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Dual-self Model

Fudenberg and Levine (EM 2012), "Timing and self-control"

In dual-self models, the agent acts to maximize expected discounted utility subject to a cost of self-control that is derived from the preferences of a more impulsive short run self.

Model:

discrete-time model with periods $n = 1, 2, \ldots$ and period length τ.

long-run self "planner" : discount rate $\delta = e^{-\rho\tau}$.

short-run self "temptation selves "doers" : discount rate $\delta\mu$, $\mu = e^{-\eta\tau}$.

$Y \subset R^{N_1}$: space of states, $A \subset R^{N_2}$: space of actions

$h_n = (y_1, a_1, y_2, a_2, \ldots, y_{n-1}, a_{n-1}, y_n)$, $h_1 = y_1$ exogenously given.

A strategy for the long-run self is a measurable map $\boldsymbol{a}$ from h_n to a, i.e., $\boldsymbol{a}(h_n) \in A\,(y_n)$

Dynamics are Markovian: they are given by probability distributions $\pi\,(y_{n-1}, a_{n-1})\,[dy_n]$ over states next period, conditioned by the current state and action.

The shorter-run self (or selves) has utility $u(y_n, a_n)$ in period n if the action an is taken in the state y_n.

We work with average present values, so that we hold $u(y_n,\, a_n)$ fixed as we vary the length τ of the time period.

The expected average present value of the shorter-run self from period n on under $\boldsymbol{a}$ is given by

$$U\left(h_n, \boldsymbol{a}\right) \equiv E_{\boldsymbol{a},h_n}\left(1-\delta\mu\right)\sum_{l=0}^{\infty}\left(\delta\mu\right)^l u\left(y_{n+l}, a_{n+l}\right)$$

ASSUMPTION SR0:

$$\begin{aligned} & E_{\boldsymbol{a},h_n}\left(1-\delta\mu\right)\sum_{l=0}^{\infty}\left(\delta\mu\right)^l u\left(y_{n+l}, a_{n+l}\right) \\ & = \left(1-\delta\mu\right)\sum_{l=0}^{\infty}\left(\delta\mu\right)^l E_{\boldsymbol{a},h_n} u\left(y_{n+l}, a_{n+l}\right) \end{aligned}$$

and $U\left(h_n, \boldsymbol{a}\right)$ has a maximum for each n and h_n.

Because the problem of the shorter-run self is Markov, this maximized value only depends on the state:

THEOREM 1: $\max_{\boldsymbol{a}} U(h_n, \boldsymbol{a}) = \bar{U}(y_n)$

$\bar{U}(y_n)$: the temptation value in period n starting at state y_n.

The foregone value is then

$$\triangle(y_n, a_n) = \bar{U}(y_n) - \left((1 - \delta\mu) u(y_n, a_n) + \delta\mu \int_Y \bar{U}(y_{n+1}) \pi(y_{n+1} | y_n, a_n) [dy_{n+1}] \right)$$

THEOREM 2: $\triangle(y_n, a_n) \geq 0$ and

if $\hat{\boldsymbol{a}} \in \arg\max_{\boldsymbol{a}} U(h_n, \boldsymbol{a})$, then $\triangle(y_n, \hat{a}_n) = 0$

Suppose the self-control cost is $\Gamma\triangle(y_n, a_n)$, i.e., linear in the foregone value, where $\Gamma \geq 0$.

The agent's objective function is

$$V(h_n, \boldsymbol{a}) \equiv E_{\boldsymbol{a},h_n} \sum_{l=0}^{\infty} \delta^l \big((1-\delta)\, u(y_{n+l}, a_{n+l}) - \Gamma\triangle(y_{n+l}, a_{n+l}) \big)$$

Page 9: "The objective function reduces to the linear-costs version of Fudenberg and Levine (2006, 2010) when $\mu = 0$. It is also a special case of the functional forms considered in Noor (2007, 2011); unlike Noor, we assume that the long-run and short-run selves have the same underlying per-period utility function, and we specify how preferences change with the period length."

Page 6: " The shorter-run self (or selves) has utility $u\left(y_{n,}a_{n}\right)$ in period n. We work with average present values, so that we hold $u\left(y_{n,}a_{n}\right)$ fixed as we vary the length τ of the time period. The objective of the long-run player is the average present value of these shorter-run self utilities, minus a cost of self-control that is defined with reference to the maximum possible average present value for the shorter-run self."

Question: Suppose $\Gamma = 0$. We have $U\left(h_{n}, \boldsymbol{a}\right) = V\left(h_{n}, \boldsymbol{a}\right)$. Is this reasonable?