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Equilibrium Product Bundling*

This article offers an equilibrium theory of product bundling by rival firms. In several models where a primary good is produced in a duopoly market and one or more other goods is produced under perfectly competitive conditions, bundling is shown to emerge as an equilibrium strategy of one or both of the duopolists for its role as a product-differentiation device. When the rival firms can commit to bundling sales, their profits are higher, but social welfare is reduced.

I. Introduction

Product bundling, the practice of selling two or more products in a package, is a widely observed business phenomenon. The existing literature on bundling falls into two broad categories: the price discrimination theory and the leverage theory. The former views bundling as a strategy by a monopoly firm to engage in price discrimination. This view was first suggested by Stigler (1968) and was further analyzed by Adams and Yellen (1976), Schmalensee (1982, 1984), and others. More recently, McAfee, McMillan, and Whinston (1989) have provided sufficient conditions under which bundling is an optimal selling strategy for a multiproduct monopolist. According to the leverage theory, on the other hand, bundling is viewed as a mechanism that enables a firm with monopoly power in one market to use the leverage provided by this power to foreclose sales in, and thereby monopolize, a second market. A formal argument of this is elegantly made in Whinston (1990).

This article takes a different approach in studying the practice of product bundling. Rather than focusing on firms that are monopolists in their primary markets (products), I assume that

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the primary market of interest here is a duopoly[1] and that there is perfect competition in the production of other goods with which the primary good can be bundled. Such an approach is motivated by observations that in many markets product bundling is actually used by one or several competing firms. Examples are abundant: credit-card issuers bundle the use of their cards with varieties of goods that are awarded to customers; rival computer firms produce and sell computers bundled with varieties of software; competing long-distance telephone companies, as well as competing hotel chains, offer free frequent-flyer miles when customers purchase their primary products. To account for such market phenomena, it appears warranted to have an equilibrium theory of product bundling by rival firms.[2]

In several models that we shall study in the following sections, I show that at least one firm in the duopoly market chooses the strategy of bundling in equilibrium, and both firms earn positive profits even though they produce a homogeneous product there and compete in prices. The central idea is that bundling enables competing firms to differentiate their products and thus reduce price competition in their primary market. In the absence of this strategic effect, a firm in the models considered hère has no incentive to bundle its primary product with a second product, which is, after all, produced competitively. The strategic effect that bundling creates on the primary market, however, makes it a profitable strategy. (It is interesting to compare this with Whinston [1990], where bundling has a strategic effect on an oligopoly market, which is the second market for the monopoly producer of a good.) I also show in my models that bundling, while benefiting the firms, always reduces social welfare, which is measured as the sum of consumer and producer surpluses.[3] This is in contrast to the ambiguous welfare effects of bundling that are usually found in the literature (see Whinston [1990] for a discussion of the reasons for the ambiguity there).

My basic model is studied in Section II, in which two firms, named A and B, produce a homogeneous product in a market named X. There is another market, named Y, which is perfectly competitive.[4] Consumers have homogeneous preference for good X but heterogeneous preferences for good Y. The competition between A and B is modeled as

1. It will become clear later that the intuition underlying my analysis is also valid if the primary market is of a more general oligopoly structure. The focus on a duopoly is mainly for technical convenience.

2. McAfee et al. (1989) contains a discussion on extending their monopoly model to a duopoly.

3. See, however, discussions in Sec. VI about possible alternative reasons for product bundling that may suggest a positive welfare effect associated with bundling and that are not modeled in this article.

4. The setup here follows Schmalensee (1982), except that the primary market of interest here is a duopoly instead of a monopoly.

a two-stage game. In the first stage, both firms simultaneously make product choice decisions: each can either produce X only or produce Y as well as X and then bundle X with Y (pure bundling). In the second stage, both A and B compete in prices. For this extremely simple model, I show that the only possible subgame-perfect equilibrium outcome is for one firm to offer X only and the other firm to offer both X and Y as a (pure) bundle; I exhibit the sufficient conditions for equilibrium existence. The welfare implications of the analysis are provided. Section III extends the basic model to include the strategy of mixed bundling, that is, selling X and Y as a bundle while offering X for separate purchase. It is well know that in monopoly models of product bundling, mixed bundling (weakly) dominates pure bundling. Here we obtain an opposite result: mixed bundling is a weakly dominated strategy. The simpler setup of our basic model that excludes mixed bundling is thus justified. Section IV relaxes the assumption of homogeneous consumer preference for good X of the basic model. With heterogeneous consumer preferences for good X, the essential results of our basic model continue to hold, even though the conditions for equilibrium existence are different now and have to be expressed in more implicit terms in general. Section V extends the basic model to the case with more than one second good. The main insights of the basic model are again valid, although it is now possible that both firms will choose (different) bundles in equilibrium. Section VI concludes my discussion.

II. The Basic Model and Its Equilibrium

There are two firms, A and B, producing a homogeneous product in a market named X. We call X the primary market of A and B, to be distinguished from other products (markets) that will be introduced. The constant marginal cost of both A and B in producing X is $c \geq 0$. There is a continuum of consumers of measure 1 in market X, each having a reservation value of r for one unit of x (but zero valuations for extra units).

There is another product (market), named Y, that may also be desired by the consumers in market X.[5] Their valuations for Y are independent and are identically distributed realizations of the random valuable V. The density function of V is $g(v)$, and $g(v) > 0$ on V's support $[\underline{v}, \overline{v}]$. The cumulative distribution function is $G(v)$. There is a sufficiently large number of perfectly competitive firms in the Y market such that Y is always available for the price c_y, where c_y is the constant marginal cost of producing Y for all firms, and $\underline{v} < c_y < \overline{v}$. We model

5. There can also be consumers who are not market X participants but may nevertheless buy good Y. I assume throughout the article that resale by consumers is not possible.

the production and pricing decisions of firms A and B as a two-stage game.

Stage 1. A and B decide simultaneously what products to offer. Since there is perfect competition in market Y, there is zero profit to be made in product Y if Y is to be produced and sold separately by either firm. For convenience, we therefore assume that neither A nor B will produce Y independently of X. This leaves us with the following choice set for both A and B: sell (produce) X only (denoted as X); sell (produce) X and Y as a (pure) bundle (denoted by XY); or offer X and XY at the same time (mixed bundling, denoted as X&XY).[6] To avoid unnecessary complications, however, I shall further exclude mixed bundling from each firm's choice set in the basic model here, and I shall justify this treatment in the next section by showing that mixed bundling is a (weakly) dominated strategy for each firm even when it is allowed. Each firm's strategy at the first stage is to choose an action in its choice set. We assume that each firm's product-choice decision cannot be reversed once it is made in a time period relevant for this model (see, e.g., Whinston [1990] on how such a commitment may be made).

Stage 2. For each pair of actions that are chosen by A and B at stage 1, there corresponds a subgame in stage 2 where each firm chooses a price (a real number) for either the single good (X) or the bundle (XY). We call those the pricing subgames. Thus, for example, if A's first-stage choice is X and B's first-stage choice is XY, then A's choice at the second stage is to select a price for good X, and B's choice is to select a price for bundle XY. Altogether, there are four possible subgames at the second stage, each denoted by (a, b), where $a \in \{X, XY\}$ is A's product choice and $b \in \{X, XY\}$ is B's product choice.

I shall use subgame-perfect equilibrium as my equilibrium concept. An equilibrium in this game is therefore defined as a pair of firm A's and B's strategies that constitutes a Nash equilibrium in each pricing subgame as well as in the full game. It should be emphasized that in my definitions of strategies at the first and second stages, only pure strategies are considered. Without this restriction on strategies, the insights of this discussion are still valid, but the analysis would be much lengthier.[7] Hence, unless otherwise indicated, by equilibrium I shall mean pure-strategy equilibrium. (Later in this section, though, I shall provide a result that allows mixed strategies.) The equilibria of the model can be found using the familiar method of backward induc-

6. I assume that both firms will remain in market X. If a firm wants to produce zero quantity of X, it can always do so by charging an infinitely high price for X.

7. Much more work would be needed in order to characterize the complete set of equilibria and their corresponding profits in all pricing subgames when mixed strategies are allowed, especially in the next section where there are nine such subgames.

tion: I first characterize (pure-strategy) Nash equilibria for all the pricing subgames and the firms' profits associated with them; then I find each firm's product choices at the first stage that constitute a Nash equilibrium. Since each firm's choice set at the first stage is finite, an equilibrium of the full model exists if and only if a Nash equilibrium exists for each of the second-stage pricing subgames.

We start from the subgame where $a = b = \text{X}$. From the standard argument in Bertrand competition (see, e.g., ch. 5 in Tirole [1988]), it is obvious that a unique pure-strategy Nash equilibrium exists in this subgame. At this equilibrium, both firms set prices equal to c and thus both earn zero profit. Similarly, in subgame $a = b = \text{XY}$, the unique pure-strategy Nash equilibrium is for each firm to charge $c + c_y$ for the bundle and also obtain zero profit. We therefore have

LEMMA 1. For any pricing subgame where $a = b$, both firms earn zero profit at the unique pure-strategy Nash equilibrium of the subgame.

Next, we turn to the subgames where one firm offers X and the other firm offers XY. Suppose the price of good X is p_{X} and the price of bundle XY is p_{XY}. The consumer with $V = v$ on the X market that will be indifferent between buying X and buying bundle XY is determined by

$$r - p_{\text{X}} + \max\{v - c_y, 0\} = r + v - p_{\text{XY}}.$$

Thus, the number of consumers purchasing X is

$$q_{\text{X}}(p_{\text{X}}, p_{\text{XY}}) = \begin{cases} G(p_{\text{XY}} - p_{\text{X}}) & p_{\text{XY}} - p_{\text{X}} < c_y \text{ and } p_{\text{X}} \leq r, \\ 1 & p_{\text{XY}} - p_{\text{X}} \geq c_y \text{ and } p_{\text{X}} \leq r, \\ 0 & p_{\text{X}} > r. \end{cases}$$

The number of consumers purchasing XY is $q_{\text{XY}} = 1 - q_{\text{X}}$, assuming $p_{\text{X}} \leq r$. Notice that $q_{\text{X}} = 1$ when $p_{\text{XY}} - p_{\text{X}} \geq c_y$ because market Y is assumed to be perfectly competitive.

The profits of the two firms selling X and XY are, respectively,

$$\begin{aligned} \pi_{\text{X}} &= (p_{\text{X}} - c)q_{\text{X}}(p_{\text{X}}, p_{\text{XY}}); \\ \pi_{\text{XY}} &= (p_{\text{XY}} - c - c_{\text{Y}})q_{\text{XY}}(p_{\text{X}}, p_{\text{XY}}). \end{aligned} \tag{1}$$

LEMMA 2. At any pure-strategy Nash equilibrium of subgame (X, XY) or (XY, X), $p_{\text{X}} > c$, $p_{\text{XY}} > c + c_y$, and both firms earn positive profits.

Proof. See the appendix.

The intuition behind lemma 2 is straightforward. Since some consumers have valuations for good Y that are below c_y, and since $p_{\text{XY}} \geq c + c_y$ in equilibrium, the firm offering X only can sell to a positive amount of consumers even with a p_{X} that is slightly higher than c.

Thus in equilibrium $p_X > c$ and $\pi_{XY} > 0$. This in turn allows the firm offering bundle XY to raise price above $c + c_y$ and to obtain a positive profit as well. When an equilibrium exists in subgame (X, XY) or (XY, X), the equilibrium profits for the firm offering X and the firm offering XY are denoted as π_X^* and π_{XY}^*, respectively.

From lemmas 1 and 2, we immediately have the following:

PROPOSITION 1. At any pure-strategy equilibrium of the full model, one firm offers X and the other firm offers XY.

I now show how to find a Nash equilibrium in subgames (X, XY) or (XY, X). From (1), the first-order conditions for an equilibrium are

$$p_X - c - \frac{G(p_{XY} - p_X)}{g(p_{XY} - p_X)} = 0 \tag{2}$$

and

$$p_{XY} - c - c_y - \frac{1 - G(p_{XY} - p_X)}{g(p_{XY} - p_X)} = 0. \tag{3}$$

PROPOSITION 2. Assume that

$$\frac{d\left[\dfrac{g(v)}{G(v)}\right]}{dv} \leq 0 \tag{i}$$

and

$$\frac{d\left[\dfrac{g(v)}{1 - G(v)}\right]}{dv} \geq 0. \tag{ii}$$

Then there is a unique pair (p_X^*, p_{XY}^*) that solves (2) and (3), where $p_X^* > c$, and $p_{XY}^* > c + c_y$. And if in addition $p_X^* \leq r$ and

$$G(p_{XY}^* - p_X^*) \geq (\sqrt{5} - 1)/2, \tag{4}$$

then (p_X^*, p_{XY}^*) is the unique pure-strategy Nash equilibrium in subgame (X, XY) or (XY, X), and $p_X^* > p_{XY}^* - c_y$.

Proof. See the appendix.

Assumption (i) is the log concavity condition of the distribution of V, and (ii) is the monotonic hazard-rate condition. (These two conditions differ only slightly.) They are both familiar assumptions in the literature and are satisfied by many well-known distributions, such as uniform, exponential, and normal distributions (see Bagnoli and Bergstrom 1989). Condition (4) is part of the sufficient conditions as well as a necessary condition. The profit of the firm offering X has an upside jump at $p_X = p_{XY}^* - c_y$; (4) is needed to ensure that this firm cannot benefit from lowering its price from p_X^* to $p_{XY}^* - c_y$. Since this firm's equilibrium output of X is $G(p^*_{XY} - p^*_X)$, condition (4) has a simple economic interpretation: to prevent the firm offering X only from undercutting the other firm and selling to all consumers in the X market, the former's equilibrium output cannot be too low.

Since $(\sqrt{5} - 1)/2 \doteq 0.62$ and $p^*_{XY} - c - c_y < p^*_X - c$, the firm selling X only has more customers and also a higher price markup than the firm selling bundle XY in equilibrium. We therefore have

COROLLARY 1. In equilibrium, the firm selling X only earns a higher profit than the firm selling bundle XY.

The equilibrium structure of the model thus resembles that of a battle-of-the-sexes game. Each firm would like the other to bundle products so that the resulting price competition will be softened. However, a firm would bundle its own products if it does not expect the other firm to do so.

Notice that when one firm offers X and the other firm offers XY in equilibrium, all the consumers whose v's are above $p^*_{XY} - p^*_X$ buy bundle XY. But $p^*_{XY} - p^*_X < c_y$. Therefore, while there occur only surplus transfers from consumers to firms in the X market since $c < p^*_X$ in equilibrium, there is a dead-weight loss that occurs in equilibrium when consumers with $v < c_y$ buy the bundle XY. We thus have the following:

COROLLARY 2. When bundling sales are allowed, producers of good X are better off, and consumers of good X are worse off. Furthermore, social welfare, measured as the sum of consumer surplus and producer surplus, is reduced.

The issue of whether a firm can increase its market power through product bundling has received much attention recently. It has been shown that a monopoly firm can never gain by (pure) bundling with another product that is produced competitively (Schmalensee 1982). Whinston (1990), however, demonstrates that if the second market has an oligopoly structure, then the monopolist in the original market can benefit from bundling because of the strategic effect on the second market. The result here is that even if the second market is in perfect competition, firms in the primary market can still benefit from bundling with the second product because of the strategic effect on the primary market. That is, bundling provides a useful way for firms to differentiate their primary products and thus gain market power in their primary market.

Our result is also related to the literature on strategic substitutes and complements, such as in Bulow, Geanakopolos, and Klemperer (1985) and Fudenberg and Tirole (1984). In the terminology of that literature, the prices of the two firms in our model are strategic complements. In this case, a firm has incentives to commit itself to being a less aggressive competitor, so that the other firm will be less aggressive as well. Or to use the term in Fudenberg and Tirole (1984), there are incentives for firms to play the ''puppy dog'' strategy. By offering only the bundled product XY, a firm softens its competitiveness in a pricing game where the other firm offers only X, which in turn leads to higher prices for both firms. In Whinston (1990), by contrast, bundling is a ''top dog'' strategy: when the monopolist bundles its monopoly product with its product in a duopoly market, it becomes more aggressive in the duopoly market and may thus exclude its rival in that market.

When (4) does not hold, subgame (X, XY) has no pure-strategy Nash equilibrium. This is a well-known problem in games with discontinuous payoff functions. Interestingly, if we remove our restriction that firms choose only pure strategies, the main result (theorem 5) in Dasgupta and Maskin (1986) can be applied here to show the following:

PROPOSITION 3. If mixed strategies are allowed, then a Nash equilibrium always exists in subgames (X, XY) or (XY, X). In this case, the full model has subgame-perfect equilibria where either one firm chooses X and the other firm chooses XY or each firm randomizes between choosing X with probability α and XY with probability $1 - \alpha$ in the first stage, where $\alpha = \pi_X^*/(\pi_X^* + \pi_{XY}^*)$.

Proof. See the appendix.

Our equilibrium analysis of bundling is thus valid in a quite general setting. We conclude this section with an example.

EXAMPLE 1. Assume $g(v) = 1$, $0 \le v \le 1$; $c + 2/3 < r$; and $(3\sqrt{5} - 5)/2 \le c_y < 1$.

Equations (2) and (3) become

$$p_X - c = p_{XY} - p_X$$

and

$$p_{XY} - c - c_y = 1 - (p_{XY} - p_X).$$

Solving these two equations, we obtain

$$p_X^* = c + (1 + c_y)/3$$

and

$$p_{XY}^* = c + 2(1 + c_y)/3.$$

One can easily verify that $p_X^* < r$ and condition (4) holds. The equilibrium profits for the two firms are

$$\pi_X^* = [(1 + c_y)/3]^2$$

and

$$\pi_{XY}^* = [(2 - c_y)/3]^2.$$

Notice that since those consumers with $(1 + c_y)/3 < v < c_y$ purchase XY and thus consume Y, there is social inefficiency in equilibrium.

III. Allowing Mixed Bundling

We now extend the basic model to include the possibility of mixed bundling. That is, in addition to offering only X or the pure bundle XY, each firm can also choose to offer both X and XY at the same time (denoted by X&XY). Thus at the second stage of the game, there are nine possible pricing subgames, each denoted by (a, b), where $a \in \{X, XY, X\&XY\}$ and $b \in \{X, XY, X\&XY\}$. Everything else is the same as in the basic model. In subgame (X, XY) or (XY, X), we shall continue to use π_X^* and π_{XY}^* to denote the equilibrium profits of the firm offering X and the firm offering XY, respectively. The subgame-perfect Nash equilibria in this extended game will therefore be transparent if we can find the Nash equilibria and their associated profits in the subgames where at least one firm engages in mixed bundling. There are five such subgames: (X&XY, X&XY), (X, X&XY), (X&XY, X), (XY, X&XY), and (X&XY, XY).

Proposition 4. In any of the pricing subgames where at least one firm offers mixed bundling, a pure-strategy Nash equilibrium always exists. Furthermore, in the subgames where both firms offer X&XY or one firm offers X and the other firm offers X&XY, both firms earn zero profit at any pure-strategy Nash equilibrium; and in the subgames where one firm offers XY and the other firm offers X&XY, the former obtains zero profit and the latter obtains $\pi_{X\&XY}^*$ in any pure-strategy Nash equilibrium, where $0 \leq \pi_{X\&XY}^* < \pi_X^*$.

Proof. See the appendix.

The intuition behind proposition 4 is simple. When a firm offers both X and bundle XY, there will be Bertrand-type competition either in X if the other firm offers X, or in XY if the other firm offers XY, or in both X and XY if the other firm also offers the mixed bundle. As a result, either the price of X will be c, or the price of XY will be $c + c_y$, or both the price of X will be c and the price of XY will be $c + c_y$. But if X is separately available on the market at price c, then no firm will be able to obtain positive profits.

Combining proposition 4 with lemma 1 and lemma 2 of Section II, we obtain the strategic form of the first stage of the extended game with mixed bundling, as is shown in figure 1. Notice that if A chooses X, then its payoff will be $\pi_X^* > 0$ if B chooses XY and zero if B chooses otherwise. On the other hand, if A chooses X&XY, then its payoff will be $\pi_{X\&XY}^* < \pi_X^*$ if B chooses XY and zero if B chooses otherwise. Similarly, this holds for firm B. We therefore have:

Proposition 5. Mixed bundling is a (weakly) dominated strategy for each firm in the first-stage game assuming equilibrium continuation in the second stage.

This result is in sharp contrast to that in monopoly models of bundling where mixed bundling (weakly) dominates pure bundling and

		B		
		X	XY	X&XY
	X	(0, 0)	(Π_X^*, Π_{XY}^*)	(0,0)
A	XY	(Π_{XY}^*, Π_X^*)	(0, 0)	$(0, \Pi_{X\&XY}^*)$
	X&XY	(0, 0)	$(\Pi_{X\&XY}^*, 0)$	(0, 0)

FIG. 1.—The first-stage game with mixed bundling

often strictly dominates separate sales (see, e.g., McAfee et al. 1989). The intuition for the result here is straightforward. By adopting mixed bundling, the product differentiation role that is played by bundling is undermined. This leads to more severe price competition and can leave both firms worse off.

There is another equilibrium of the game shown in figure 1 in which both firms choose mixed bundling, a weakly dominated strategy, with zero payoffs for both firms. It is natural to dismiss this as an equilibrium outcome. however, based on generally accepted equilibrium refinements.[8]

IV. Heterogeneous Consumer Preferences for Good X

We now extend the basic model in another direction: relaxing the strong assumption that consumers have homogeneous preference for good X. While each consumer is still assumed to demand at most one unit of X, consumers' valuations of X are now the realizations of a random variable, U. The joint distribution of U and V is $h(u, v)$, which

8. According to Kreps (1990), e.g., the only situations where the equilibrium play of weakly dominated strategies can be justified are certain models where a player's choice set is continuous, such as the Bertrand model.

is continuous and strictly positive on its support: $\underline{u} \leq u \leq \overline{u}$; $\underline{v} \leq v \leq \overline{v}$. Assume $\underline{u} < c < \overline{u}$ and $\underline{v} < c_y < \overline{v}$. The joint cumulative distribution function (cdf) is $H(u, v)$, and the marginal cdf of U is $F(u)$. All other aspects of the model, as well as the equilibrium definition, are the same as those of Section II.

Using similar arguments as in Section II, one can easily show that lemma 1, lemma 2, and proposition 1 still hold here. Hence, if there is a pure-strategy Nash equilibrium in subgame (X, XY) or (XY, X), the full model has an equilibrium where one firm offers the single product X and the other firm offers pure bundle XY.

The situations under which a Nash equilibrium exists for subgames (X, XY) or (XY, X) are, however, different now. In particular, not all consumers will make purchases in equilibrium.

Suppose one firm has chosen X and the other has chosen XY, and their prices are p_X and p_{XY}, respectively. Since no customer would buy XY if $p_{XY} - p_X \geq c_y$, in equilibrium it must be that $p_{XY} - p_X < c_y$. (This implies that those buying X only must have $v < c_y$.) Thus to find a Nash equilibrium in the subgame, we need only restrict attention to situations where $p_{XY} - p_X < c_y$. The consumers who would buy X are those whose u's and v's satisfy

$$u - p_X \geq u + v - p_{XY}$$

and

$$u > p_X.$$

The consumers that would buy XY are those whose u's and v's satisfy

$$u - p_X < u + v - p_{XY}$$

and

$$\max\{v - c_y, 0\} < u + v - p_{XY}.$$

The demands for X and XY are, respectively,

$$q_X(p_X, p_{XY}) = \iint\limits_{\substack{v < p_{XY} - p_X \\ p_X < u}} h(u, v)\, du\, dv, \qquad p_{XY} - p_X < c_y;$$

$$q_{XY}(p_X, p_{XY}) = \iint\limits_{\substack{v > c_y \\ u > p_{XY} - c_y}} h(u, v)\, du\, dv$$

$$+ \iint\limits_{\substack{c_y > v > p_{XY} - p_X \\ u + v > p_{XY}}} h(u, v)\, du\, dv, \quad p_{XY} - p_X < c_y.$$

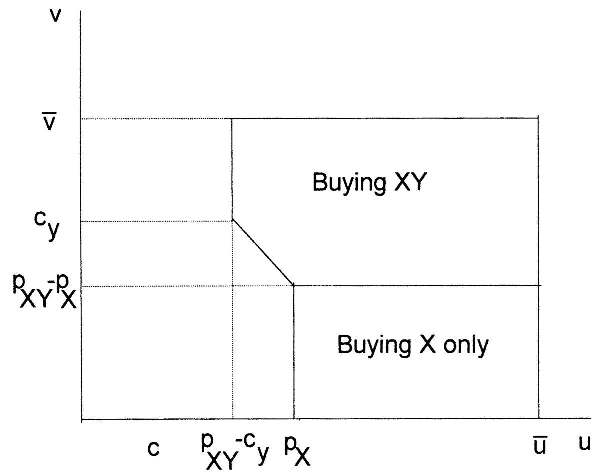

FIG. 2.—Demand under heterogeneous consumer preferences for X

Figure 2 illustrates q_X and q_{XY}. The consumers with u and v in the bottom corner of the diagram would purchase X only, and the consumers with u and v in the top corner of the diagram would purchase XY. Thus q_X and q_{XY} are obtained by integrating $h(u, v)$ over those two areas, respectively. Now suppose

$$p_X^* = \text{argmax}(p_X - c)q_X(p_X, p_{XY}^*) \tag{5}$$

and

$$p_{XY}^* = \text{argmax}(p_{XY} - c - c_y)q_{XY}(p_X^*, p_{XY}). \tag{6}$$

Since the demand for X is discontinuous at $p_X = p_{XY} - c_y$, and if $p_X \leq p_{XY} - c_y$, the firm selling X only would receive $(p_X - c) \times [1 - F(p_X)]$. Thus in order for (p_X^*, p_{XY}^*), as is defined above, to be a Nash equilibrium, we also need

$$\max_{p_X \leq p_{XY}^* - c_y} (p_X - c)[1 - F(p_X)] \leq (p_X^* - c)q_X(p_X^*, p_{XY}^*). \tag{7}$$

We therefore have

PROPOSITION 6. If (p_X^*, p_{XY}^*) satisfies (5), (6), and (7), then there is an equilibrium for the full model where one firm chooses X and charges p_X^*, and the other firm chooses XY and charges p_{XY}^*.

Obviously, corollary 1 in Section II continues to hold here as well. That is, when bundling sales are allowed, both firms are better off, consumers of good X are worse off, and social surplus is reduced. There are, however, two important differences as compared to the basic model so far as welfare is concerned. First, there are now two sources of welfare losses due to bundling: a deadweight loss in the X market due to higher prices there (some consumers with $u > c$ will no longer consume X), and a deadweight loss in the Y market for the consumption of Y by those with $v < c_y$.

Second, it is now more difficult to assess whether social welfare would be higher if market X were under monopoly as compared to a duopoly that can engage in bundling sales. On one hand, under duopoly, the price in the X market is generally lower than that under monopoly,[9] thus reducing efficiency losses on the X market compared to the monopoly. On the other hand, however, competition by the duopolists creates efficiency losses in the consumption of good Y, which would not occur if the X market were a monopoly. Such trade-offs may have important implications for public policies.

If $h(u, v)$ is known, we can find (p_X^*, p_{XY}^*) explicitly, as in the following example (the technical details of the calculations for this example and the examples in the next section are available from the author upon request).

EXAMPLE 2. Assume that $h(u, v) = 1, 0 \leq u < 1; 0 \leq v < 1$. In addition, $c = 0.4$ and $c_y = 0.9$. Then, $p_X^* = 0.646$, $p_{XY}^* = 1.451$; and $q_X^* = 0.285$, $q_{XY}^* = 0.058$.

V. Multiple Nonprimary Goods

We now extend the basic model to the case where there is more than one other good beyond the primary product X. Specifically, we assume that there are two other goods, Y and Z, that are produced competitively.[10] The constant marginal cost of Y is again c_y, as in the previous sections, and the constant marginal cost of Z is c_z. A consumer demands, at most, one unit of Y and one unit of Z, and the valuations of consumers for Y and Z are realizations of random variables V and W. The joint density of V and W is $m(v, w)$, and $m(v, w) > 0$ on the entire support $\underline{v} < v < \overline{v}$, $\underline{w} < w < \overline{w}$. We again assume that $\underline{v} < c_y < \overline{v}$, and each consumer in market X demands only one unit of X with valuation r that is sufficiently high so that market X will be covered in equilibrium. In the product-choice stage, we assume that firm A chooses $a \in \{X, XY, XZ\}$, and firm B chooses $b \in \{X, XY, XZ\}$.[11] Thus there are nine possible pricing subgames.

As in Section II, we can again show that in any pure-strategy Nash equilibrium of the pricing subgames where $a = b$, both firms have

9. To see why, notice that at the duopoly equilibrium, when p_X becomes higher, there are both the dropout effect (some consumers will no longer buy X) and the switching effect (some will switch to buying XY from the other firm), as is clear from appendix fig. A1, while under monopoly only the dropout effect is present. Thus a monopolist has more incentives to raise prices.

10. Generalizing our analysis to more than two nonprimary goods seems also possible, although the calculations become more complicated.

11. For computational convenience, I assume that a firm cannot produce and sell more than two goods together and that no mixed bundling will be used.

zero profit; and in any pure-strategy Nash equilibrium of subgames (X, XY), (X, XZ), (XY, X), and (XZ, X), both firms have positive profits. However, now it is possible for both firms to offer different bundles.

The calculations of pure-strategy Nash equilibrium profits at any pricing subgame where one firm has chosen a bundle and another has not are entirely similar to those in Section II. To compute the pure-strategy Nash equilibrium profits at the pricing subgames where one firm has chosen XY and the other has chosen XZ, notice that the consumer who is indifferent between XY and XZ is located at

$$r + v - p_{XY} = r + w - p_{XZ}.$$

For sufficiently high r, the demands for XY or XZ are, respectively,

$$q_{XY} = \iint_{v - w \geq p_{XY} - p_{XZ}} m(v, w)\, dv\, dw, \quad -c_z < p_{XY} - p_{XZ} < c_y,$$

$$q_{XZ} = 1 - q_{XY}.$$

Without specifying the functional form of $m(v, w)$, the equilibrium conditions can only be given in general terms, and I shall spare the readers from such details. Notice, however, the following: if $p_{XY} > p_{XZ}$ in equilibrium, there are consumers with $w < c_z$ buying XZ; if $p_{XY} < p_{XZ}$, there are consumers with $v < c_y$ buying XY; and if $p_{XY} = p_{XZ}$, there are both consumers with $w < c_z$ buying XZ and consumers with $v < c_y$ buying XY. Thus the welfare effects here will be similar to those in the basic model, involving deadweight losses due to inefficient consumption. The following two examples show that, depending on the form of $m(v, w)$, in equilibrium either one firm offers X only and the other firm offers a bundle, or both firms offer different bundles.

EXAMPLE 3. Assume $m(v, w) = 1$, $0 \leq v \leq 1$, $0 \leq w \leq 1$; $c + 3/2 < r$; and $(3\sqrt{5} - 5)/2 \leq c_y = c_z < 1$. In this case, the pure-strategy equilibrium outcome of the full game is that one firm chooses X and the other chooses XY or XZ, and then they charge $c + (1 + c_y)/3$ and $c + 2(1 + c_y)/3$, respectively.

EXAMPLE 4. Assume $m(v, w) = 1$, $0 < v < 1$, $w = 1 - v$, $(3\sqrt{5} - 5)/2 \leq c_y = c_z < 1$, and $c + c_y + 1 < r$. The pure-strategy equilibrium outcome of the full game is that one firm chooses XY and the other firm chooses XZ, and then both set prices equal to $c + c_y + 1$.[12] We thus have

PROPOSITION 7. In the presence of multiple nonprimary goods, it is possible that in equilibrium one or both firms engage in bundling and that each firm earns positive expected profits.

12. This example is motivated by a Hoteling model in which consumers are uniformly distributed along a unit line, their reservation values for good Y (or Z) are r, two firms are located on the two end points of the line with constant unit production cost $c + c_y$, and the unit transportation cost is 1.

The preceding analysis also suggests a possibility to extend the basic model to the case where there are more than two firms in the primary market. Suppose, for instance, our model is the same as the one in this section except that there are three competing firms in the X-market. Then it is not difficult to show that, provided a pure-strategy Nash equilibrium exists in each of the pricing subgames, it is a subgame-perfect equilibrium for one firm to choose X, another firm to choose XY, and a third firm to choose XZ and that all firms can earn positive profits at this equilibrium. The intuition is again the same as before: bundling enables firms to achieve product differentiation and thus reduces price competition.[13]

VI. Conclusion

This article offers an equilibrium theory of product bundling. Bundling is shown to emerge as an equilibrium strategy of competing firms for its role as a product-differentiation device. The profit and welfare implications of product bundling under oligopoly differ sharply from those under monopoly. Mixed bundling, while dominating pure bundling under monopoly, becomes a dominated strategy in the presence of competing firms. When it is possible for firms to commit to (pure) bundling,[14] the profits of all firms in the industry are higher, but social surplus is unambiguously reduced.

I have modeled the choice of bundling as a simultaneous decision by the rival firms. The results of this article, however, can still hold if firms make sequential product choices. In fact, sequential decision on bundling by firms can narrow the set of equilibria, so that in the context of Section II, for example, the only equilibrium will be for the firm moving first to offer only X and the other firm to offer bundle XY. Thus, according to our analysis, if firms move sequentially in their product choices, then the firm moving first has an advantage, and the latecomer is more likely to adopt the strategy of bundling sales.[15] Another possible direction of extending our analysis is to allow entry in the primary market with perhaps some entry cost, so that in addition

13. If the number of firms is greater than or equal to the number of nonprimary goods +2, then it can still be an equilibrium that only one firm offers X and all other firms offer some type of bundles, although in this case a firm offering a bundle may not have positive profits if the bundle is also offered by another firm. There can also be a mixed-strategy equilibrium where each firm randomizes between X and different bundles. Each firm could then have positive expected profits in such a mixed-strategy equilibrium.

14. All we essentially need is that firms set prices after their decisions about product bundling are made. In a dynamic model, we might allow firms to revise their product choices from time to time. I expect that this one-period model can yield insights about the outcomes in such a dynamic model as well. At each period, there are incentives for at least one firm to bundle products so as to achieve product differentiation.

15. Incidently, Compaq, a latecomer competitor of IBM and a major personal computer producer today, was the industry pioneer of bundling computers with software.

to the two-stage decisions described here, there is the additional stage where firms make entry decisions. It is likely that inefficient entries will occur in this case, but in light of the results in Mankiw and Whinston (1986), it does not appear that much new insight would emerge from such an extension.

To focus on the strategic use of bundling as a product differentiation device, I have assumed away other possible reasons for bundling to occur. It is important, however, to keep some alternative perspectives when exploring the results. For instance, in a dynamic context, producers of X may be able to collude on prices. In this case, bundling X with some goods may actually enable a firm to engage in some form of price cutting and yet not to create a general price war. It is also possible that bundling may create real convenience for consumers. Or, as is analyzed in Salinger (1995), bundling may lead to cost savings.[16] The equilibrium bundling theory suggested here can be complementary to these alternative explanations.

There are, of course, other ways competing firms can differentiate their products, such as advertising and quality choices.[17] In many markets, firms not only compete in prices but also compete in all those other dimensions, and product bundling is only one of these strategic choices. It seems desirable to know why and when the competition in a particular market may concentrate more on a particular form. One might expect that bundling is more likely to be used as a strategic device when there is little room for competing firms to differentiate a particular product through advertising or quality choices.[18] It remains an area of future research to establish a theory of multidimensional competition that would address such issues.

Appendix

Proofs

Proof of lemma 2. At subgame (X, XY) or (XY, X), let π_X^* and π_{XY}^* denote the equilibrium profits of the firms offering X and XY, respectively. Without

16. Turning to the examples of product bundling that were mentioned in the beginning of the article, the alternatives to our explanation would then suggest that bundling the use of credit cards with free goods could be a form of price cutting, bundling computers with software might be a strategy to create real convenience for consumers, and bundling telephone uses with frequent-flyer miles might have cost savings since the phone companies can obtain frequent-flyer miles at a lower cost.

17. See, e.g., Shaked and Sutton (1982) on the selection of different product qualities by competing firms to achieve product differentiation so as to relax price competition. My analysis shares some similar intuition to their model. In particular, one might interpret Y as an increment to quality above the standard product X. By assuming Y to be a separate product and available on a separate market, however, I focus on a firm's product bundling decision.

18. To most people, for instance, there could hardly be any difference in their valuations between a long-distance call connected through AT&T or through MCI or between the service provided by a Visa card issued through, say, Citibank or Chemical Bank.

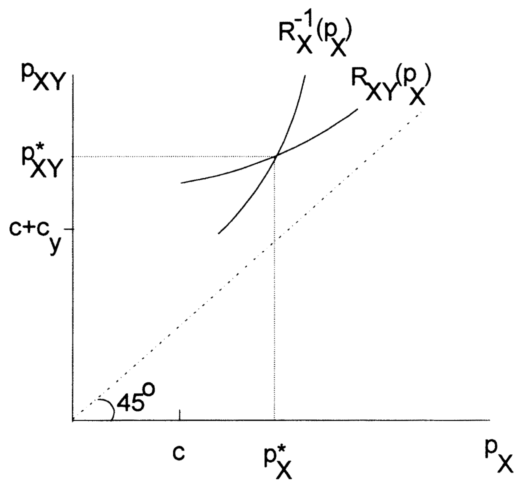

FIG. A1.—Equilibrium in subgame (X, XY)

loss of generality, suppose the subgame is (X, XY), that is, A's product choice is X and B's is XY. Let A's price be p^*_X and B's p^*_{XY} at a Nash equilibrium. Then clearly $p^*_X \geq c$ and $p^*_{XY} \geq c + c_y$. Given p^*_{XY}, if A's price is $c + \epsilon$, where $\epsilon > 0$, the number of consumers that would buy from A is at least $G[p^*_{XY} - (c + \epsilon)] \geq G(c_y - \epsilon)$, which is positive for sufficiently small ϵ. Thus $p^*_X > c$ and $\pi^*_X > 0$. Now given $p^*_X > c$, if B's price is $c + c_y + \delta < p^*_X + c_y$, where $\delta > 0$, the number of customers who would buy from B is at least $1 - G(c_y) > 0$. Thus $p^*_{XY} > c + c_y$ and $\pi^*_{XY} > 0$ as well. Q.E.D.

Proof of proposition 2. For any $p_{XY} \geq c + c_y$, the left-hand side of (2) < 0 when $p_X = c$, and the left-hand side of (2) > 0 when $p_X = p_{XY} - \underline{v}$. By assumption i, the left-hand side of (2) increases monotonically in p_X. Thus (2) defines a single-valued reaction function $p_X = R_X(p_{XY})$ for $p_{XY} \geq c + c_y$, and clearly $0 \leq dR_X(p_{XY})/dp_{XY} < 1$. Denote the inverse function of $R_X(p_{XY})$ by $p_{XY} = R_X^{-1}(p_X)$. Then $dR_X^{-1}(p_X)/dp_X > 1$. On the other hand, given any $p_X \geq c$, the left-hand side of (3) < 0 when $p_{XY} = c + c_y$; the left-hand side of (3) > 0 when $p_{XY} = p_X + \bar{v}$. By assumption ii, the left-hand side of (3) monotonically increases in p_{XY}. Thus (3) defines a single-valued reaction function $p_{XY} = R_{XY}(p_X)$, and clearly $0 < dR_{XY}(p_X)/dp_X < 1$. Notice further that $R_X(c + c_y) > c$, and $R_{XY}(c) > c + c_y$. Thus there exists a unique pair (p^*_X, p^*_{XY}) at which the two reaction functions intersect (see fig. A1 for an illustration).

Next, the second-order condition for the firm selling X only is

$$-2g(p_{XY} - p_X) + (p_X - c)g'(p_{XY} - p_X) < 0.$$

When the first-order condition is satisfied, the inequality above becomes

$$-2g(p^*_{XY} - p^*_X) + [G(p^*_{XY} - p^*_X)/g(p^*_{XY} - p^*_X)]g'(p^*_{XY} - p^*_X) < 0,$$

which holds if assumption i holds. Similarly, assumption ii ensures that the second-order condition for the firm that sells bundle XY is satisfied. In addition, the X market is covered since $p^*_X \leq r$.

Finally, since the firm selling X only has a jump in its profit function at $p_X = p^*_{XY} - c_y$ (the only discontinuous point), to prevent it from deviating to this price, we need

$$p^*_{XY} - c - c_y \leq (p^*_X - c)G(p^*_{XY} - p^*_X),$$

which also implies that $p^*_{XY} - p^*_X < c_y$. Using equations (2) and (3), the inequality above becomes

$$1 - G(p^*_{XY} - p^*_X) - G^2(p^*_{XY} - p^*_X) \leq 0.$$

Condition (4) then follows.

Proof of proposition 3. Using the notations in Dasgupta and Maskin (1986), define $A^*(X) = A^*(XY) = \{(p_X, p_{XY}) | p_X \in [c, r], p_{XY} \in [c + c_y, r + \bar{v}],$ and $p_{XY} - p_X = c_y\}$. Let $A^{**}(X)$ and $A^{**}(XY)$ denote the discontinuity sets of the two firms offering X and XY, respectively. Then $A^{**}(X) \subset A^*(X)$, and $A^{**}(XY) \subset A^*(XY)$. From (1), assuming r to be sufficiently high, the sum of the two firms' profits are:

$$\pi_X + \pi_{XY} = \begin{cases} p_{XY} - c - c_y - (p_{XY} - p_X - c_y)G(p_{XY} - p_X) & \text{if } p_{XY} - p_X < c_y, \\ p_X - c & \text{if } p_{XY} - p_X \geq c_y. \end{cases}$$

Thus, although π_X and π_{XY} are discontinuous on $A^*(X)$, the sum of π_X and π_{XY} is continuous on this set. Furthermore, for all $p'_X \in A^{**}(X)$, and for all $p_{XY} \in A^{**}_{-1}(X)$,

$$\lim_{p_X \xrightarrow{-} p'_X} \inf \pi_X(p_X, p_{XY}) \geq \pi_X(p'_X, p_{XY}).$$

Thus the payoff function of the firm offering X is weakly lower semicontinuous in its own strategies in the sense described by Dasgupta and Maskin (1986). One can verify that this is also true for the payoff function of the firm offering XY.

Therefore, according to theorem 5 in Dasgupta and Maskin (1986), there is always a mixed-strategy Nash equilibrium in subgames (X, XY) or (XY, X). Moreover, at any such mixed-strategy equilibrium, the firm offering X must have a positive probability to sell a positive quantity at a price higher than c, and the firm offering XY must have a positive probability to sell a positive quantity at a price higher than $c + c_y$. Thus both firms must have positive expected profits at (X, XY) or (XY, X), with π^*_X being the expected profit of the firm offering X only, and π^*_{XY} the expected profit of the firm offering XY. From lemma 1, the rest of the proposition is obvious. Q.E.D.

Proof of proposition 4. First, in (X&XY, X&XY), or in subgames where one firm offers X only and the other firm offers X&XY, both firms must earn zero profit in any pure-strategy Nash equilibrium. The arguments are similar to the standard argument in Bertrand competition and are thus omitted.

Next, consider the subgames where one firm offers XY and the other firm offers X&XY. One Nash equilibrium in each of the two subgames is for the firm offering XY to charge $c + c_y$ and the other firm to charge $c + c_y$ for XY and p^1_X for X, where

$$p^1_X = \underset{c \leq p_X \leq r}{\text{argmax}} \ (p_X - c)G(c + c_y - p_X).$$

Since $(p_X - c)G(c + c_y - p_X)$ is continuous in p_X, p_X^1 exists and so does the equilibrium. Furthermore, one can easily verify that, at any pure-strategy Nash equilibrium of the subgames, no firm charges more than $c + c_y$, for XY, and thus the unique pure-strategy equilibrium profit for the firm offering XY is zero and that for the firm offering X&XY is $\pi^*_{X\&XY}$, where

$$\pi^*_{X\&XY} \equiv (p_X^1 - c)G(c + c_y - p_X^1) \geq 0.$$

Finally, notice that π^*_X, the equilibrium profits for the firm offering X only at subgames (X, XY) and (XY, X), is higher than $\pi^*_{X\&XY}$. This is because $p^*_{XY} > c + c_y$ at subgame (X, XY) or (XY, X), and thus at these two subgames if the firm selling X only charges p_X^1, more consumers will buy X than in subgame (XY, X&XY) or (X&XY, XY). (Prices for X and XY are strategic complements.) Q.E.D.

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