MESOSCOPIC MODELING OF SLOW CRACK PROPAGATION

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The formation of crack networks in slow fracture of thin layer of materials is addressed from a statistical physics point of view. Analytic and simulation results of a mesoscopic model are described. Motivated by experiments, the model captures the effects of pinning by friction and stress concentration at crack tips, which favors nucleation and propagation of cracks, respectively. The morphology of the emerging patterns depends on how the stresses are imposed. For homogeneous stresses, the patterns exhibit a cellular stucture which can be characterized by the growth of stress correlation and a scaling behavior in the thickness and friction. For inhomogeneous stresses with a propagating front, the crack path grows linearly and exhibit various instabilities in agreement with experiments.

1 Introduction

Nature is full of fascinating forms and patterns[1,2]. Many examples spring to mind, from clouds, plants, skin patterns of animals, waves in sand flows, vortices in fluids, to flockings of fishes and slime mould cells. Curiosity in those objects have inspired the studies of fractals[3], reaction-diffusion systems[4], granular materials[5], fluid dynamics[6], and self-propelled particles[7]. A no less ubiquitous class of patterns is generated by the fracture of solids[8], as manifested by cracks in rocks, old paintings, battered roads, dried-out fields, and tectonic plates. Indeed, people have long been interested in crack patterns in various contexts[1,9,10], but systematic studies are relatively few[11]. The reason is probably that the problem is mathematically too difficult. While it is intuitively obvious when and how things break[12], to understand it on a quantitative basis requires unraveling the complexity of many interacting, dynamically evolving cracks. Thus, dynamic fracture mechanics remains a discipline scattered with unsolved problems, even for a lone crack propagating in a homogeneous medium[13]. The difficulty with a mechanical treatment has prompted a different approach.

In many ways, fracture processes are similar to phase transitions[14]. For instance, energy balance plays an important role: to propagate a crack, one trades elastic potential energy for surface one, precisely as in the nucleation of droplets during first order phase transitions. The fact that crack patterns appear to be so similar over an exceedingly wide range of scales, from μm to km, largely independent of the materials involved, strongly suggests the existence of some universal mechanism. Universality – the irrelevance of microscopic details – is one of the central themes of modern statistical physics. Hence, analogous to phase transitions, a coarse-grained description at a mesoscopic level is expected to be more fruitful than a microscopic one, on the one hand, and on the other hand may yield more insight than a purely phenomenological approach.

With this expectation in mind, we may ask questions like: What is the underlying mechanism responsible for the diversity of scales and the apparent similarity of morphology? Does fracture occur abruptly like in a first order phase transition, or smoothly like in a continuous one? We try to answer these and similar questions by comprehensive studies of simple mesocopic models using concepts and methods of statistical physics, along with simple experiments.

2 Mesoscopic Model

To study cracking in 'slowly driven' fracture processes, such as by desiccation, we proposed, analyzed, and simulated a model system that describes the fracture of a brittle layer in contact with a frictional substrate[15,16]. In a mesoscopic approach, we identify the grains in a material as the smallest units relevant to crack formation, and represent them as blocks on a lattice. Neighboring blocks interact by short-range forces through interconnecting bundle of springs. The essence of the substrate is to provide friction and hence a stick-and-slip manner of relaxation. It hinders block movement and favors nucleation of cracks. Competing with it is the stress amplification at a crack tip, due to deformation of the surrounding medium in response to new free surfaces. Amplification tends to favor crack propagation. The morphology of the resulting crack paths can be accounted for largely by the relative dominance of these two factors, which in turn depends crucially on how the system is 'driven' (i.e., how the stresses are imposed), in addition to properties like the thickness of layer and the magnitude of friction.

3 Results

3.1 Homogeneous Stresses

To describe simple in-plane fracture in which the fragments remain planar (cf. Sec. 3.2), we initially prestrain the system and then relieve it by increasing the stresses very slowly compared to relaxation. In a typical desiccation process, the stresses are induced by evaporation of the liquid from the upper free surface, causing the material to get weaker and stiffer. Since the evaporation is supposed to be spatially uniform, we impose homogeneous stresses. By analyzing and simulating this case, we obtained the following results[16]:

1. the cracking is preceded by a growth of correlation length ξ of the stress field, and the area S of coorporative slippings;

2. the growth of ξ and S are described by power laws with exponents related by a 'scaling relation';

3. friction and thickness play complementary roles via a scaling variable;

4. the correlation length ξ at the onset of cracking naturally selects the final, mean fragment area A, such that A increases with increasing thickness and decreasing friction.

Typical stationary patterns are shown in Fig. 1. Our results show that similar patterns over vastly different scales can be obtained by tuning the thickness and substrate coupling.

We have also explored the nature of fracture processes from the viewpoint of phase transitions (between an intact and a ruptured phase). Depending on the values of prestrain and friction, the rupture can be either continuous or abrupt[15]. This agrees with a related analytic study of the effect of disorder using a mean-field type model known as the democratic fiber bundle model[17]. Between the abrupt and continuous behavior lies a novel tricritical point controlled by disorder. Since friction and disorder have similar effects on fracture, both limiting the stress amplification, our conclusion is quite general: while some rupture modes are preceded by precursor events, some have none and hence are inherently unpredictable. This has important implications for failure prediction in industries and for the feasibility of earthquake predictions.

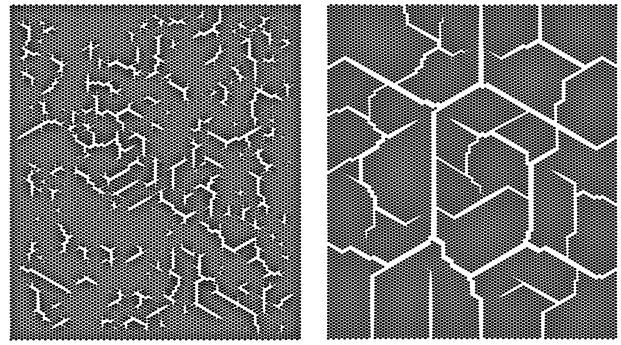

Figure 1: Typical crack patterns at intermediate stage, obtained by simulation of the mesoscopic model: Left: Small thickness and large friction, pattern dominated by crack nucleation. Right: large thickness and small friction, pattern dominated by crack propagation.

3.2 Inhomogeneous Stresses With a Front

Very recently, we obtained by experiment a novel and unusual class of crack paths in the form of spiral *within* desiccated fragments in drying precipitates[18], as shown in Fig. 2. The observation is not accidental because we

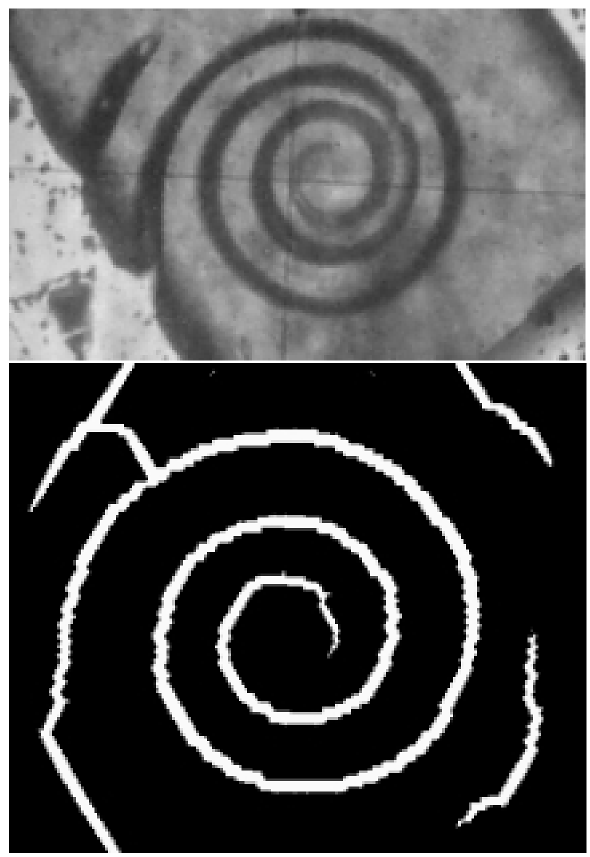

Figure 2: Upper: Spiral crack in a fragment of dried $Ni_3(PO_4)_2$ precipitate, viewed from below. The size of the spiral is about 1 mm. Lower: A spiral obtained by computer simulation of the mesoscopic model. Note prematured ending of earlier attempts in both pictures.

have observed them with various compounds. Though the present study is by no means comprehensive, the necessary conditions for spirals to appear seem to be (i) fine grains in the precipitate, (ii) small layer thickness (sub *mm*), and (iii) the horizontal components of stresses have strong gradients versus the depth of layer. Due to (iii), the fragments are prone to fold up and detach from the substrate as they are drying, causing a large radial component of stresses along the detachment rim which shrinks in time. Computer simulation, using the above model with inhomogeneous stresses imposed on an advancing front, successfully reproduces the observed spirals.

Finally, when driven by a planar stress front which sweeps across the system slowly at constant speed v, at zero friction the crack undergoes turning and branching instabilities as v is increased, in agreement with experiments on the quenching of a heated glass strip[19]. As friction is increased, the linear crack path turns into a dense network. These results[20] will be reported elsewhere.

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