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Spring 2009

NTU Homework 01: Hold-up problem

Q: Consider a potential trading relationship in which a supplier may provide one unit of a good to a buyer. At time $T = 0$, the buyer makes an investment of I dollars. At $T = 1$, the seller's cost to produce the good c is drawn from [0,1] according the the distribution function $F(c)$. After the realization of c, both parties decide whether to trade. If trade occurs, the two parties write a contract that stipulates the transfer price p, and the supplier must produce the good at cost c and deliver it to the buyer. Let $v(I)$ be the buyer's valuation of the good. Suppose $v(I)$ is increasing and concave for all $I \geq 0$, i.e., $\frac{\mathrm{d}v}{\mathrm{d}I} > 0$, and $\frac{\mathrm{d}^2v}{\mathrm{d}I^2} < 0$. The two parties are risk neutral. Suppose that I, v, and c are observable to both parties whenever they are realized; however, they are not verifiable by a court, so that a contract cannot be written contingent on them. (Assume that an interior solution for the investment exists for any problems that you encounter.)

a) Characterize the (efficient) trading rule and the investment level that maximizes the total expected gains from trade.

b) Suppose after the realization of v the two parties bargain over the gains from trade according to the Nash bargaining solution. Characterize the equilibrium trading and investment decision.

c) What if the buyer makes a take-it-or-leave-it offer in the bargaining? Repeat the same exercise as in (b).

Prior to the investment decision, the two parties write a contract specify (p, d), where p is the price the buyer pays to the seller in case of trade, and d is the damage that the seller must pay to the buyer in case of his breaching the contract (i.e., refusing to produce the good at trading price p). There is no damage against the buyer if she breaches the contract.

d) Evaluate the performance of the expectation damages rule.

e) Evaluate the rule in which both parties can specify any fixed pair (p, d) in their best joint interest. Compare the outcome with that of (d).

Answer

a) First best outcome

- Trade is efficient:

$$\text{Trade iff } v(I) \geq c$$

- Investment is efficient (I^*):

$$\begin{aligned} W(I) &= \int_0^{v(I)} (v(I) - c)\mathrm{d}F(c) - I \\ &= (v(I) - c)F(c)]_0^{v(I)} + \int_0^{v(I)} F(c)\,\mathrm{d}c - I \\ &= \int_0^{v(I)} F(c)\,\mathrm{d}c - I \\ W'(I) &= v'(I)\,F(v(I)) - 1 = 0 \end{aligned}$$

Lets call the solution I^*. Hence,I^* satisfies

$$v'(I^*) = \frac{1}{F(v(I^*)))} > 1 \tag{1}$$

b) Let $B(I)$ and $S(I)$ be the buyer's and the seller's surplus, respectively Nash Bargaining solution implies that they share the surplus from trade equally. Hence,

$$\begin{aligned} B(I) &= -I + \frac{1}{2}(v(I) - c) \text{ if } v(I) - c > 0 \\ S(I) &= \frac{1}{2}(v(I) - c) \text{ if } v(I) - c > 0 \end{aligned}$$

Rewrite the above as $\begin{cases} B(I) = -I + \frac{1}{2}\int_0^{v(I)}(v(I) - c)\mathrm{d}F(c) \\ S(I) = \frac{1}{2}\int_0^{v(I)}(v(I) - c)\mathrm{d}F(c) \end{cases}$

First period, the buyer chooses I^W satisfying

$$\begin{aligned} B'(I) &= -1 + \frac{1}{2}v'(I)\,F(v(I)) = 0 \\ v'(I) &= \frac{2}{F(v(I)))} > v'(I^*) \end{aligned}$$

Since $v'' = \frac{\mathrm{d}v'}{\mathrm{d}I} < 0$, we know $v'(\cdot)$ is decreasing in I. Therefore, we have $I^* > I^W$, which implies underinvestment.

c) Since the buyer makes a take-it-or-leave-it offer, the buyer will enjoy all the surplus from trade. Hence, $B(I) = -I + \int_0^{v(I)} (v(I) - c)\mathrm{d}F(c)$. Therefore, the investment level is efficient.

d) $d = v(I) - p$ (Note that this requires that the court is able to evaluate $v(I)$.)

- The seller makes trade-decision. We will show that trade-decision is efficient:

$$\begin{aligned} \text{Trade iff } p - c &\geq -d \\ p - c &\geq -v(I) + p \\ v(I) &\geq c \end{aligned}$$

- The buyer makes investment decision. We will show that the buyer will overinvest, i.e., $I^{ED} > I^*$:

$$\begin{aligned} B(I) &= \int_0^{v(I)} (v(I) - p)\mathrm{d}F(c) + \int_{v(I)}^1 d\mathrm{d}F(c) - I \\ &= v(I) - p - I \\ B'(I) &= v'(I) - 1 = 0 \end{aligned}$$

Lets call the solution I^{ED}. Hence,I^{ED} satisfies

$$v'\left(I^{ED}\right) = 1 < v'(I^*)$$

Therefore, $I^{ED} > I^*$

e)

- The seller makes trade decision. Trade iff $p - c \geq -d$

- The buyer makes investment decision. I^{LD}

$$\begin{aligned} B(I) &= \int_0^{p-d} (v(I) - p)\mathrm{d}F(c) + \int_{p-d}^1 d\mathrm{d}F(c) - I \\ B'(I) &= v'(I) F(p - d) - 1 = 0 \end{aligned}$$

Let $I_x(d)$ be the solution that satisfies

$$v'(I)\left\{F(p - d) - 1 = 0 \qquad (2)$$

Given that they can choose any d they like on the contracting day, in order to have efficient trade-decision, they have to select $d = v(I_x) - p$ in the contract. However, when they choose

$d = v(I_x) - p$, equation (2) reduces to equation (1). Hence, $I_x = I^*$. Since I^* is the efficient investment level, we have that the optimal d is $v(I^*) - p$.

Note that fixing p, the damages payment under ED rule is $v\left(I^{ED}\right) - p$, which is higher than $v(I^*) - p$. Hence, the probability of trade $F(v(I))$ is higher under ED rule. The buyer is better off and the seller is worse off under ED rule.