both firms making sales (so that our earlier assumptions hold in the relevant range), it is straightforward to show that equilibrium

prices and profits (gross of fixed costs) in an independent pricing game are given by

$$P_A^0 = \gamma$$

$$P_{Bi}^0 = c_{Bi} + (1/3)$$
$$\times \left[3\alpha_j + (\alpha_i - \alpha_j) + (c_{Bj} - c_{Bi}) \right]$$

$$\Pi_1^0 = (\gamma - c_A) + \left[1/9(\alpha_1 + \alpha_2) \right]$$
$$\times \left[3\alpha_2 + (\alpha_1 - \alpha_2) + (c_{B2} - c_{B1}) \right]^2$$

$$\Pi_2^0 = \left[1/9(\alpha_1 + \alpha_2) \right]$$
$$\times \left[3\alpha_1 - (\alpha_1 - \alpha_2) - (c_{B2} - c_{B1}) \right]^2.$$

In contrast, profits (gross of fixed costs) for firm 2 when firm 1 bundles are given by

$$\Pi_2^0 = \left[1/9(\alpha_1 + \alpha_2) \right] \left[3\alpha_1 - (\alpha_1 - \alpha_2) \right.$$
$$\left. - (c_{B2} - c_{B1}) - (\gamma - c_A) \right]^2,$$

which is lower than in the independent pricing case. Note also that firm 2's profits fall as the surplus associated with good A, $(\gamma - c_A)$, rises. In order to illustrate the other points made above, I consider three special cases of this model in turn: $\alpha_1 = 0$, $\alpha_1 = \alpha_2$, and $\alpha_2 = 0$.

Consider first the case where $\alpha_1 = 0$. In this case, firm 1 always increases its profits by excluding firm 2 (that is, monopoly profits with bundling are greater than duopoly profits with independent good pricing). This is because as a monopolist firm 1 suffers no loss from bundling. Furthermore, the monopoly bundle price of $(w + \gamma)$ leaves all consumers with zero surplus. While all consumers are made worse off, aggregate welfare may rise or fall: if $c_{B2} > c_{B1}$ aggregate welfare must rise since all consumers are still served, and production costs fall. When $c_{B2} < c_{B1}$, the change in aggregate efficiency is given by $\Delta W = K_2 - \alpha_2 (c_{B1} - c_{B2})^2$.

For simplicity, assume now that $c_{Bi} \equiv c_B$. When $\alpha_1 = \alpha_2 \equiv \alpha$, the independent pricing equilibrium has full coverage of market B whenever $w > c_B + (3/2)\alpha$. For simplicity, I

also assume that $(\alpha/2) < (\gamma - c_A)$, which implies that firm 1 will always sell its bundle to all consumers when it is a monopolist (the qualitative results in the other case are similar). In that case, firm 1's price and profits are given by

$$\bar{P}^0 = w + \gamma - \alpha$$

$$\Pi_1^0 = w + \gamma - \alpha - c_A - c_B.$$

Comparison of these expressions with those for the independent pricing game (setting $\alpha_1 = \alpha_2$) reveals that firm 1 always gains from exclusion. The effective price of good $B1$ $(\bar{P} - \gamma)$, however, falls whenever $(c_B + (3/2)\alpha) \leq w \leq (c_B + 2\alpha)$, so that some consumers (for example, those who were already buying $B1$) are made better off in these cases. Aggregate consumer surplus, however, never rises here: with an independent pricing duopoly, consumer surplus is $w - c_B - (5/4)\alpha$, while it is $(\alpha/2)$ with a bundling monopolist.

Finally, when $\alpha_2 = 0$, firm 1 profits in an independent goods pricing duopoly are given by $\Pi_1^0 = (\gamma - c_A) + (1/3)\alpha_1$ (an interior solution arises whenever $w > c_B + (2/3)\alpha_1$). Then, assuming again that $(\alpha_1/2) < (\gamma - c_A)$, we have that firm 1's profits rise from this exclusion if and only if $w > c_B + (4/3)\alpha_1$. Notice that exclusion is more likely to be profitable as the value of monopolizing market B rises (increases in $(w - c_B)$ and decreases in α_1) and as the competitive constraint that firm 2 imposes when it is in the market becomes more severe (decreases in α_1).   ∎

II. Heterogeneous Consumer Preferences for Good A

The results of Section I provide two important lessons. First, tying can be profitably used as an exclusionary device. Second, there may be important differences in the likelihood of its use, depending on whether a commitment to tying is possible. A feature of that model, however, was the strong assumption that all consumers have the same valuation of the tying good. In this section, I investigate the effects of relaxing

that assumption. Two points emerge from this investigation. First, a commitment to tying need not always result in foreclosure as it did in the model of Section I. Second, when consumer valuations for the tying good differ, tying can be a profitable strategy for firm 1 even in the absence of an ability to commit, and when it is, it may lower firm 2's profitability in a similar manner to that observed earlier. In the following two subsections I consider first the case of commitment and then that of no commitment.

A. Commitment

In the model of Section I, firm 1's commitment to offering only a bundle lowered firm 2's sales because it created an incentive for firm 1 to price more aggressively, lowering its bundle price below $P_{B1}^*(P_{B2}) + \gamma$ (strategic foreclosure). More generally, when consumers have heterogeneous preferences for good A, the impact of tying on firm 2's profits can be determined by asking whether, at the bundle price $\bar{P}'$ such that firm 2's sales equal its independent pricing level (i.e., $\bar{P}'$ such that $x^2(\bar{P}' - \gamma, P_{B2}) = x^2(P_{B1}^*(P_{B2}), P_{B2})$), firm 1 has an incentive to lower its price further.[11] This will be true when

$$(3) \quad (\bar{P}' - c_A - c_B)\frac{d\text{ Bundle Sales}}{d\bar{P}}$$
$$+ x^1(P_{B1}^*(P_{B2}), P_{B2}) < 0.$$

With homogeneous preferences for good A, for example, the inequality in condition (3) is satisfied because $(\bar{P}' - c_A - c_B) > P_{B1}^*(P_{B2}) - c_{B1}$ and $(d\text{ Bundle Sales}/d\bar{P}) = x_1^1(P_{B1}^*(P_{B2}), P_{B2})$.

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[11] Here, unlike in Section I, firm 1 may prefer to commit to producing the bundle plus one of the two goods independently as part of an exclusionary strategy (i.e., Lemmas 1 and 2 do not hold here). I focus on the case of a commitment to pure bundling here to provide a comparison with the result in Section I. These other strategies may also lower firm 2's equilibrium profits. If they do so sufficiently to exclude firm 2 from the market, then they will actually be preferred by firm 1 to pure bundling, since they restrict its pricing to a lesser degree when firm 2 is out of the market.

Condition (3) indicates that, with heterogeneous valuations for good A, a commitment to offering only a bundle may fail to lower firm 2's profits for two distinct reasons. First, enough consumers may find good A unattractive (may have valuations below the cost of production) so that firm 1 may have a lower, rather than a higher, margin at price $\bar{P}'$. In such a case, firm 1's monopoly of good A is too weak for bundling to be an effective exclusionary threat in market B; bundling would help rather than hurt firm 2. This effect, of course, is exactly what one should expect a priori.

The second reason is a bit more subtle. As noted above, with homogeneous valuations, the derivative of bundle demand at price $\bar{P}'$ is identical to that arising in market B with independent goods pricing. With heterogeneous valuations, however, this demand derivative can change when firm 1 bundles, potentially counteracting the price-cost margin effect. The clearest example of this occurs in the limiting case where products $B1$ and $B2$ are nearly homogeneous.[12] Then bundling essentially transforms a nearly homogeneous market B into a setting with vertical differentiation (since all consumers value the bundle more than $B2$, but they differ in how large this valuation difference is—see, for example, Avner Shaked and John Sutton, 1982) and can thereby raise firm 2's profits.

The following example, which is an extension of Example 2, illustrates these points.

Example 3. The model considered here is identical to that in Example 2 except that I now allow there to be different possible levels of consumer valuations for product A. I assume that the distribution of γ in the population is described by $F(\gamma)$ and that for all d, $\text{Prob}(\gamma \le s|d) = F(s)$ (i.e., types are independently distributed across the two markets). In the discussion that follows, I assume that w is large enough so that (in the relevant range) all consumers purchase product B from one of the firms.

Suppose that firm 1 commits to tying by producing only a bundle. A consumer of type (γ, d) will buy the bundle if and only if $d \le (1/\alpha)[(\bar{P} - \gamma) - P_{B1}]$. It is useful to first assume that for any level of γ some consumers of that type are buying from each of the firms ("interior equilibria"). For interior equilibria, equilibrium profits for the two firms are given by

$$\Pi_1^0 = [1/9(\alpha_1 + \alpha_2)][3\alpha_2 + (\alpha_1 - \alpha_2)$$
$$+ (c_{B2} - c_{B1}) + (E\gamma - c_A)]^2$$
$$\Pi_2^0 = [1/9(\alpha_1 + \alpha_2)][3\alpha_1 - (\alpha_1 - \alpha_2)$$
$$- (c_{B2} - c_{B1}) - (E\gamma - c_A)]^2,$$

where $E\gamma = \int s\, dF(s)$. Comparing firm 2's profits to its level under independent goods pricing (derived in Section I), we see that firm 2's profits are lower in the bundling equilibrium as long as $E\gamma > c_A$. Thus, as one would expect, if there are enough consumers who dislike product A (have γ levels below c_A), tying raises rather than lowers firm 2's profits. Relating this finding to condition (3), we see that bundling has no effect on the demand derivative, but that $\bar{P}'$ is now given by $\bar{P}' = P_{B1}^*(P_{B2}) + E\gamma$ so that the inequality in (3) holds if and only if $E\gamma > c_A$.

It is worth noting, however, that the lack of any effect on the demand derivative term in (3) relies heavily on the linearity of the demand structure assumed here. If bundling were to lower this average derivative (in absolute value), this would work against the incentive for more aggressive pricing that arises in this linear model. In fact, this is why interiority of the equilibrium is important for the characterization above. When bundling causes all consumers with some values of γ strictly to prefer either the bundle or good $B2$, this lowers the derivative of firm 1's demand with respect to its bundle price (since none of these consumers are marginal). Intuitively, this effect seems more likely to occur when the dispersion of valuations for good A increases and the

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[12]Note that this requires that the K_i's are close to zero if independent pricing would result in a duopoly.

TABLE 1

	$\beta \geq \alpha$	$\beta < \alpha$
$\gamma \geq 3\|\beta - \alpha\|$	$\Pi_2^{\text{BUND}} < \Pi_2^{\text{IND}}$	$\Pi_2^{\text{BUND}} < \Pi_2^{\text{IND}}$
$\gamma \leq 3\|\beta - \alpha\|$	$\Pi_2^{\text{BUND}} < \Pi_2^{\text{IND}}$ if and only if: $\left(\dfrac{3\beta - \gamma}{3}\right)^2 < \alpha\beta$	$\Pi_2^{\text{BUND}} < \Pi_2^{\text{IND}}$

differentiation between products $B1$ and $B2$ decreases.

To investigate this effect further, consider the special case where γ is uniformly distributed on the interval $(\gamma - \beta, \gamma + \beta)$, where $\beta \leq \gamma$, $c_{B1} = c_{B2} = 0$, and $\alpha_1 = \alpha_2 \equiv \alpha > 0$.[13] In this example, the issue addressed above, that some consumers may value good A at less than its production cost, does not arise (here $E\gamma > c_A$). Rather, the focus here is on the effects of the level of valuation dispersion for good A and the level of product differentiation in market B. Tedious calculations (an example of which is provided in Appendix B) reveal that the effect of a commitment by firm 1 to offering only a bundle on firm 2's profits can be summarized as in Table 1. Examination of the condition in the' lower left-hand box (the only case where firm 2's profits are not necessarily lowered by firm 1's bundling) confirms that high levels of dispersion of valuations for good A and low levels of differentiation in market B are necessary for firm 2's profits to rise when firm 1 bundles. Interestingly, though, even when α is close to zero, we need γ not to be too large for this to occur (so that the incentive to make sales of A does not outweigh the

differentiation effect).[14] Note also that firm 1's profits may now rise with bundling even if bundling does not drive firm 2 from the market. In fact, in this example, whenever bundling causes firm 2's profits to rise, firm 1's profits rise as well and, further, firm 1's rise in some cases where firm 2's profits fall (this is shown in Appendix B).   ∎

B. No Commitment

The presence of heterogeneous valuations of product A can also cause tying to be firm 1's optimal strategy even in the absence of an ability to commit to this strategy. To see this more clearly, consider first the no commitment game analyzed in Section I. In that game, when both firms are active in market B, firm 1 selects its prices taking firm 2's price as given and acting as a monopolist on the residual demand structure. Given the literature on bundling by multiproduct monopolists (Adams and Yellen, 1976; McAfee, McMillan, and Whinston, 1989), which has found bundling to be a profitable strategy quite generally, it should not be surprising that firm 1 may now find some form of bundling to be its best response to firm 2's price choice. What is interesting from our perspective, however, is that this tying strategy by firm 1 may have detrimental effects on firm 2's profits since, when firm 1 does decide to bundle, it may have an incentive to foreclose sales in market B in a manner similar to that discussed in Section I.

In Whinston (1987), for example, I considered the structure described in Example 3 with two types of valuations for good A, γ_L and γ_H with $\gamma_H > c_A$ and $\text{Prob}(\gamma_H) = \lambda$. For this case I showed that any equilibrium of the no precommitment game is equivalent to an equilibrium of a game where firm 1 is allowed to either sell A and $B1$ independently or to offer only the bundle and

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[13]My investigation of this example is motivated in part by the example analyzed in independent work by J. Carbajo, D. DeMeza, and D. J. Seidmann (1987). They illustrate the differentiation effect in an example with homogeneous goods in market B and valuations for goods A and B that are perfectly correlated and uniformly distributed across consumers. Earlier versions of this paper pointed out the implications of noninteriority for the derivative of demand in the context of a two-type (of γ) example.

[14]The reader may be puzzled by this point since it seems that when $\alpha = 0$ firm 2's profits would always rise with bundling. In fact, when $\alpha = 0$, the upper left box would have $\Pi_2^{\text{BUND}} = \Pi_2^{\text{IND}}$ and firm 1 making all sales when it bundles.

product A at price $P_A \in (\gamma_L, \gamma_H].$[15] In addition, the equilibrium may involve firm 1 pursuing the latter (bundling) strategy, though a necessary condition for this is that $\gamma_L > c_A$ (see Whinston, 1987, for details). When the equilibrium does involve bundling and is "interior" in the sense discussed above, firm 2's profits are

$$\Pi_2^0 = [1/9(\alpha_1 + \alpha_2)][3\alpha_1 - (\alpha_1 - \alpha_2)$$
$$- (c_{B2} - c_{B1}) - (1 - \lambda)(\gamma_L - c_A)]^2.$$

Thus, when firm 1 does tie here, it forecloses firm 2's sales in a similar manner to that observed earlier. Note, though, that firm 2's equilibrium profits are larger here than when firm 1 commits to only offering a bundle. The reason is that when firm 1 also offers product A independently, it is assured of making sales of product A to all type H consumers regardless of whether they buy product $B1$; thus, here the incentive for foreclosure arises only from the L types and firm 2's profits fall only if $\gamma_L > c_A$.

Though the effect of firm 1's tying here may be exclusionary (firm 2, anticipating that firm 1 will tie, may choose to be inactive), one might argue that its motives are in some sense "innocent" since its decision to tie is never affected by the possibility that firm 2 might be excluded from the market. Such dynamic considerations, however, may be important even when firm 1 cannot precommit to tying. For example, if firm 2 faces a financial constraint that it must meet in order to remain active in the market (as in the work of J. P. Benoit, 1984; and Drew Fudenberg and Jean Tirole, 1986), firm 1

may be led to use tying in order to lower firm 2's profits and increase the likelihood that firm 2 will be forced to exit the market, even when tying is not profit-maximizing in a static sense.

To formalize this idea, consider a simple extension of the earlier no commitment model in which there are two production periods. If firm 2 decides to be active and incurs the set-up cost K_2 prior to period 1, it may face a financial constraint that it must meet after period 1 in order to be able to remain in the market in period 2. In particular, suppose that with probability $1 - \theta$ firm 2 will not face a financial constraint, while with probability $\theta f(\Pi)$ firm 2 will face a constraint that prohibits continued participation if first-period profits were less than Π and assume that $f'(\Pi) \geq 0$ (there is a diminishing marginal return to predation).

In this setting, what is the effect of an increase in θ on the attractiveness of tying for firm 1? It is not difficult to see that for the two type example if $\gamma_L > c_A$ (and outcomes are "interior") then increases in θ make tying a relatively more attractive policy for firm 1 in period 1 for any given level of P_{B2}. The central (and very general) idea is that increases in θ make firm 1 care more about foreclosure relative to current profits.

To see this more formally, let G denote the benefit to firm 1 if firm 2 does not meet its financial constraint and fix some initial level of θ and P_{B2}. Suppose, first, that firm 1 pursues its best independent pricing policy and that this results in a profit level for firm 2 of Π_2^I. Then, firm 1's price choices are equal to the level that it would choose in the simple one production period model if its marginal costs of production for $B1$ were $c_{B1} - \theta G f(\Pi_2^I)(P_{B2} - c_{B2})$ instead of c_{B1}. Likewise, if firm 1 pursues its optimal bundling strategy and thereby gives firm 2 profits of Π_2^B, then its prices are equal to those it would pick in the static game if its marginal cost was

$$c_{B1} - \theta G f(\Pi_2^B)(P_{B2} - c_{B2}).$$

Since we have seen that the optimal bundling strategy in the one period no pre-

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[15]That is, we can without loss of generality restrict firm 1's pricing strategy choices to one of these two forms. This equivalence actually holds for any market B structure that satisfies the assumptions made in Section II (a proof of this fact is available from the author upon request). It is worth noting that a bundling strategy of this sort may not appear to be tying at all since firm 1 does offer to sell product A at a price that some consumers are willing to pay. For type L consumers, however, this offer is unattractive, putting them in exactly the same situation as when firm 1 offers only a bundle.

commitment game results in lower profits for firm 2 then does the optimal independent pricing policy for any given level of c_{B1} (since $\gamma_L > c_A$), it must be that $\Pi_2^B < \Pi_2^I$ (that is, that bundling leads to foreclosure). But the envelope theorem then implies that a small increase in θ raises the profits from the optimal bundling best response by more than it raises the profits from the optimal independent pricing best response (since the derivative of firm 1 profits with respect to θ is $GF(\Pi_2)$). Thus, in this example, increases in θ strictly increase the likelihood that firm 1 will find bundling to be its best response (since bundling is never optimal if $\gamma_L < c_A$).[16]

III. Complementary Products

I now turn to the case of complementary products used in fixed proportions. I first consider a model of fixed proportions that is essentially an extension of the simple example quoted above from Posner (1976) to the case where the tied good market involves differentiated products with scale economies in production and an oligopolistic, rather than a competitive, market structure. Despite these differences, I show that Posner's central contention continues to hold: a monopolist of one component never finds it worthwhile to tie in order to reduce the level of competition in the market for the other component. The key point is that with complementary products used in fixed proportions, the monopolist can actually derive *greater* profits when its rival is in the market than when it is not because it can benefit through sales of its monopolized product from the additional surplus that its rival's presence generates (due to product differentiation).

Nevertheless, I then show that in two natural extensions of this model in which the monopolized product is no longer essential for all uses of other components, tying once again emerges as a profitable exclusionary strategy. In one case, the presence of an inferior, competitively supplied alternative to the "monopolized" component leads to results that parallel those of the independent products case. In the other case, the existence of a second use for the nonmonopolized product (such as a replacement part market) can give the monopolist an incentive to tie in order to eliminate competition in this other market.

The discussion in the text focuses on the case of precommitment. In fact, for each of the models considered here, any no precommitment outcome is equivalent to an equilibrium of the independent pricing game.[17] Of course, this is therefore also true when firm 1 produces A and $B1$ independently in the commitment game. In order to simplify the exposition, in Parts B and C below, I will use this fact and simply compare bundling outcomes to the independent pricing game equilibria when investigating whether firm 1 would find a commitment to bundling to be a profitable exclusionary device.

A. *The Basic Model*

Consider the following simple model. There are two components needed to comprise a system, A and B: a system consists of one unit of each. As before, firm 1 is a monopolist of component A, and two different versions of component B could potentially be available, $B1$ and $B2$. The production technology for these products is as before.

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[16]The two-type of γ example considered here is special in one sense. With more general distributions of γ, firm 1's best response in the one period no precommitment game will quite generally involve some form of bundling (see McAfee, McMillan, and Whinston, 1989). In such cases, one would have to examine how increases in θ affected the degree of bundling (i.e., the difference between $\bar{P}$ and $P_A + P_{B1}$).

[17]The proofs of this fact for the three models presented in this section are available from the author upon request. For the model of Part C, the result requires the use of Selten's (1975) notion of trembling-hand perfection in order to eliminate the use of weakly dominated strategies. In Parts B and C this equivalence is a consequence of the homogeneity of valuations assumed there (as in Section I).

The set of consumers is the same as in Section I. Each consumer demands at most one unit of the system. A consumer of type d's valuation of a system with product Bi is $v_{A/Bi}(d)$. When goods A, $B1$, and $B2$ are independently priced, consumers' demand for an A/Bi system is given by some function $x^i(P_A + P_{B1}, P_A + P_{B2})$, where $x_j^i(\cdot,\cdot) \geq 0$ if $i \neq j$ and ≤ 0 if $i = j$, with strict inequalities whenever $x^i(\cdot,\cdot) \in (0,1)$, and where $(x_i^1(\cdot,\cdot) + x_i^2(\cdot,\cdot)) \leq 0$.

In the case of independent products we implicitly assumed that purchase of a produced bundled unit allowed the independent use of either of the products (the proof of Lemma 1, for example, uses this fact). Though natural in the case of independent products, this assumption is less so when products must be used together. For example, the bundling of a stereo tuner and a stereo amplifier into a stereo receiver may not allow the buyer to use just the amplifier in conjunction with another manufacturer's tuner. Thus, here I assume that production of a bundled good does not allow the user to use only part of the bundle.

In this model, since component A is essential to any system, firm 1 is trivially able to exclude firm 2 by committing to produce only a bundle. Nevertheless, as the following proposition indicates, firm 1 never finds it worthwhile to tie in order to exclude firm 2.

PROPOSITION 3: *If a commitment to tying causes firm 2 to be inactive, firm 1 can do no worse — and possibly better — by committing to producing only independent components.*

PROOF:
Suppose that firm 1's precommitment to tying (by not producing one or both of the components individually) causes firm 2 to be inactive. In this case, since only firm 1's bundle price is relevant once firm 2 is inactive, firm 1's profits given its optimal bundle price of $\bar{P}^*$ are

$$(\bar{P}^* - c_A - c_{B1})x^1(\bar{P}^*, \infty).$$

Suppose that firm 1 instead commits to only

producing components A and $B1$ independently. One pricing policy that it can always follow, regardless of whether firm 2 is active, is to set individual component prices of $\hat{P}_{B1} = c_{B1} - \varepsilon$ (where $\varepsilon > 0$) and $\hat{P}_A = \bar{P}^* - \hat{P}_{B1}$. If firm 2 is inactive, this pricing scheme leads to exactly the same level of profits as did the bundling outcome. If firm 2 is active, however, firm 1's profits will be at least as large as those in the bundling outcome since they are given by

$$(\bar{P}^* - c_A - c_{B1})\big[x^1(\bar{P}^*, \hat{P}_A + P_{B2})$$
$$+ x^2(\bar{P}^*, \hat{P}_A + P_{B2})\big]$$
$$+ \varepsilon x^2(\bar{P}^*, \hat{P}_A + P_{B2}),$$

when firm 2 names price P_{B2} (since $x^1(\cdot,\cdot) + x^2(\cdot,\cdot)$ weakly increases when prices fall and $x^2(\cdot,\cdot)$ is nonnegative).   □

The basic idea behind this result is fairly simple to see. If firm 2 did not exist, firm 1 could do as well as it does through bundling by setting independent prices that had component $B1$ priced at or below cost and component A's price set at a high level; it would simply earn all of its profits on sales of component A (consumers' purchases depend only on the sum of the prices). But, if pricing in this manner leads firm 2 to be active, this can only raise firm 1's profits since firm 1 would then sell more component A's (on which it makes profits) and fewer component B's (on which it has a negative margin). Intuitively, firm 1 is able to benefit through sales of its product A from the increase in surplus generated by firm 2's presence.

While firm 1 never gains from committing to tying here if this forces firm 2 to be inactive, firm 1 may commit to tying in order to price discriminate. For example, suppose that some set of consumers get positive benefits only out of an $A/B1$ system, while the remainder get positive benefits only out of an $A/B2$ system, and that the latter group's valuation of its desired system is much higher. Then firm 1 will want to set a

very high price for good A in order to extract surplus from this latter group, and a very low price for good $B1$ in order to get an optimal $A/B1$ system price for the former group. If this attempt hits the nonnegativity constraint on P_{B1}, however, then firm 1 will find it worthwhile to tie by offering a bundle with price $\bar{P} < P_A$.[18]

Firm 1's lack of desire to use tying as an exclusionary device can change dramatically, however, when firm 1's monopolized component is not essential for all uses of product $B2$. I now consider two natural extensions of the above model in which tying can prove to be not only an effective exclusionary device but also a profitable one.

B. An Inferior, Competitively Supplied Component A: Strategic Foreclosure

Suppose that there exists a uniformly inferior, competitively supplied alternative to firm 1's product A, denoted as product $A2$ (henceforth, firm 1's product A will be denoted by $A1$). The cost of component $A2$ is also c_A, but compared with the valuations described above for $A1/B1$ and $A1/B2$ systems, a consumer's valuation for a system that has product $A2$ in it rather than $A1$ is $(\gamma - c_A)$ lower (i.e., $v_{A2/Bi}(d) = v_{A1/Bi}(d) - (\gamma - c_A)$) where $\gamma > c_A$.

Consider, first, the independent pricing game (which, as noted above, yields an outcome identical to what occurs if firm 1 produces A and $B1$ only independently). In this game, firm 1 always sets $P_{A1} \le \gamma$ and makes all component A sales. When firm 1 sets $P_{A1} < \gamma$ in this equilibrium, the inferior alternative (product $A2$) is irrelevant for pricing and profits. In the case where $P_{A1} = \gamma$, however, the presence of the infe-

rior product $A2$ constrains firm 1's equilibrium pricing and profits. This could mean that, contrary to Proposition 3, firm 1 would prefer to have firm 2 out of the market (firm 1 can no longer necessarily benefit through its component $A1$ sales from the surplus created by the presence of firm 2).[19] Example 4 illustrates this point and shows how the presence of component $A2$ can make competitive interaction here look very much like the independent products case considered earlier.

Example 4. Suppose that $v_{A1/Bi}(d) = w - \alpha d$, $c_A > 0$ and $c_{B1} = c_{B2} \equiv c_B > 0$, and that $w \ge 2\alpha + c_A + c_B$ (to ensure that all consumers buy a system; note the parallel to Example 2). Ignoring the constraint imposed by the presence of product $A2$, the independent pricing equilibrium level of P_{A1} is increasing in w.[20] When $w > \gamma + c_B + (3/2)\alpha$, the unique equilibrium involves prices of $P_A^0 = \gamma$ and $P_{B1}^0 = P_{B2}^0 = c_B + \alpha$, and all consumers receive positive surplus (see Figure 2). Profits (gross of fixed costs) are given by

$$\Pi_1^0 = (\gamma - c_A) + (\alpha/2)$$

$$\Pi_2^0 = (\alpha/2).$$

Note that this equilibrium essentially replicates the independent goods outcome from Section II (Example 2 with $\alpha_1 = \alpha_2$ and $c_{B1} = c_{B2}$). That is, the presence of a competitive constraint from product $A2$ serves to "uncouple" the two component markets. As in the independent products case, if w is large, firm 1's profits are increased by firm 2 being inactive (firm 1 then acts as a systems

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[18]This point is analogous to the observation that an upstream monopolist may wish to integrate vertically forward into one of the industries that uses its product in order to achieve price discrimination across users (see, for example, Tirole, 1988, p. 141). Note that bundled production is essential for this purpose since otherwise the second set of consumers would buy the bundle to get their component A whenever the bundle price was lower than the price of good A alone.

[19]The fact that the presence of an inferior competitively supplied product A can potentially prevent firm A from deriving maximal (two-product monopoly) profits has also been noted in Ordover, Sykes, and Willig (1985).
[20]More precisely, though multiple equilibria exist in the game when component $A2$ does not exist (corresponding to a range of values for P_{B1} and P_{B2} that is independent of w), in any such equilibrium the level of P_A is given by $P_A^* = w - (1/2)[\alpha + c_B - 3P_{B1}^*]$.
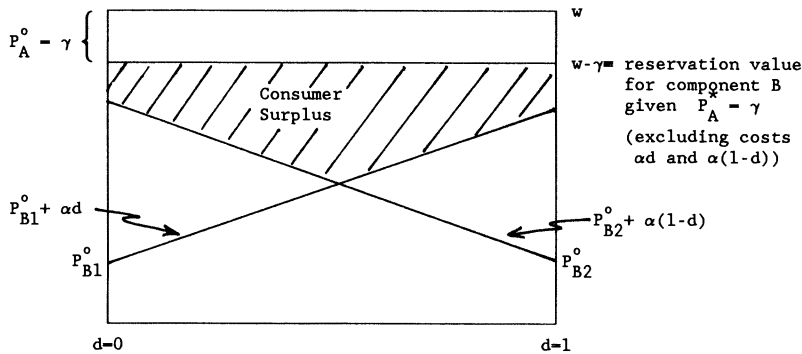
FIGURE 2

monopolist setting an $A1/B1$ system price of w).[21] ∎

Now consider the commitment game. When firm 1 would prefer firm 2 to be out of the market, can a commitment to tying by firm 1 force firm 2 out of the market (note that firm 1's component $A1$ is not essential)? The answer is yes, and for the same basic reason as in Section I: when it is only able to sell a bundle, firm 1 can only gain its profits from component $A1$ if it also sells component $B1$; this causes firm 1 to foreclose sales in the component B market. To see this formally, suppose that firm 1 can only produce a bundle. When the presence of product $A2$ constrains firm 1's pricing in an independent pricing game (so that $P_{A1} = \gamma$), firm 1's price for component $B1$ given firm 2's price P_{B2} is given by $P_{B1}^*(P_{B2})$ such that

$$(4) \quad \{[P_{B1}^*(P_{B2}) - c_{B1}] x_1^1(\gamma + P_{B1}^*, \gamma + P_{B2})$$
$$+ x^1(\gamma + P_{B1}^*, \gamma + P_{B2})\}$$
$$+ (\gamma - c_A)[x_1^1(\gamma + P_{B1}^*, \gamma + P_{B2})$$
$$+ x_1^2(\gamma + P_{B1}^*, \gamma + P_{B2})] = 0.$$

The first term of this expression represents the effect on sales of component $B1$ of marginally changing P_{B1}, while the second is the effect on sales of component $A1$. Note that this second change is due to the *total* change in system sales. In contrast, when firm 1 commits to bundling, its optimal bundle price given firm 2's price, P_{B2}, is given by $\bar{P}^*(P_{B2})$ such that

$$(5) \quad [\bar{P}^*(P_{B2}) - c_A - c_{B1}] x_1^1(\bar{P}^*, \gamma + P_{B2})$$
$$+ x^1(\bar{P}^*, \gamma + P_{B2}) = 0.$$

At $\bar{P} = P_{B1}^*(P_{B2}) + \gamma$, this expression becomes

$$(6) \quad \{[P_{B1}^*(P_{B2}) - c_{B1}] x_1^1(\gamma + P_{B1}^*, \gamma + P_{B2})$$
$$+ x^1(\gamma + P_{B1}^*, \gamma + P_{B2})\}$$
$$+ (\gamma - c_A) x_1^1(\gamma + P_{B1}^*, \gamma + P_{B2}).$$

Note that $x_1^2(\cdot, \cdot)$ does not appear in the second term of (6). This represents the fact that when firm 1 bundles, only by increasing sales of $A1/B1$ systems does it increase the sales of component $A1$. Firm 1 is therefore led to set $\bar{P}^*(P_{B2}) < P_{B1}^*(P_{B2}) + \gamma$, foreclosing sales in market B and lowering firm 2's profits.[22] Thus, by committing firm 1 to

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[21]Unlike the independent products case, firm 1 suffers no loss from bing restricted to bundle when it is a monopolist. Rather, here the cost of exclusion of firm 2 is that firm 1 is unable to capture any of the surplus created by firm 2 (through firm 1's sales of component A).

[22]This incentive for foreclosure is similar to the effects studied in Carmen Matutes and Pierre Regibeau (1986). They study product compatibility in a symmetric duopoly and identify a collusive incentive to

"strategic foreclosure," tying can exclude firm 2 from market B and thereby raise firm 1's profits.[23,24]

Example 4 *cont.* If firm 1 commits to only producing a bundle, firm 2's equilibrium profit when both firms are active is given by

$$\Pi_2^0 = \max\left\{0, (1/2\alpha)\left(\alpha - \frac{\gamma - c_A}{3}\right)^2\right\},$$

which is lower than its profit under independent pricing. Note that if firm 1 bundles and forces firm 2 to be inactive, all consumers receive zero surplus here (although, as usual, aggregate welfare may either fall or rise). ∎

C. *An Alternative Use for Product B: Direct Foreclosure*

Next, consider an alternative variation in the basic model. Suppose that there exists an alternative use for component B that does not rely on the simultaneous purchase of component A. One example of such a use is a replacement parts market for existing owners of a system who need to replace only component B. Because component A is not essential for the use of product B in that market, firm 1 is not able to benefit from firm 2's presence in this market through sales of good A and the logic of Proposition 3 therefore breaks down. Firm 1 may now find it worthwhile to exclude firm 2, if it can, in order to monopolize this other market for product B. Furthermore, because component A is still essential for certain uses of product B, firm 1 may have the means to accomplish this end: by offering to sell compoent A only in a bundle with component $B1$, firm 1 directly forecloses firm 2's sales in the joint use market (foreclosure of these sales is complete regardless of firm 1's bundle price), which may drive firm 2's profits below the level that justifies its continued operation. The following simple example illustrates these points.

Example 5. Suppose that there are two types of consumers. Type I consumers desire a system. There are a continuum of type I consumers indexed by the uniformly distributed variable $d \in [0,1]$ with total measure 1. Consumer d has valuations for the two possible systems of $v_{A/B1}(d) = w \cdot d$ and $v_{A/B2}(d) = w \cdot d + \gamma_1$. Type II consumers, of which there are a total measure of θ, only desire product B. Each type II consumer has valuations for products $B1$ and $B2$ of $v_{B1} = \varphi$ and $v_{B2} = \varphi + \gamma_2$. The firms are unable to discriminate (in a third degree sense) across these consumers in their pricing. The cost structure has $c_A > 0$, $c_{B1} = c_{B2} \equiv c_B > 0$, $K_2 > 0$, and $K_1 = 0$. Finally, I make two further assumptions:

(A1) $\qquad (1 + \theta)\gamma_2 > \gamma_1 > \gamma_2$

and

(A2) $\quad w > \max\{4\gamma_2 - \gamma_1 + c_A + c_B,$

$$\gamma_1 + c_A + c_B\}.$$

---

have compatibility. This corresponds to the "puppy dog" strategy in the Fudenberg and Tirole (1984) taxonomy for the case of accommodation under price competition, in contrast to the "top dog" strategy that I focus on here (see fn. 7).

[23] The discussion in the text has only compared producing A and $B1$ independently with producing only a bundle. One might wonder about other alternatives. In fact, it can be shown that as long as the sort of price discrimination motivation discussed in Part A is not present, any of the other alternatives are either equivalent to independent pricing (Bundle and $A1$; Bundle, $A1$, and $B1$), equivalent to producing only a bundle (Bundle and $B1$), or clearly inferior to these options ($A1$ only, $B1$ only).

[24] A long-standing issue in the legal treatment of tying is when to treat the tying and tied products as distinct products. A common argument is that the tied product must be one that consumers might want to purchase separately, without also purchasing the tying product. In Justice O'Connor's concurrence in *Jefferson Parish Hospital District No. 2 v. Hyde*, 466 U.S. 2 (1984), for example, she argues this position because "When the tied product has no use other than in conjunction with the tying product, a seller of the tying product can acquire no *additional* market power by selling the two products together." The model analyzed here illustrates that this view is incorrect unless one defines "other uses," contrary to Justice O'Connor's meaning, to include use with other producers' component A's.

Consider, first, the outcome of the independent pricing game. The unique equilibrium outcome when both firms are active involves prices of[25]

$$P_{B1}^0 = c_B$$

$$P_{B2}^0 = c_B + \gamma_2$$

$$P_A^0 = [w + (\gamma_1 - \gamma_2) + c_A - c_B]/2.$$

In this equilibrium, all consumers buying a component B buy product $B2$, and profits for the two firms are given by

$$\Pi_1^0 = [w + (\gamma_1 - \gamma_2) - c_A - c_B]^2/4w$$

$$\Pi_2^0 = \gamma_2 \cdot \left[\theta + \frac{w + (\gamma_1 - \gamma_2) - c_A - c_B}{2w}\right] - K_2.$$

Suppose, instead, that firm 1 commits to producing only a bundle and product $B1$ alone. In this case the unique equilibrium prices when firm 2 is active are given by

$$P_{B1}^0 = c_B$$

$$P_{B2}^0 = c_B + \gamma_2$$

$$\bar{P}^0 = (w + c_A + c_B)/2,$$

and profits are

$$\Pi_1^0 = (w - c_A - c_B)^2/4w$$

$$\Pi_2^0 = \gamma_2 \cdot \theta - K_2.$$

Thus, by committing to tie, firm 1 denies firm 2 its profitable sales to type I consumers, lowering firm 2's profits, and possibly forcing firm 2 to be inactive. Furthermore, if tying does force firm 2 to be inactive, firm 1's profit is $((w - c_A - c_B)^2/$

$4w) + \theta(\varphi - c_B)$, which is larger than its independent pricing profits if φ, the gain from monopolizing the type II market, is large.[26] Finally, if firm 1 does exclude firm 2 in this manner, all consumers are made worse off here, although aggregate welfare may either fall or rise.   ∎

IV. Conclusion

The above results demonstrate, in my view, that the leverage hypothesis can be formally modeled in a coherent and appealing way. Once one allows for scale economies and strategic interaction, tying can make continued operation by a monopolist's tied market rival unprofitable by leading to the foreclosure of tied good sales. As the models above have indicated, such a strategy can be a profitable one for a monopolist, often precisely because of this exclusionary effect on market structure.

While the analysis vindicates the leverage hypothesis on a positive level, its normative implications are less clear. Even in the simple models considered here, which ignore a number of other possible motivations for the practice, the impact of this exclusion on

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[25] I am ignoring equilibria here that involve firm 1 pricing its component $B1$ below cost and making no sales. As earlier, these equilibria involve the use of a weakly dominated strategy by firm 1 and can be eliminated through the use of Selten's (1975) notion of trembling-hand perfection.

[26] The reader may be wondering about other alternatives available to firm 1. A commitment to producing A only, $B1$ only, or just a bundle is worse as an exclusionary strategy for firm 1 than committing to produce the bundle and $B1$ since firm 2's profits are higher when it is active under these strategies than when firm 1 commits to produce the bundle and $B1$, and firm 1's profits are lower under these options if firm 2 is inactive. They also are less attractive as an accommodation strategy for firm 1 than independent production of A and $B1$. Producing A, $B1$, and a bundle yields an outcome equivalent to the independent production outcome (restricting attention to trembling-hand perfect equilibria). Finally, producing a bundle and A is less effective as an exclusionary strategy than producing a bundle and $B1$ (it gives firm 2 higher profits if it is active and firm 1 lower profits when firm 2 is not active), and when firm 2 is active, no pure strategy (trembling-hand perfect) equilibrium with this product offering can give firm 1 higher profits than when it produces A and $B1$ independently. However, a pure strategy equilibrium may not exist here. A sufficient condition for a pure strategy equilibrium to exist is that $\gamma_2 < (\varphi - c_B)$. Thus, when this condition holds, firm 1 can effectively limit itself to the two options considered in the text.

welfare is uncertain. This fact, combined with the difficulty of sorting out the leverage-based instances of tying from other cases, makes the specification of a practical legal standard extremely difficult.

Finally, it should be noted that the leverage debate is not limited to the practice of tying, but rather arises in numerous areas of antitrust analysis. With the practice of reciprocity, for example, a monopsonistic buyer of some product refuses to buy from his suppliers unless they also buy a product (in which he may face competition) from him. Alternatively, when a vertically integrated monopolistic input supplier can sell his input to both his own downstream manufacturer and to a rival manufacturer, a refusal to supply this rival manufacturer is similar to the tying of complementary goods. The results here raise the possibililty that the use of leverage as an effective and profitable exclusionary device could arise in these other settings as well.[27]

APPENDIX A

PROOF OF LEMMA 2: Suppose, first, that firm 2 is active and that the equilibrium prices are $((P_{B1}^0, \bar{P}^0); P_{B2}^0)$. There are two cases to consider. If $\bar{P}^0 \le P_{B1}^0 + \gamma$, then all consumers prefer the bundle to buying only good $B1$ from firm 1 (again, for expositional reasons, I assume here that consumers buy the bundle when they are indifferent). If so, then firm 1 selling only the bundle at price $\bar{P}^0$ generates identical sales and profits for both firms for all P_{B2}. If $\bar{P}^0 \ge P_{B1}^0 + \gamma$, then it must be that firm 1 is making no sales since otherwise it could do better by setting $\bar{P} = P_{B1}^0 + \gamma$ (it would make exactly the same number of sales of the bundle as it did of $B1$, but at a larger margin since $\gamma > c_A$). In this case, firm 1 selling only the bundle at price $P_{B1}^0 + \gamma$ generates identical sales and profits for both firms when $P_{B2} = P_{B2}^0$ and for firm 2 for all P_{B2}.

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[27]In fact, the models analyzed above can frequently be reinterpreted to apply to these other settings. Some differences do arise, however, in modeling the various practices. For example, the extent to which commitment is possible is likely to vary by practice. Also, in the case of vertical integration discussed in the text, the upstream monopolist sells his component (input) to the unintegrated final goods producer, who then sets a price for the entire system (finished good) rather than selling directly to consumers who put a system together themselves. Furthermore, in such a setting, more complicated wholesale contracts may be possible than the simple linear pricing considered here.

A similar argument holds if firm 2 is not active. Therefore, any perfect equilibrium outcome (including decisions regarding activity in market B) is equivalent to one that arises after firm 1 has committed to producing only the bundle.

PROOF OF PROPOSITION 2: The argument is a simple comparative statics exercise. Letting $\phi \equiv \bar{P}_1 - \gamma$, firm 1's problem, given P_{B2}, can be written

$$\max_{\phi} \left[(\phi - c_{B1}) + (\gamma - c_A) \right] \cdot x^1(\phi, P_{B2}).$$

The bundling equilibrium is then characterized by the following two equations, which have a unique solution (with positive sales by both firms) under our assumptions.

$$[(\phi^{**} - c_{B1}) + (\gamma - c_A)] x_1^1(\phi^{**}, P_{B2}^{**})$$
$$+ x^1(\phi^{**}, P_{B2}^{**}) = 0$$

$$(P_{B2}^{**} - c_{B2}) x_2^2(\phi^{**}, P_{B2}^{**}) + x^2(\phi^{**}, P_{B2}^{**}) = 0.$$

Note that if $\gamma = c_A$, then $(\phi^{**}, P_{B2}^{**}) = (P_{B1}^*, P_{B2}^*)$, the independent pricing equilibrium. Now define (omitting arguments of functions):

$$A \equiv 2x_1^1 + [(\phi^{**} - c_{B1}) + (\gamma - c_A)] x_{11}^1$$

$$B \equiv 2x_2^2 + (P_{B2}^{**} - c_{B2}) x_{22}^2$$

$$C \equiv x_2^1 + x_{12}^1 [(\phi^{**} - c_{B1}) + (\gamma - c_A)]$$

$$D \equiv x_1^2 + x_{21}^2 (P_{B2}^{**} - c_{B2}).$$

The assumption that $P_{Bi}^{*\prime}(P_{Bj}) \in (0,1)$ implies that $(A, B) \ll (-C, -D) \ll 0$. This then implies that

$$\text{sign} \frac{d\phi^{**}}{d\gamma} = \text{sign}\{-Bx_1^1\} < 0$$

$$\text{sign} \frac{dP_{B2}^{**}}{d\gamma} = \text{sign}\{Dx_1^1\} < 0,$$

so that both firms' profits fall relative to the independent pricing equilibrium.   □

APPENDIX B

Here I work out the example for the cases where $\beta \ge \alpha$. In this class of cases, the division of consumers between the two firms can be represented diagrammatically as in Figure 4. Consider first equilibria that are in region (i), that is, that satisfy $\bar{P} \le (\gamma - \beta) + (P_{B2} + \alpha)$. In this region, the first-order conditions for the two firms are as follows (these conditions are sufficient for a maximum since at any point where these conditions
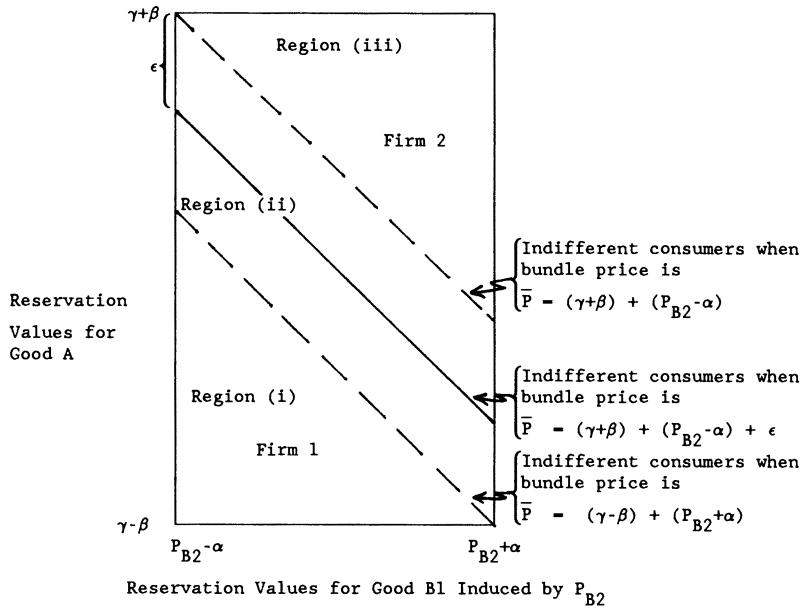
FIGURE 3

hold, the firms' profit functions are concave):

Firm 1: $4\alpha\beta - (1/2)[\bar{P} - (\gamma - \beta) - (P_{B2} - \alpha)]^2$

$$- \bar{P}[\bar{P} - (\gamma - \beta) - (P_{B2} - \alpha)] = 0$$

Firm 2: $[\bar{P} - (\gamma - \beta) - (P_{B2} - \alpha)] - 2P_{B2} = 0.$

From firm 2's first-order condition we see that we are in region (i) if and only if $P_{B2} \le \alpha$. Solving the two first-order conditions for P_{B2} yields the following expression:

$$-8(P_{B2})^2 + 2[\alpha - (\gamma - \beta)]P_{B2} + 4\alpha\beta = 0.$$

This expression is strictly concave and is nonnegative at $P_{B2} = 0$. Hence, $P_{B2} \le \alpha$ if and only if the value of this expression is nonpositive at $P_{B2} = \alpha$. Substituting yields the requirement that $\gamma \ge 3(\beta - \alpha)$. Firm 2's profits in this region under bundling are $(1/2\alpha\beta)(P_{B2})^3$ compared with its profits of $(\alpha/2)$ under independent goods pricing. Since $P_{B2} \le \alpha$ in this region, firm 2's profits must fall.

Consider now bundling equilibria that fall in region (ii), that is, where $\bar{P} \in ((\gamma - \beta) + (P_{B2} + \alpha), (\gamma + \beta) + (P_{B2} - \alpha))$. Straightforward analysis of the firms' first-order conditions reveals that in equilibrium we must have $3P_{B2} = 3\beta - \gamma$. In addition, to be in region (ii), P_{B2} must satisfy $2\beta - \alpha \ge P_{B2} \ge \alpha$, or substituting for P_{B2}: $3(\beta - \alpha) \ge \gamma \ge 3(\alpha - \beta)$. The first of these inequalities is just the reverse of our region (i) condition, while the second, which assures that we are not in

region (iii), is always satisfied since $\beta \ge \alpha$ (in fact, the bundling equilibrium can never be in region (iii)). Firm 2's profits under bundling in this region are given by $(1/2\beta)(P_{B2})^2$ compared with $(\alpha/2)$ under independent goods pricing. Substituting for P_{B2} yields the condition in the text. Firm 1's profits under bundling in this region are given by

$$\Pi_1 = \bar{P}\{1 - (1/2\beta)[\bar{P} - (\gamma - \beta) - P_{B2}]\}$$

$$= \left(\frac{3\beta + \gamma}{3}\right)\left(\frac{1}{2} + \frac{\gamma}{6\beta}\right),$$

while under the parameter values of this region its independent goods pricing profits are given by

$$(\alpha/2) + \left(\frac{\gamma + \beta}{2}\right)^2 (1/2\beta).$$

Bundling then yields firm 1 larger profits than independent pricing (assuming that firm 2 remains active) if and only if

$$\left(\frac{9\beta + 5\gamma}{6}\right)\left(\frac{3\beta - \gamma}{6}\right) \ge \alpha\beta.$$

But the expression on the left side of this inequality is strictly larger than $((3\beta - \gamma)/3)^2$, which implies that whenever firm 2's profits are higher under bundling, so are firm 1's. There is also clearly an area of the parameter space where firm 1 is better off and firm 2 is

worse off under bundling compared to independent goods pricing.

The analysis of cases where $\alpha > \beta$ proceeds in a similar manner.

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