Designing Random Allocation Mechanisms: Discussion

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February 19, 2011

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- Many agents
- Agents should be treated symmetrically – hence reliance on randomization

Good Ordinal Mechanisms

- Strategy-proof
- Ordinally efficient
- Symmetric (equal treatment of equals)

Two Well-Studied Ordinal Mechanisms Coincide in the Limit!

- Random Priority (Abdulkadiroglu and Sonmez 1999)
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Core from Random Endowments (Top Trading Cycles) coincide with Random Priority already in small markets (Abdulkadiroglu and Sonmez 1999, Pathak and Sethuraman 2011)

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Theorem (Liu and Pycia 2011). In the continuum economy, every mechanism that is strategy-proof, ordinally efficient, and symmetric coincides with Random Priority/Probabilistic Serial for almost every preference distribution.

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If we restrict attention to "continuous" mechanisms then:

- In the continuum economy, Random Priority/Probabilistic Serial is the only strategy-proof, ordinally efficient, and symmetric mechanism.
- If a sequence of symmetric and strategy-proof mechanisms is asymptotically OE as we replicate the economy, then the sequence converges to Random Priority/Probabilistic Serial.

Intensive vs Extensive Margins

Manea 2009: Random Priority does not become ordinally efficient in the limit as the number of object types grows.

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- Cardinal mechanisms offer the premise of eliciting preference intensity, and hence implement Pareto-efficient outcomes

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- Boston mechanism: good efficiency properties in equilibrium (Abdulkadiroglu, Che, and Yasuda 2011) but fails strategy-proofness even in the limit (Abdulkadiroglu and Sonmez 2003, Kojima and Pathak 2009) and has bad redistributive properties (Pathak and Sonmez 2008)

How to Decompose a Random Allocation?

Budish, Che, Kojima, Milgrom 2010 show how to decompose allocations while preserving conjunctions of elementary constraints of the form

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