

IO (I) 323 M2510  
Homework #3

Two players are deciding whether to invest. There is a safe action  $\beta$  (not invest); there is a risky action  $\alpha$  (invest) which gives a higher payoff if the other player invests. Payoffs are given by the following matrix.

	Invest ( $\alpha$ )	Not Invest ( $\beta$ )
Invest ( $\alpha$ )	$\theta, \theta$	$\theta - 1, 0$
Not Invest ( $\beta$ )	$0, \theta - 1$	$0, 0$

If there were complete information about  $\theta$ , there would be three cases to consider.

- If  $\theta > 1$ , each player has a dominant strategy to invest.
- If  $\theta \in [0, 1]$ , there are two pure strategy Nash equilibria: both invest and both not invest.
- If  $\theta < 0$ , each player has a dominant strategy not to invest.

1. Using the notation in Carlsson and van Damme (EM 1993), find the following *Intervals* :  $G^\gamma$ ,  $D_i^\alpha$ ,  $D_i^\beta$ ,  $R^\alpha$ ,  $R^\beta$ , and  $G_i^+$ .

Suppose there is incomplete information about  $\theta$ . Player  $i$  observes a private signal  $x_i = \theta + \varepsilon_i$ . Each  $\varepsilon_i$  is independently uniformly distributed over the interval  $[-\varepsilon, \varepsilon]$ . We assume that  $\theta$  is randomly drawn from the interval  $[a, b]$ , with each realization equally likely.

2. For  $a = -1$ ,  $b = 2$  and  $\varepsilon \leq \frac{1}{8}$ , find  $S_i^{\varepsilon, n}$ ,  $A_i^{\varepsilon, n+1}$  and  $B_i^{\varepsilon, n+1}$  for  $n = 1, 2, \dots, \infty$ .
3. For  $a = -1$ ,  $b = 1$  and  $\varepsilon \leq \frac{1}{8}$ , find  $S_i^{\varepsilon, n}$ ,  $A_i^{\varepsilon, n+1}$  and  $B_i^{\varepsilon, n+1}$  for  $n = 1, 2, \dots, \infty$ .
1. For  $a = -\frac{1}{2}$ ,  $b = \frac{3}{2}$  and  $\varepsilon = 1$ , find  $S_i^{\varepsilon, n}$ ,  $A_i^{\varepsilon, n+1}$  and  $B_i^{\varepsilon, n+1}$  for  $n = 1, 2, \dots, \infty$ .