Meng-Yu Liang

Dual-self Model : note 2
Fudenberg and Levine (EM 2012), "Timing and self-control"

Simple Temptation : a choice between either utility 0 in every period or a flow of $u_g > 0$ that is received for a number of periods N, with $-u_b < 0$ forever after.

The average present values S for the short-run self and P for the long-run self of this temptation are

$$S = \left(1 - (\delta\mu)^N\right) u_g - (\delta\mu)^N u_b$$

$$P = \left(1 - \delta^N\right) u_g - \delta^N u_b$$

Suppose $P < 0 < S$, a conflict between short-run self and long-run self.

Example 1:

a). Temptation will be resisted if

$$P < -\Gamma S$$

b). Future temptation is easier to resist. Future temptation (n periods) will be resisted at date 1 if

$$\delta^n P < -\Gamma \left(\delta\mu\right)^n S$$

or

$$P < -\Gamma \mu^n S$$

c). Persistent temptation is harder to resist. If resist, the temptation is still there the next period. (for example, a bottle of wine instead of a cake). Resisted if

$$(1 - \delta) P < -\Gamma (1 - \delta\mu) S$$

Question 2: Suppose $\mu = 0$ and $\tau \to 0$, i.e., $\delta \to 1$. Then $(1-\delta) P \to 0$ and $-\Gamma(1-\delta\mu) S = -\Gamma u_g$ This implies that the planner will always take the persistent temptation if we take the calendar length of time τ to 0. Is this result reasonable?

Example 2: Nonmonotonicity of the Value of Commitment: At time 0 the agent's action is to pick a menu from the list ({0},{0,1}).

{0} : commitment, temptation is not available at time 1.

{0, 1} : temptation is available at time 1.

Suppose $u_g = u_b = 1$ and $T = 1$. $\Gamma = 3$ and $\rho = \ln\left(\frac{3}{2}\right)$.

$$P = -\frac{1}{3} \text{ and } S = 1 - \frac{4}{3}e^{-\eta}$$

$S > 0$ iff $\eta > \ln\left(\frac{4}{3}\right)$

under choice $\{0,1\}$, planner's decision at time 1.

$\eta \in [0, \ln\frac{4}{3}]$: $S < 0$ no self-control problem.

$\eta \in (\ln\frac{4}{3}, \ln\frac{3}{2})$: resist $U_1 = -\Gamma S$

$\eta \in (\ln\frac{3}{2}, \infty)$: take temptation $U_1 = P$

η	$U_0(\{0\})$	$U_0(\{0,1\})$	choice at time 0
$[0, \ln\frac{4}{3})$	0	0	$\{0\}$ and $\{0,1\}$
$(\ln\frac{4}{3}, \ln\frac{3}{2})$		$-\Gamma e^{-\rho} S$	$\{0\}$
$(\ln\frac{3}{2}, \bar{\eta})$	$-\Gamma e^{-(\rho+\eta)} S$	$e^{-\rho} P$	$\{0,1\}$
$(\bar{\eta}, \infty)$		$e^{-\rho} P$	$\{0\}$

Example 3: Declining Marginal Interest Rates

The incremental intest rate between times t_{i-1} and t_i.

$$\rho_i = \ln(c_{t_i}/c_{t_{i-1}})/\left(t_i - t_{i-1}\right).$$

Note the definition of c_t at page 14 is in-consistent with ρ_i but the definition of c_t at page 15 is fine.

Long-run self is indifferent between 1 unit now and c_n units later, then since $\mu < 1$, the initial shorter-run self strictly prefers 1 unit now.

The temptation is to consume now.

The initial short-run self gets an average present value of $1-\delta\mu$from consuming at time 1 and gets $(1-\delta\mu)(\delta\mu)^{n-1}c_n$ from the delayed option, so the control cost of the delayed option is $\Gamma(1-\delta\mu)(1-(\delta\mu)^n - 1c_n)$. Hence, we can solve c_n :

$$1 - \delta = (1 - \delta)\delta^{n-1}c_n - \Gamma(1 - \delta\mu)(1 - (\delta\mu)^{n-1}c_n)$$

Compute $\frac{c_{n+1}}{c_n}$to find ρ_n. Take larger value of μ and get a more gradual decline (Further reading: please check hyperbolic or quasi-hyperbolic discounting model and the comments of Levine's book "Is Behavioral Economics Doomed? The Ordinary versus the Extraordinary ")

Willpower as a stock:

Consider the case that willpower is depleted and replenished.

Beginning of period n, we have willpower w_n at the end of the period $\tilde{w}_n = f(w_n, \triangle(y_n, a_n)) \leq w_n$, $f_1 \geq 0$, $f_2 \leq 0$,and $f(w_n, 0) = w_n$.

$w_{n+1} = r(\tilde{w}_n) \geq \tilde{w}_n$. $m(y_n, \tilde{w}_n)$: extra utility from $\tilde{w}_n$. If $r(\tilde{w}_n) = \tilde{w}_n$, the objective function of the long-run self is to maximize

$$V(h_n, \boldsymbol{a}) \equiv E_{\boldsymbol{a},h_n}(1-\delta)\sum_{l=0}^{\infty}\delta^l(u(y_{n+l}, a_{n+l}) + m(y_{n+l}, f(w_{n,}\triangle(y_n, a_n))))$$

Assumption i) $r(\tilde{w}_n) = \bar{w}$ ii) $\mu = 0$.

Question 3: How to define c(page 20).to get the objective funtion to be

$$V\left(h_n, \boldsymbol{a}\right) \equiv E_{\boldsymbol{a},h_n}\left(1-\delta\right)\sum_{l=0}^{\infty}\delta^l(u\left(y_{n+l}, a_{n+l}\right)$$
$$+ m\left(\bar{w}\right) - c(\triangle\left(y_{n+l}, a_{n+l}\right)))$$

Question 4: Is $g\left(\triangle_n\right)$(page 21) consistent with Theorem 3?