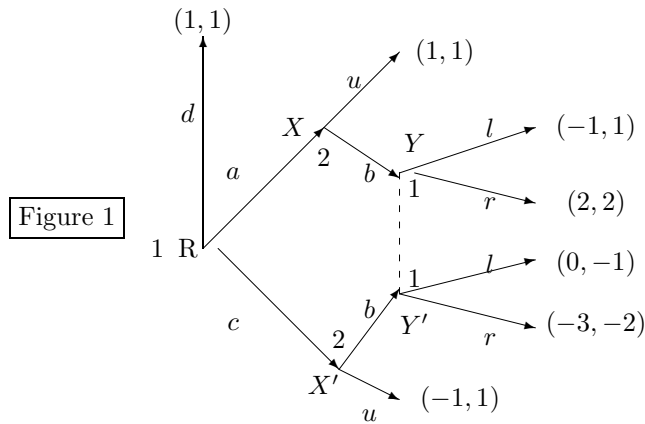
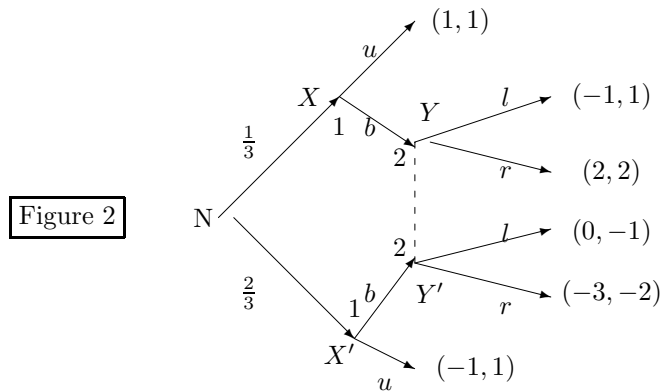


1. (10 points) Consider Figure 1.

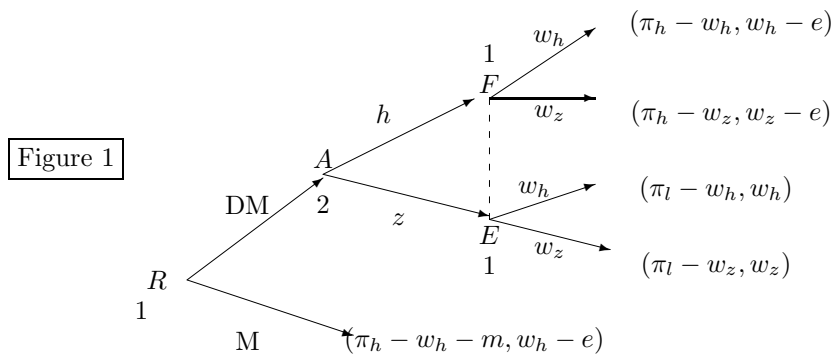


- (a) (2 points) How many **subgames** are there in the sequential game (including the original game)?
- (b) (2 points) Is this a game with imperfect information?
- (c) (3 points) How many information sets are owned by player 1?
- (d) (3 points) How many strategies does player 1 have?

2. (10 points) Find all the pure strategy Bayesian Nash equilibrium for the sequential game in Figure 2.



3. (10 points) Find all the pure-strategy Nash equilibrium and subgame perfect Nash equilibria for the sequential game in Figure 3. (Note: I am **not** asking for equilibrium outcome.)



4. (10 points) Consider the sequential game in Figure 4. Find all the possible sequential equilibria in which both types choose L .

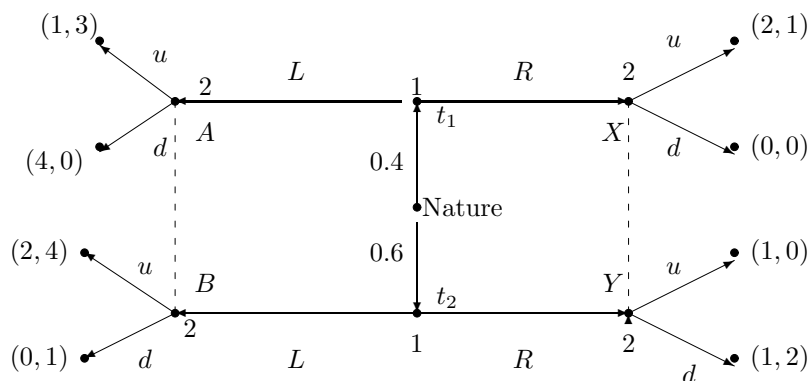


Figure 4

5. (10 points) Suppose a parent and a child play the following game. First, the child takes an action, A , that produces income for the child, $I_c(A) = 2A - 5A^2$, and income for the parent, $I_p(A) = A - A^2$. Second, the parent observes the incomes I_c and I_p and then chooses a bequest, B , to leave to the child. The child's payoff is $U(I_c + B)$; the parent's payoff is $V(I_p - B) + \frac{1}{4}U(I_c + B)$. Assume that: $A \geq 0$; the income functions $I_c(A)$ and $I_p(A)$ are strictly concave and are maximized at $A_c > 0$ and $A_p > 0$, respectively; the bequest B can be positive or negative; and the utility functions $U(x) = V(x) = \sqrt{x}$ are increasing and strictly concave.

- (a) (2 point) Solve the following maximization problem:

$$\max_{A \geq 0} I_c(A) + I_p(A)$$

- (b) (8 points) Solve the backwards induction outcome of this game.

6. (0 points) Suppose there are only one worker and one firm in the market. With probability 0.4, the worker is Qualified and with probability 0.6 the worker is Unqualified. If the firm hires a Qualified worker, the firm's net payoff is 30. If the firm hires an Unqualified worker, the firm's net payoff is -10 . If the firm doesn't hire, the firm's payoff is 0. The worker knows his own type. The only signal the worker can use is through education $n \in \{0, 1, 2, 3, \dots\}$. The cost of education n is $\frac{n^2}{2}$ for a Qualified worker and for an Unqualified worker, is n^2 . The payoff for both types of workers is 100 if he gets a job, and 0 otherwise. The game is as follows: The nature moves first. Then the worker picks n . After the firm observes n , the firm will choose hire or not. Then the game ends. Suppose the firm must use simple strategy which has the following form: $s^F(n) = \begin{cases} Hire & \text{if } n \geq n_0 \\ No & \text{otherwise} \end{cases}$. Draw the sequential game and write down players' information sets and strategies. Find all the possible pooling equilibria and separating equilibria.