Due Oct.26, 2011

IO (I) Homework

1.1. Let $X = R^2$ and $X' = \{(0,0), (2,1), (1,2), (3,3)\}$. Is X' a lattice ? Is X' a sublattice of X? Please explain your answers.

1.2. Let $X_i = \{0,1\}$ for $i = 1,2,3$, $X = \times_{i=1}^{3} X_i$, $f(0,0,0) = 3$, $f(0,0,1) = f(0,1,0) = f(1,0,0) = 2$, $f(0,1,1) = 4$, $f(1,0,1) = f(1,1,0) = 0$, and $f(1,1,1) = 1$. Show that $f(x)$ is quasisupermodular in each pair of components of $x = (x_1, x_2, x_3)$ but not quasisupermodular in x. Is $f(x)$ supermodular in each pair of components of x?

1.3. let $X_i = \{0,1\}$ for $i = 1,2$, $X = \times_{i=1}^{2} X_i$, and define $f_k(x)$ on X for $k = 1,2,\ldots$ such that $f_k(0,0) = 0$, $f_k(0,1) = f_k(1,0) = 1$, and $f_k(1,1) = 1 + 1/k$. Let $f(x) = \lim_{k\to\infty} f_k(x)$ for each x in X. Show that each $f_k(x)$ is quasisupermodular on X, but $f(x)$ is not quasisupermodular on X.

1.4. Consider a duopoly game where each player has a compact interval as strategy space. The payoff to player 1 is $\pi_1(a_1, a_2; t)$ and to player 2, $\pi_2(a_1, a_2)$. Show that if the game is supermodular and $\pi_1(a_1, a_2; t)$ has increasing differences in (a_1, t), then extremal equilibria are increasing in t. Show that if the game is submodular, then extremal equilibrium strategies for firm 1 (2) are increasing (decreasing) in t if $\pi_1(a_1, a_2; t)$ has increasing differences in (a_1, t). (Hint: For the submodular game consider the transformed game $s_1 = a_1$ and $s_2 = -a_2$.)