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NTU Homework 03: Durable Goods Monopoly

Q: Consider the durable goods monopoly selling problem as in class. Suppose

$$f(q) = \begin{cases} 10, & \text{if } q \in [0, 0.8] \\ 2, & \text{if } q \in (0.8, 1] \end{cases}, \text{ and } c = 0..$$

Find the stationary subgame perfect equilibrium of this game. Show that $\lim_{\delta \to 1} P_\delta(0) = 2$.

Ans: Since the lowest price that the monopolist will charge in equilibrium is $f(1) = 2$, we know the consumer's acceptance function must be $P(q) = \begin{cases} 2 & \text{for } q \in (0.8, 1] \\ 10(1-\delta) + 2\delta & \text{for } q \in (\bar{q}_1, 0.8] \end{cases}$, where $\bar{q}_1$ is such that the monopolist is indifferent between charging $P(0.8)$ and $P(1)$. Hence, we have

$$P(0.8)(0.8 - \bar{q}_1) + 0.2\delta P(1) = P(1)(1 - \bar{q}_1)$$
$$(10(1-\delta) + 2\delta)(0.8 - \bar{q}_1) + 0.4\delta = 2(1 - \bar{q}_1)$$
$$\bar{q}_1 = 0.75$$

Therefore, we obtain $t(q) = \begin{cases} 1 & \text{for } q \in (0.75, 1] \\ 0.8 & \text{for } q \in [\bar{q}_2, 0.75] \end{cases}$, and $P(q) = 10(1-\delta) + \delta P(0.8) = 10(1-\delta^2) + 2\delta^2$ for $q \in (\bar{q}_2, 0.75]$, where $\bar{q}_2$ is such that the monopolist is indifferent between charging $P(0.75)$ and $P(0.8)$. Hence, we have :

$$P(0.75)(0.75 - \bar{q}_2) + \delta R(0.75) = P(0.8)(0.8 - \bar{q}_2) + \delta R(0.8)$$

$R(0.75) = P(1)(1 - 0.75) = 0.5$

$R(0.8) = P(1)(1 - 0.8) = 0.4$

$$\left(10(1-\delta^2) + 2\delta^2\right)(0.75 - q) + 0.5\delta = (10(1-\delta) + 2\delta)(0.8 - q) + 0.4\delta$$
$$\bar{q}_2 = 0.75 - \frac{0.062\,5}{\delta}$$

$P(q) = 10(1-\delta^3) + 2\delta^3$ for $q \in (\bar{q}_3, \bar{q}_2]$. $R(\bar{q}_2) = \left(10(1-\delta^2) + 2\delta^2\right)\left(\frac{0.062\,5}{\delta}\right) + 0.5\delta = \frac{0.625}{\delta}$

$$\left(10(1-\delta^3) + 2\delta^3\right)\left(0.75 - \frac{0.062\,5}{\delta} - \bar{q}_3\right) + 0.625 = \left(10\left(1-\delta^2\right) + 2\delta^2\right)(0.75 - \bar{q}_3) + 0.5\delta$$

$$\bar{q}_3 = 0.75 - \frac{7.8125 \times 10^{-2}}{\delta^3} - \frac{0.0625}{\delta}$$

Using the same arguments as in the classnote, we obtain $\bar{q}_{k-1} - \bar{q}_k = (\hat{q} - \bar{q}_1)(\frac{\underline{v}}{\overline{v} - \underline{v}})^{k-1} \delta^{-k(k-1)/2} = 0.05(\frac{5}{4})^{k-1} \delta^{-k(k-1)/2}$ for $k \geq 2$. $P(q) = 10(1 - \delta^k) + 2\delta^k$ for $q \in (\bar{q}_k, \bar{q}_{k-1}]$

Let m be the minimum number such that $\sum_{k=1}^{k=m} (\bar{q}_{k-1} - \bar{q}_k) > 1$, i.e.,$m = \min\{k : \bar{q}_k < 0, k \in \boldsymbol{N}\}$ Since $\bar{q}_{k-1} - \bar{q}_k \geq 0.05$, we have $m \leq \frac{0.75}{0.05} + 1 \leq 16$.

Given $\{\bar{q}_k\}_{k=1}^m$ is well defined, we can write down the stationary subgame perfect equilibrium as follows:

Buyers' strategy:

If the current price p is in $[0, P(\bar{q}_i)]$, then the consumers who are still in the market and satisfy $q \in [0, \bar{q}_i]$ buy one unit of goods and the rest consumers choose not to buy.

The monopolist strategy:

In the first period, choose $p_1 = P(\bar{q}_{m-2})$ if $\bar{q}_{m-1} > 0$ and choose $p_1 = \pi P(\bar{q}_{m-2}) + (1 - \pi) P(\bar{q}_{m-3})$ for any $\pi \in [0, 1]$ if $\bar{q}_{m-1} = 0$.

If the histry at period i satisfies $p_{i-1} = P(\bar{q}_{k+1})$, then $p_i = P(\bar{q}_k)$.

If $p_{i-1} \in (P(\bar{q}_{k+1}), P(\bar{q}_k))$ then $p_i = P(\bar{q}_k)$ with probability π and $p_i = P(\bar{q}_{k-1})$ with probability $1 - \pi$, where π satisfies $p_{i-1} = 10(1 - \delta) + \delta[\pi P(\bar{q}_k) + (1 - \pi) P(\bar{q}_{k-1})]$

If $p_{i-1} > P(0)$, then $p_i = P(\bar{q}_{m-2})$.

To show that $\lim_{\delta \to 1} P_\delta(0) = 2$, observe that $\lim_{\delta \to 1} P_\delta(0) \leq \lim_{\delta \to 1} P_\delta(\bar{q}_{m-1}) \leq \lim_{\delta \to 1}(10(1 - \delta^{16}) + 2\delta^{16}) = 2$. Since $P_\delta(q) \geq 2$ for all q, we conclude that $\lim_{\delta \to 1} P_\delta(0) = 2$.