**Sornette, Leung, and Andersen Reply:** We argue that the existence of abrupt failure in the democratic fiber bundle model (DFBM) is more general than concluded by da Silveira in his Comment [1]. In this goal, we reformulate his Eq. (1) in a much more intuitive way: $fx_{i+1} \equiv (F/N_0)(N_{i+1}/F) = 1 - P(1/x_i) \equiv 1 - P(F/N_i)$, which leads to $F/N_0 = (F/N_{i+1})[1 - P(F/N_i)]$ At equilibrium, $N_{i+1}$ and $N_i$ are replaced by the number $n$ of remaining intact fibers and we retrieve the usual equation used in [2]:

$$F/N_0 = (F/n)[1 - P(F/n)] \equiv x_n[1 - P(x_n)], \quad (1)$$

which expresses that the total bundle will not break under a load $F$ if there are $n$ fibers in the bundle, each of which can withstand the stress $F/n$. In contrast, the iterative Eq. (1) of [1] is just a numerical scheme and has no physical interpretation. It may be misinterpreted as representing the sequence of fiber ruptures given small increments of the applied force. See [3] for proper derivations of the power law distributions of the genuine fiber rupture burst sizes.

The bundle breaks down when $F$ reaches the maximum of $N_0 x_n[1 - P(x_n)]$. In the case when $N_0 x_n[1 - P(x_n)]$ has a single maximum, we showed [2] that the rate of fiber failure diverges with a square root singularity on the approach towards global failure (critical behavior) when the above function is quadratic at the maximum. Note that the distinction between the cases (i), (ii), and (iii) in [1] are immaterial since the onset of failure is qualitatively the same, all being critical. Instead, an abrupt "first-order" rupture occurs when the maximum happens to be at the minimum strength $x_{\min}$. This condition comprises the case studied in [2] and is the same as stated in [1], i.e., $p(x_{\min}) > 1/x_{\min}$, strictly equivalent to $d\{x_n[1 - P(x_n)]\}/dx_n < 0$ at $x_n = x_{\min}$, which follows directly from our analysis [2].

We can generalize this further. Indeed, the most general condition for a brutal rupture is that

$$\pm d\{x_n[1 - P(x_n)]\}/dx_n|_{x_n = x^{*\pm}} < 0\,, \quad (2)$$

i.e., that the function $x_n[1 - P(x_n)]$ has a discontinuity with a change of sign in its slope at its maximum $x^*$. Explicitly, this gives

$$p(x^{*-}) < \frac{1 - P(x^*)}{x^*} < p(x^{*+})\,, \quad (3)$$

i.e., the differential distribution $p(x) = \frac{dP}{dx}$ must have a jump that is sufficiently large at $x^*$. The previous case corresponds to the situation where the jump occurs at $x_{\min}$ but this is a very particular case. This condition accounts for the more subtle cases where the discontinuity at $x_{\min}$ does not tell the whole story even when $P(x)$ is monotonous. Consider, for instance, a Weibull distribution $P(x) = 1 - \exp\{-[(x - x_{\min})/D]^m\}$, for $x > x_{\min}$, and $P(x) = 0$, for $x < x_{\min}$. Condition (3) gives $x^* = x_{\min}$ and $m < 1$. The rupture is first order for $m < 1$, critical for $m > 1$, and, for $m = 1$, it is first order for $D < x_{\min}$ and critical otherwise. More complicated scenarios occur, because condition (3) only expresses the existence of a jump, and not the fact that this is a global jump. Another condition must ensure that $x^*$ is the global maximum and not only a local one. For instance, for $m = \frac{1}{2}$, $x[1 - P(x)]$ has another maximum at $x = 2D(1 + \sqrt{1 - x_{\min}/D})$ in addition to the "ridge" at $x_{\min}$. This implies that the abrupt rupture at $x_{\min}$ is only partial. After that, the applied force has to increase to climb a barrier whose peak corresponds to a critical rupture down to $n = 0$.

This class of condition (3) is probably relevant for real materials that do not have a continuous distribution. The habit to use continuous distributions, such as the Weibull law and others, stems from their ability to fit rupture data of large macroscopic systems. These fits, as in most statistical analysis, are controlled by the regions where the data are plentiful and not by the extreme tails. Without exploring the tails, it is, thus, very difficult to assert statistically whether the fiber strength distribution is smoothed or exhibits jumps.

As for the role of disorder, in addition to the explicit example discussed in [2], we have also studied the cases where $P(x)$ is a tanh, a power law, and the Weibull distribution ($m = 1$) above a minimum strength. In these case, the discontinuity condition at $x_{\min}$ for first-order rupture to occur consistently translates into a requirement of small disorder which is represented by the width of the relevant distribution.

In summary, we have refuted the claim in [1] that the nature of the rupture process in the DFBM depends on the "disorder distribution only via its large $x$ behavior."

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