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LETTER TO THE EDITOR

Anomalous interfacial correlations in non-equilibrium anisotropic systems

K-T Leung† and R K P Zia‡

† Institute of Physics, Academia Sinica, Taipei, Taiwan 11529, Republic of China
‡ Center for Stochastic Processes in Science and Engineering and Physics Department, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061, USA

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Abstract. Low-temperature interfacial properties of several driven, non-equilibrium lattice gases in steady states are studied with computer simulations. The drives are directed along the tangent to the interface. By focusing on the small-momentum behaviour of the height–height correlation, $G(q)$, we determine that the longitudinal spatial isotropy symmetry is relevant to the long-distance behaviour. For the case where the sign of the drive is random in time, the analytically predicted $G(q) \sim q^{-1}$ is observed for the first time. For uniform drive, we determine numerically $G(q) \sim q^{-2+\eta}$, with $\eta \approx 1.33$. This suggests that the effective stiffness of the interfaces is enhanced on large scales, consistent with roughness being severely suppressed.

It has been recognized in recent years that when a system is maintained in a non-equilibrium steady state by certain external fields, its behaviour can be drastically different from that in equilibirum. If the constituents of the system are driven to move in a specific direction, the system acquires spatial anisotropies. Notable examples are driven lattice gases [1], which may model superionic conductors [2], and binary fluids under shear flow [3]. Novel features such as generic singularities at all temperatures and anomalous dimensions at criticality which are distinct from the equilibrium ones can all be attributed to the breakdown of the fluctuation-dissipation theorem and anisotropies at one level or another [4, 5].

In addition to the effects on the bulk, anisotropies also dictate the behaviour of the interfaces at low temperature. For example, the rough interface of an Ising lattice gas in two dimensions [6] was found to become smooth [7, 8] (i.e. finite statistical width or localized) under a *constant* drive that forces particles in the bulk to move in a direction parallel to the interface. When the drive is random in time but unidirectional [9], interfacial relaxation and correlation can be calculated from the dynamics [10]. Strong anisotropy is found in the height–height correlation $G(q)$, which behaves according to $1/q_{\parallel}$ for small momenta along the drive, and according to the usual $1/q_{\perp}^2$ transverse to the drive. This analytical result has three implications: (i) the interfacial roughness, obtained by $\int_q G(q)$, is also suppressed as in the case with constant drive; (ii) the correlation remains gapless, differing from the massive modes of equilibrium interfaces smoothed by, for example, gravity in a binary fluid [6]; and, most importantly, (iii) the mechanism of roughness suppression is achieved by violating the fluctuation-dissipation theorem.

As a continuation of and extension to the above studies, we report in this brief letter extensive Monte Carlo simulations for the interfaces of several non-equilibrium lattice gases with attractive nearest-neighbour interactions:

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(i) *Randomly driven model* (RDM) [9]. The drive E is a uniform, annealed variable random in time with equal probability to be in the $+y$ or $-y$ direction. It is incorporated into the jump rates by a factor $e^{\pm E/k_B T}$ for jumps along or against the drive (see [9] for details).

(ii) *Two-temperature model* (2TM) [11]. In this model, configurations are updated in the usual way but with different temperatures T and αT for jumps along x and y, respectively. Being generally distinct from (i), it becomes identical in the limit $\alpha \to \infty$ to a randomly driven model with $E = \infty$ and anisotropic sampling rates between x and y directions.

(iii) *Uniformly driven model* (UDM) [1]. This is the model in which roughness suppression was first observed [7, 8]. We re-examine it in greater depth for comparison purposes. The only difference from the first model is that the drive is constant here.

We will focus on probing the structure factor. Our motivations for this are three-fold: first, we test the predicted form $G(q) \sim 1/q$ which is analytically obtained for the RDM [10]. Second, we determine the relevance of parity symmetry in the y direction, which is violated in the third model but intact in the first two. Third, by analogy with equilibrium systems, we infer the effective stiffness of the interface from its correlation at small momenta.

The computer simulations used two-dimensional square lattices with a periodic boundary condition along the horizontal edges, and fixed boundary conditions (particle density $\equiv 1$ and 0) along the vertical edges [7, 8]. This results in an interface that runs parallel to y and wanders about its mean position: $L_y/2$. To obtain a well-defined interface and to allow for sufficient sampling of its fluctuations, simulations are done at certain well-chosen temperatures below bulk criticality. In this letter, we will only report the data obtained for $T = 0.9$ in units of the Onsager critical temperature [12]. Since the bulk correlations decay algebraically rather than exponentially [13, 11], some care must be exercised in choosing the transverse size, L_x, to avoid boundary effects. We have ensured that L_x is sufficiently large in our studies. By a suitable coarse graining of the particle configurations, we are able to probe the *local* structure of the interface, exemplified by the height–height correlation function $\langle h(x)h(x')\rangle - \langle h\rangle^2$. The structure factor $G(q)$ is its direct Fourier transform.

We scan across a wide range of field strengths ($2 \leqslant E \leqslant 50$, $4/3 \leqslant \alpha \leqslant 10^6$) and find no qualitative difference between strong and weak fields. Our main results are summarized in figures 1–4. First, at large momenta (small length scales), the fluctuations 'see' no drive, and we expect the inverse structure factor for each model to behave as q^2, just as in equilibrium. This is clearly shown in figure 1, which also displays the crossover from q^2 to $q^{2-\eta}$ for $q < q_c$, with $\eta \sim 1$. As anticipated, the crossover wavenumber q_c diminishes with decreasing drive, and vanishes at zero drive.

For small q, the RDM indeed exhibits a linear region in $1/G(q)$ (see figure 2), consistent with an analytical calculation based on the bulk dynamics of a continuum model [10]. However, the picture in the $q \to 0$ limit is less clear. There appears to be two alternatives: one a straight extrapolation to a gap, the other a crossover to a lower power in q. The data very near $q = 0$ are too noisy for us to resolve the differences, although those for large drive seem to favour the second scenario (see figure 3). Now, we argue that the second is the more plausible alternative. We found that the interfacial relaxation rate is proportional to q^2 [10]. Being diffusive and relaxing as q^2 also, the bulk modes are equally 'soft', and may not decouple fully from the interface modes. Contrast this situation with that in equilibrium cases where the interface relaxes as q^3 [14]. These bulk modes could systematically modify the infrared behaviour of the interfacial correlations through nonlinear couplings. Another possible source of anomalous powers of q is nonlinear and/or non-local interactions of $h(x)$ itself. Such effects are absent from our previous linear stability analysis, which was essentially a Gaussian theory. On the other hand, a gap in the spectrum is reminiscent of

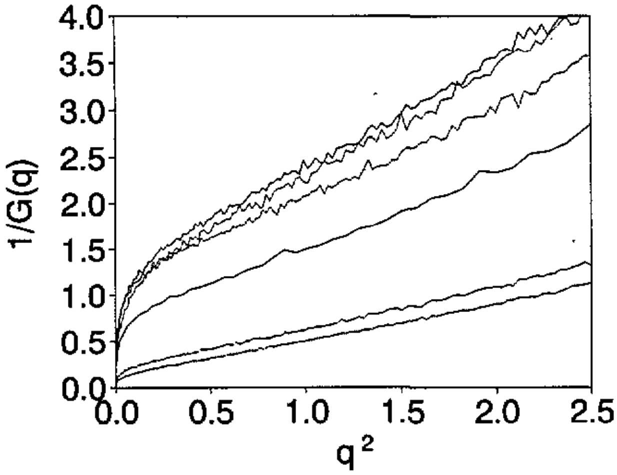

Figure 1. Inverse structure factor $1/G(q)$ plotted against wavenumber squared, q^2, showing the crossover from equilibrium-like behaviour to non-equilibrium one. The curves are, from top to bottom, for $E = 50$(RDM), $E = 50$(UDM), $\alpha = 10^6$(2TM), $E = 5$(RDM), $\alpha = 4/3$(2TM) and $E = 2$(RDM). All E are in unit of nearest-neighbour coupling; and $E = 50$ is practically infinite.

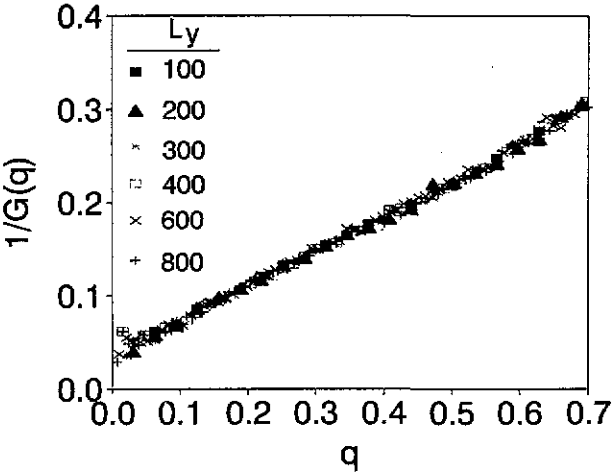

Figure 2. Inverse structure factor $1/G(q)$ against wavenumber q, for randomly driven model at $E = 2$. The linear behaviour is predicted by theory. Finite-size (L_y) effects are negligible, but the limit $q \to 0$ seems perplexing.

the effects of gravity on capillary waves [6]. But, unlike that case, the drive here does not break translational invariance. Thus, we believe that $1/G(q \to 0) \to 0$ is the more plausible outcome.

While the 2TM behaves very much like the RDM, the trend of the UDM is more conclusive: $1/G(q)$ crosses over from q^2 to $q^{2-\eta}$, and is apparently gapless at $q = 0$. Numerically we find $\eta \approx 1.33$ for both strong and weak drive (see figure 4). The difference in the small-q behaviour between the uniformly driven case and the others shows that breaking the parity

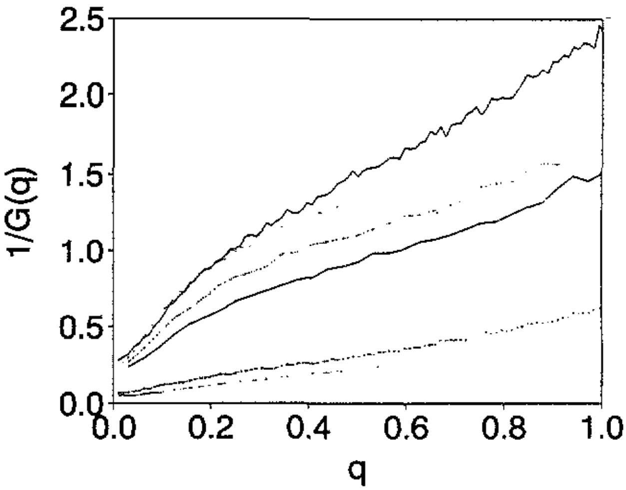

Figure 3. Randomly driven model and two-temperature model with different drive. From top: $E = 50$, $\alpha = 10^6$, $\alpha = 5$, $E = 5$, $\alpha = 4/3$ and $E = 2$. The trend suggests possible crossover from $1/G(q) \propto q$ to q^p with $p < 1$ as q gets very small. $L_y (= 600$ here) is still not large enough to settle whether a gap is present.

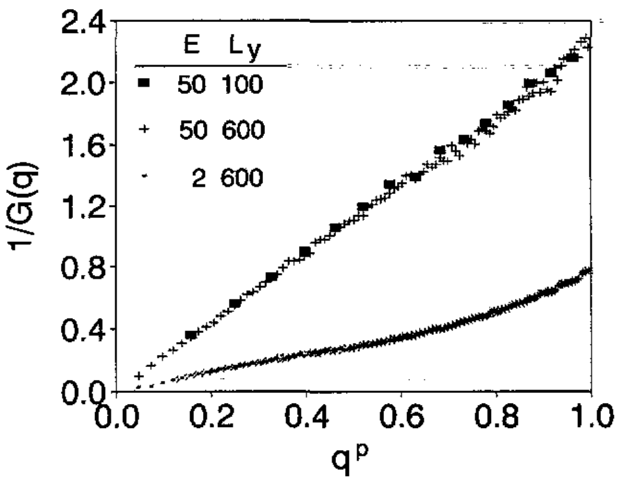

Figure 4. Uniformly driven model with different drive and system sizes. The small-q behaviour is simpler than the other two models, as the structure factor displays a weak singularity: $G(q) \sim 1/q^p$ with $p \approx 0.67$.

in y is relevant to the long-distance properties of the interface.

The gapless mode implies an infinite correlation length, despite the interface being smooth. In this respect, our situation is closer to an equilibrium interface in a quasiperiodic potential, which is governed by an *effective* Hamiltonian of the form [15]

$$\mathcal{H}\{h\} = \frac{1}{2} \int_q \Sigma_{\text{eff}}(q) q^2 |h(q)|^2$$

with an effective stiffness

$$\Sigma_{\text{eff}}(q) \approx K q^{-\eta} \qquad \text{for small } q.$$

For certain quasiperiodic potentials in two bulk dimensions, $\eta > 0$ so that the stiffness grows on large scales: $\Sigma_{\text{eff}} \sim L^{\eta}$ [15]. Thus, by analogy, we are led to an intuitively appealing picture which suggests that a uniform drive suppresses the roughness of an interface by enhancing its effective stiffness on large scales.

To conclude, we have computed numerically the structure factor for interfaces in several non-equilibrium systems. The suppression of roughness is manifested as a modification of the long-distance correlation and effective stiffness, which are different in the two cases with and without parity symmetry. We end by identifying several open questions. First, it is necessary to obtain more precise data for *very small* q in the case with random field and two temperatures, in order to determine whether a mass gap is present. On the other hand, an analysis of the nonlinear effects of the soft, bulk modes on the interfaces under a random field is desirable, which will undoubtedly lead to a better understanding of not only the mechanism of roughness suppression, but also the general interplay between interfaces and the bulk with comparable relaxation rates. Finally, it will be a challenging task to determine the index η analytically under uniform drive, e.g. by starting from the non-Hermitian operator involved in the linearized bulk dynamics.

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