

1. Similar to the nonlinear-prices example in class, a monopolist offers a menu of bundles $\{q, T\}$ to consumers, where q is the quantity sold and T is the fixed fee. Consumers with type θ receive utility $\theta\sqrt{q} - T(q)$ if they purchase a quantity q and 0 otherwise. Suppose (q_1, T_1) is directed at type- θ_1 consumers (in proportion λ), and (q_2, T_2) is directed at type- θ_2 consumers (in proportion $1 - \lambda$). The unit cost of producing the good is 1. Suppose $\theta_1 = 1$ and $\theta_2 = 4$.
 - (a) Is it possible that the monopolist's best strategy is to sell to type- θ_1 consumers only?
 - (b) If the monopolist only sells to type- θ_2 consumers, what is the optimal bundle (q_2^*, T_2^*) ?
 - (c) If the monopolist sells to both types, what are the optimal bundles (q_1^*, T_1^*) and (q_2^*, T_2^*) ?
 - (d) What is the monopolist optimal decision?
 - (e) Suppose θ is uniformly distributed over $[1, 4]$. Find the optimal nonlinear price (q, T)