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Biing-Shen Kuo, Wen-Jen Tsay

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Institute of Economics
Academia Sinica
Taipei 115, TAIWAN
http://www.sinica.edu.tw/econ/

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Abstract

This paper proposes a new class of GMM estimators to increase the efficiency of the coefficient estimate relative to the ordinary least squares (OLS) estimator when all the error term and regressors having nonparametric autocorrelation. This class of GMM estimators are built on the moments generated from the long difference (LD) operator of Griliches and Hausman (1986) and those from the multiple difference (MD) operator of Tsay (2007). Most importantly, the GMM estimator is designed to beat both OLS and first-differenced (FD) estimators when neither OLS nor FD estimator attains Gauss-Markov bound in that the proposed method merges the information inherent in the moments of the OLS estimator and those of the FD one. Thus, the GMM estimator also resolves the dilemma concerning 'to difference or not to difference' in the time series literature, because both level and differenced data are employed for the GMM estimation. The Monte Carlo experiments confirm the theoretical findings by showing that the GMM method has very good finite-sample power performance relative to both OLS and FD estimators.

Key words: Time series regression; GMM; Long difference; Multiple difference

*Wen-Jen Tsay: The Institute of Economics, Academia Sinica. 128 Academia Road, Sec. 2, Taipei, Taiwan, R.O.C. Tel.: (886-2) 2782-2791 ext. 296. Fax: (886-2) 2785-3946. E-Mail: wtsay@econ.sinica.edu.tw

1 Introduction

This paper considers the regression model consisting of stationary time series processes as follows:

$$C_t = \gamma + Z_t^\top \beta + \varepsilon_t, \quad t = 1, 2, \ldots, n, \tag{1}$$

where γ represents a scalar finite constant, Z_t is a $(K \times 1)$ random vector whose j-th element is $Z_{t,j}$, and β is a $(K \times 1)$ non-stochastic vector of unknown regression coefficients to be estimated and tested. Adaptive estimation method based on an approximate frequency-domain generalized least squares (GLS) has been considered by Hannan (1963) when ε_t in (1) is a short memory process with nonparametric autocorrelation. The methodology of Hannan (1963) has been further extended by Hannan (1965), and Hannan and Terrel (1972, 1973) to other interesting econometric models. Robinson and Hidalgo (1997) and Hidalgo and Robinson (2002) also apply the frequency-domain method of Hannan (1963) to the cases where ε_t and Z_t are long memory processes.

The most well-known time-domain method for the model in (1) is the ordinary least squares (OLS) estimator. The OLS estimator cannot achieve Gauss-Markov bound when ε is not an independently identically distributed (iid) process. Thus, GLS-type approach has been proposed to increase the efficiency of the coefficient estimate when ε_t admits a specific parametric form. For example, the Cochrane and Orcutt (1949) transformation method requires ε_t as an autoregressive process of order 1, or AR(1). Other authors, including Maeshiro (1976), Chipman (1979), and Krämer (1982) and the references therein, suggest that the first-differenced (FD) estimator can be an approximation to the GLS estimator in estimating the coefficient of the linear trend when the error term is a highly persistent AR(1) process. In fact, by the partial differences transformation or the Prais-Winsten (1954) estimator, we can see that the FD estimator is an approximate GLS estimator in the standard regression framework if the regressors are weakly stationary and ε_t is a highly persistent AR(1) process. However, when ε_t having nonparametric autocorrelation, we do not have a clear understanding about the relative performance between OLS and FD estimators in estimating β. Kuo and Tsay (2008) first fill the gap of the literature by suggesting a time-domain semiparametric Stein-like (SPSL) estimator advocated in Judge and Mittelhammer (2004) to increase the efficiency of the coefficient estimate relative to both OLS and FD estimators when Z_t and ε_t admitting nonparametric autocorrelation.

We contribute to the time series literature by proposing a new class of GMM estimators for the model in (1) by exploiting the moments from the long difference (LD) operator of Griliches and Hausman (1986) and those from the multiple difference (MD) operator of Tsay (2007). Although the number of moments employed in this paper is much larger than that considered in Kuo and Tsay (2008), the proposed estimator is easily implemented for standard statistic packages. Furthermore, the proposed GMM estimator combines the information contained in the moment conditions inherent in the OLS and the FD estimators, $\widehat{\beta}_{\mathrm{GMM}}$ is thus at least as efficient as both OLS and FD counterparts no matter whether the error term is an iid or a highly persistent stationary process. Put differently, the GMM estimator is designed to beat both OLS and FD estimators provided neither OLS nor FD methods attains Gauss-Markov bound. This feature is important and cannot be shared with the usual GMM estimators based on the moments composing of the lead or lagged values of regressors. The simulations conducted in this paper confirm the preceding predictions.

The remaining parts of this paper are arranged as follows: Section 2 presents the GMM estimator and the main results. In Section 3 the theoretical findings generated from this paper are verified through a Monte Carlo experiment. Section 4 provides a conclusion.

2 Main statistics

With a sample of size n and define $\overline{H}_{T_1:T_2}$ as the sample mean of the random variable H_t from $t = T_1$ to $t = T_2$, the usual OLS estimator for the model in (1) is to solve the following moment condition:

$$\widehat{\beta}_{n,\mathrm{OLS}} = \operatorname*{Arg\,min}_{\beta} \left[\sum_{t=1}^{n} (Z_t - \overline{Z}_{1:n})\varepsilon_t \right]^{\top} \left[\sum_{t=1}^{n} (Z_t - \overline{Z}_{1:n})\varepsilon_t \right]. \tag{2}$$

When ε_t is an iid process, the OLS estimator attains Gauss-Markov bound. Define $\triangle = 1 - L$, where L is the usual lag operator ($Lx_t = x_{t-1}$), the FD estimator for the model in

(1) is defined as:

$$\widehat{\beta}_{n,\mathrm{FD}} = \underset{\beta}{\mathrm{Arg\,min}} \left[\sum_{t=2}^{n} \left(\triangle Z_t - \overline{\triangle Z}_{2:n} \right) \triangle \varepsilon_t \right]^{\top} \left[\sum_{t=2}^{n} \left(\triangle Z_t - \overline{\triangle Z}_{2:n} \right) \triangle \varepsilon_t \right]. \tag{3}$$

When $\triangle\varepsilon_t$ is an iid process, the FD estimator achieves Gauss-Markov bound under suitable regularity conditions.

The above results reveal that whether the OLS or the FD estimator attains Gauss-Markov bound depends on the time series properties of the error term. In reality, we do not know whether the OLS or the FD approach attains Gauss-Markov bound, thus, we do not know whether we should difference or not-to-difference the data. This is the dilemma frequently encountered in dealing with the time series observations.

The key insight of this paper is to merge the information contained in the moment conditions in both OLS and FD estimators to suggest the following GMM estimator:

$$\widehat{\beta}_{\mathrm{GMM}} = \underset{\beta}{\mathrm{Arg\,min}}\, g(\beta)^{\top} \widehat{S}_n^{-1} g(\beta), \tag{6}$$

where

$$\underbrace{g(\beta)}_{2K\times 1} = \begin{bmatrix} n^{-1} \sum_{t=2}^{n} \left(Z_t - \overline{Z}_{2:n} \right) \varepsilon_t \\ n^{-1} \sum_{t=2}^{n} \left(\triangle Z_t - \overline{\triangle Z}_{2:n} \right) \triangle \varepsilon_t \end{bmatrix}, \tag{7}$$

and $\widehat{S}_n$ is the long-run variance estimator of $\sqrt{n}g(\beta)$. It is clear that both OLS and FD estimators belong to the special cases of the GMM estimator defined in (6). Since the efficiency of the GMM estimator basically improves with the number of moment conditions, $\widehat{\beta}_{\mathrm{GMM}}$ is at least as efficient as both OLS and FD counterparts even though the error term is an iid or a highly persistent stationary process. Accordingly, we view the use of level and differenced moments in (7) as a kind of insurance to help us attain Gauss-Markov bound as closely as possible even though ε_t can be an iid or a highly persistent stationary process. This also indicates that the GMM method based on the moments in (7) can defeat both OLS and FD counterparts if none of OLS and FD estimators attains Gauss-Markov bound.

There is another feature of the GMM estimator which cannot be shared with the usual GMM estimators based on the moments generated from the lead or lagged values of regressors. That is, the introduction of the GMM estimator relieves the burdens of choosing whether to difference or not to difference the data, because we employ both the level and differeced data. This paper thus resolves the dilemma concerning 'to difference or not to difference' in the time series literature.

Extending the above idea of combining the level and differenced moments of the model in (1), we propose a much more general class of GMM estimators based on the moments generated from the LD operator of Griliches and Hausman (1986) and those from the MD operator of Tsay (2007). Particularly, we can add the second-differenced moments from the model in (1) to the moments in (7), or even add all the M-th differenced moments suggested by Tsay (2007) to the moments in (7) as follows:

$$\underbrace{g(\beta, \triangle, M)}_{(M+1)K\times 1} = \begin{bmatrix} n^{-1}\sum\limits_{t=M+1}^{n}\left(Z_t - \overline{Z}_{M+1:n}\right)\varepsilon_t \\ n^{-1}\sum\limits_{t=M+1}^{n}\left(\triangle Z_t - \overline{\triangle Z}_{M+1:n}\right)\triangle\varepsilon_t \\ \vdots \\ n^{-1}\sum\limits_{t=M+1}^{n}\left(\triangle^M Z_t - \overline{\triangle^M Z}_{M+1:n}\right)\triangle^M\varepsilon_t \end{bmatrix}. \tag{8}$$

Accordingly, the GMM estimator based on the moments in (8) is basically more efficient than that based on the moments (7). The notation used in $g(\beta, \triangle, M)$ clarifies that the MD operator ($\triangle$) is employed to generate moments for the GMM estimation. It also reveals that the maximum order of differencing for the MD operators is M.

We can also generalize the moments in (7) in another direction, i.e., we add all the M-th LD moments pioneered in Griliches and Hausman (1986) and later used in Hahn et al. (2007) for panel data analysis to the moments in (7). These moments are presented as follows:

$$\underbrace{g(\beta, \triangledown, M)}_{(M+1)K\times 1} = \begin{bmatrix} n^{-1}\sum\limits_{t=M+1}^{n}\left(Z_t - \overline{Z}_{M+1:n}\right)\varepsilon_t \\ n^{-1}\sum\limits_{t=M+1}^{n}\left(\triangledown Z_t - \overline{\triangledown Z}_{M+1:n}\right)\triangledown\varepsilon_t \\ \vdots \\ n^{-1}\sum\limits_{t=M+1}^{n}\left(\triangledown^M Z_t - \overline{\triangledown^M Z}_{M+1:n}\right)\triangledown^M\varepsilon_t \end{bmatrix}, \tag{9}$$

where $\triangledown^M = 1 - L^M = (1-L)(1+L+L^2+\ldots+L^{M-1})$. Because $g(\beta, \triangle, M) = g(\beta, \triangledown, M)$ when $M = 1$, the moments in (9) reduces to be the ones in (7). Again, the GMM estimator based on the moments in (9) is basically more efficient than that based on the moments (7). Furthermore, the notation used in $g(\beta, \triangledown, M)$ clarifies that the LD operator ($\triangledown$) is employed to generate moments for the GMM estimation. It also reveals that the maximum order of differencing for the LD operators is M.

Following the spirit behind the moments in (8) and (9), we certainly can utilize all the $2M$ distinctive moments contained in (8) and (9) to improve the efficiency of the coefficient estimate relative to that of the GMM estimators based on the moments in (8) or (9), respectively. These moments are stacked in the following $(2MK \times 1)$ column vector:

$$\underbrace{g(\beta, \triangle, \triangledown, M)}_{2MK\times 1} = \begin{bmatrix} n^{-1} \sum_{t=M+1}^{n} \left(Z_t - \overline{Z}_{M+1:n} \right) \varepsilon_t \\ n^{-1} \sum_{t=M+1}^{n} \left(\triangle Z_t - \overline{\triangle Z}_{M+1:n} \right) \triangle \varepsilon_t \\ \vdots \\ n^{-1} \sum_{t=M+1}^{n} \left(\triangle^M Z_t - \overline{\triangle^M Z}_{M+1:n} \right) \triangle^M \varepsilon_t \\ n^{-1} \sum_{t=M+1}^{n} \left(\triangledown^2 Z_t - \overline{\triangledown^2 Z}_{M+1:n} \right) \triangledown^2 \varepsilon_t \\ \vdots \\ n^{-1} \sum_{t=M+1}^{n} \left(\triangledown^M Z_t - \overline{\triangledown^M Z}_{M+1:n} \right) \triangledown^M \varepsilon_t \end{bmatrix}. \tag{10}$$

The notation used in $g(\beta, \triangle, \triangledown, M)$ clearly clarifies that both LD operator and MD operator are used to generate moments in (10) for the GMM estimation.

We now discuss the asymptotic properties of the GMM estimator based on both MD and LD operators. Suppose that Z_t and U_t in (1) are weakly stationary, Tsay (2007) shows that the MD transformed regressors and the MD transformed errors can be represented with the following MA(∞) processes:

$$\triangle^M U_t = \sum_{i=0}^{\infty} \psi_i a_{t-i}, \qquad \triangle^M Z_{t,j} = \sum_{i=0}^{\infty} \varphi_{i,j} b_{t-i,j}, \qquad t = M+1, M+2, \ldots, \tag{11}$$

where both ψ_i and $\varphi_{i,j}$ in (11) are absolutely summable. Likewise, the LD transformed regressors and the LD transformed errors can be represented with the following MA(∞) processes:

$$\triangledown^M U_t = \sum_{i=0}^{\infty} \phi_i a_{t-i}, \qquad \triangledown^M Z_{t,j} = \sum_{i=0}^{\infty} \xi_{i,j} b_{t-i,j}, \qquad t = M+1, M+2, \ldots. \tag{12}$$

Because the LD operator contains a unit root when $M \geq 1$, we also observe $\sum_{i=0}^{\infty} |\phi_i| < \infty$, and $\sum_{i=0}^{\infty} |\xi_{i,j}| < \infty$.

The common feature shared with the MD and the LD transformed data is that they remain weakly stationary provided the original data are weakly stationary. Consequently, the theoretical results established in Hansen (1982) can be directly applied to the proposed GMM estimator. What we require is that the central limit theorem (CLT) holds for the moments considered in (10), i.e.,

$$\sqrt{n} g(\beta, \triangle, \triangledown, M) \Rightarrow N(0, S), \tag{13}$$

where $\Rightarrow$ denotes convergence in distribution, and S is the long-run variance matrix of $\sqrt{n} g(\beta, \triangle, \triangledown, M)$ which is assumed to be finite and positive-definite. We also need to define the following matrix:

$$\underbrace{D_n^\top}_{2MK \times K} = - \begin{bmatrix} n^{-1} \sum_{t=M+1}^{n} \left(Z_t - \overline{Z}_{M+1:n} \right) Z_t^\top \\ n^{-1} \sum_{t=M+1}^{n} \left(\triangle Z_t - \overline{\triangle Z}_{M+1:n} \right) \triangle Z_t^\top \\ \vdots \\ n^{-1} \sum_{t=M+1}^{n} \left(\triangle^M Z_t - \overline{\triangle^M Z}_{M+1:n} \right) \triangle^M Z_t^\top \\ n^{-1} \sum_{t=M+1}^{n} \left(\triangledown^2 Z_t - \overline{\triangledown^2 Z}_{M+1:n} \right) \triangledown^2 Z_t^\top \\ \vdots \\ n^{-1} \sum_{t=M+1}^{n} \left(\triangledown^M Z_t - \overline{\triangledown^M Z}_{M+1:n} \right) \triangledown^M Z_t^\top \end{bmatrix}, \tag{14}$$

which is important to describe the asymptotic properties of the GMM estimator. Theorem 1 summarizes the results concerning the asymptotic properties of $\widehat{\beta}_{\text{GMM}}$.

THEOREM 1. *Under M being a finite non-negative integer, Z_t and U_t in (1) being weakly dependent processes, the CLT in (13) holds, the spectral density function of $\sqrt{n} g(\beta, \triangle, \triangledown, M)$ at frequency zero being finite and positive-definite, $D_n^\top \xrightarrow{p} D^\top$, and the columns of $D^\top$ being linearly independent, and $\widehat{S}_n$ being a sequence of positive-definite matrices such that $\widehat{S}_n \xrightarrow{p} S$, then as $n \to \infty$, $\sqrt{n}(\widehat{\beta}_{\text{GMM}} - \beta) \Rightarrow N(0, V)$, where $V^{-1} = D S^{-1} D^\top$.*

The results in Theorem 1 are directly derived from Proposition 14.1 of Hamilton (1994). Theorem 1 reveals that $\widehat{\beta}_{\mathrm{GMM}}$ converges at the rate of $\sqrt{n}$ as usually found in the short memory time series model. Moreover, $\widehat{\beta}_{\mathrm{GMM}}$ is consistent and asymptotically normally distributed, thus, the usual t-ratio statistic associated with the GMM estimator is tested with the critical values from the standard normal distribution. Note that the preceding results do not depend on the number of differencing M used in generating the moments in (10) as long as the regularity conditions in Theorem 1 are satisfied. In addition, the potential candidates for calculating $\widehat{S}_n$ are numerous, including the methods introduced in Newey and West (1987), Andrews (1991), and Robinson (1998).

We now check whether the GMM estimator achieves Gauss-Markov bound when ε_t is an iid process. Theorem 1 shows that the asymptotic variance of the GMM estimator is $V = \left(DS^{-1}D^{\top}\right)^{-1}$. When Z_t is a univariate iid process with variance σ_Z^2 and ε_t is also a iid process with variance σ_ε^2, then $D = (\sigma_Z^2, 2\sigma_Z^2)$ when $M = 1$ and Z_t is independent of ε_t. We also observe

$$S = \begin{bmatrix} \sigma_Z^2\sigma_\varepsilon^2 & 2\sigma_Z^2\sigma_\varepsilon^2 \\ 2\sigma_Z^2\sigma_\varepsilon^2 & 6\sigma_Z^2\sigma_\varepsilon^2 \end{bmatrix},$$

thus, the associated variance of the GMM estimator is $V = \sigma_\varepsilon^2/\sigma_Z^2$ which is identical to the normalized variance of the OLS estimator under this circumstance. When $M = 2$, we also prove $V = \sigma_\varepsilon^2/\sigma_Z^2$ if the preceding conditions are fulfilled. Consequently, the asymptotic behavior of the GMM estimator corresponds exactly to our design that the GMM estimator achieves Gauss-Markov bound when ε_t is an iid process. Similarly, under suitable regularity conditions, we can apply the above arguments to show that the GMM estimator is as efficient as the FD counterpart if the error term is a highly persistent stationary AR(1) process.

The methodology developed in this paper also can be applied to the long memory regression model considered in Tsay (2007). As compared to the MD estimator of Tsay (2007) who employs only one specific but flexible value of M for estimating a class of long memory regression models, the GMM estimator developed in this paper efficiently combines all the $M + 1$ multiple-differenced moments inherent in (8) to deliver a more efficient coefficient estimate. In this sense, $\widehat{\beta}_{\text{GMM}}$ estimator based on the moments in (8) resolves the seemly arbitrary choice about the order of the MD operator in Tsay (2007), because we now use all the feasible choice of the MD operator in (8) to increase the efficiency of coefficient estimate. Another advantage of $\widehat{\beta}_{\text{GMM}}$ is that it employs the $M+1$ long-differenced moments in (9) which has never been considered in the time series literature.

Before moving on to the numerical investigation of the GMM estimator, we emphasize here that the moment based on the level data should be used cautiously under the long memory scenario, because the presence of stationary long memory regressors and errors might induce the spurious regression as discussed in Tsay and Chung (2000).

3 Monte Carlo experiment

This section focuses on the finite sample performance of the proposed GMM estimator $\widehat{\beta}_{\text{GMM}}$ for the regression model with stationary regressor and errors. Without loss of generality, only one regressor is considered in the experiment, i.e., we assume $K = 1$ throughout this section. Moreover, $\gamma = 0$ is assumed in this section.

We use the moment conditions in (7) to implement the GMM estimator. The rationale is that the number of moment conditions in (7) corresponds to that in (10) when $M = 1$, which is the minimum positive number of differencing employed in (10). If we can find the GMM estimator based on the moment conditions in (7) outperforms both OLS and FD counterparts, then we expect that the GMM estimator based on the moments in (10) with $M > 1$ will deliver a even better asymptotic performance than do the OLS and the FD approaches.

We focus on the cases where ε_t and Z_t are both generated as AR(1) processes:

$$(1 - \phi_\varepsilon L)\varepsilon_t = v_t, \qquad (1 - \phi_Z L)Z_t = w_t, \tag{15}$$

such that both v_t and w_t are zero-mean normally iid white noise processes with:

$$E(v_t^2) = \sigma_v^2, \qquad E(w_t^2) = \sigma_w^2, \quad E(v_t w_t) = 0. \tag{16}$$

The value of σ_v^2 and σ_w^2 in (16) are chosen to ensure the variance of ε_t and Z_t are both equal to 1. The values of ϕ_ε and those of ϕ_Z ranges from 0.1 to 0.9.

In the context of stochastic regressor framework, we generate 1,000 replications of Z_t and ε_t with the following model:

$$C_t^l = \beta_1 Z_t^l + \varepsilon_t^l, \quad t = 1, 2, \ldots, n, \quad l = 1, 2, \ldots, 1000, \tag{17}$$

where l denotes the l-th replication of the data. β_1 can be 1 or 0.9 for investigating the empirical powers of the GMM estimator given that the null hypothesis for β_1 is always tested as:

$$H_0 : \beta_1 = 1. \tag{18}$$

We adopt a two-step procedure to conduct the GMM estimation. The first step of the estimation procedure is to run the GMM estimation with $\widehat{S}_n^{-1}$ being replaced with a conformable identity matrix. In the second step we adopt the long-run variance estimator of Robinson (1998) to calculate $\widehat{S}_n^{-1}$, because it does not involve the difficult choices of kernel function, bandwidth parameter, or lag length of AR model typically used in the literature. Theoretically, it is legitimate to apply Robinson's (1998) approach in this context, because the conditions imposed in (15) and (16) satisfy Assumption A of Robinson (1998). Moreover, in the second step of GMM estimation the iteration of the GMM procedure will not stop until it touches the maximum iteration limit, 100, or when the change of parameter estimates is less than 10^{-7}. Furthermore, the value of $\widehat{S}_n^{-1}$ is recursively updated during the iterative process of the second step estimation procedure.

Table 1 contains the root of mean-squared-error (RMSE) of the OLS, FD, and GMM estimators in estimating the regression coefficient β_1. The results show that, for a given value of ϕ_Z, the performance of the OLS estimator deteriorates with the increasing value of ϕ_ε. This phenomenon is expected, because the OLS estimator achieves Gauss-Markov bound when the error term is a Gaussian white noise. On the other hand, the efficiency of the FD estimator improves with the increasing value of ϕ_ε. This corresponds to the findings in Chipman (1979) and those of Krämer (1982) that the FD estimator is an approximation to the GLS estimator when estimating the coefficient of the linear trend with a highly persistent AR(1) error term.

Table 1 also reveals that the difference between the RMSE of the OLS estimator and that of the FD one can be sizable under the configurations considered in Table 1. In particular, the RMSE of the OLS estimator is 0.129, while that of the FD estimator is 0.311 when $\phi_Z = 0.9$ and $\phi_\varepsilon = 0.1$. This indicates the potential power gain from using the OLS estimator relative to the FD counterpart is tremendous. On the other hand, the RMSE of the OLS estimator is 0.098, while that of the FD estimator is only 0.034 when $\phi_Z = 0.1$ and $\phi_\varepsilon = 0.9$. This implies the FD estimator should be used under this circumstance. The preceding results clearly point to the dilemma concerning "to difference or not-to-difference" in the time series literature, because the researchers might have difficulty to decide whether to use the OLS or the FD estimator in empirical applications. This dilemma motivates us to propose the GMM estimator.

As argued previously, the GMM estimator merges the information contained in both OLS and FD estimators, $\widehat{\beta}_{\text{GMM}}$ thus is at least as efficient as both OLS and FD methods although the error term is an iid or a highly persistent stationary process. This prediction is confirmed by Table 1, because we find the RMSE of the GMM estimator is almost equivalent to that of the FD estimator when ϕ_ε is close to 0.9, i.e., when the error term is a highly persistent stationary process. This observation is reasonable, because it is not easy to beat the FD estimator when it almost achieves Gauss-Markov bound. On the other hand, the RMSE of the GMM estimator is very close to that of the OLS estimator if ϕ_ε is close to 0.1, i.e., when the error term is nearly to be an iid process. Again, this phenomenon is expected, because the OLS estimator almost attains Gauss-Markov bound under this circumstance. Interestingly, we find the GMM estimator is more efficient than both OLS and FD methods when ϕ_ε is in the middle range of the parameter values studied in Table 1. This finding also confirms that the GMM estimator can beat both OLS and FD methods when neither of OLS and FD methods attains Gauss-Markov bound.

For ease of comparison, we define RMSE_ξ as the RMSE of the estimator ξ in estimating β of the model in (1), and compare the finite sample relative efficiency of the OLS estimator to its GMM counterpart as:

$$\text{Relative efficiency of GMM to OLS estimator in estimating } \beta = \frac{\text{RMSE}_{\text{OLS}}}{\text{RMSE}_{\widehat{\beta}_{\text{GMM}}}}. \tag{19}$$

The GMM estimator is more efficient than the OLS counterpart in estimating β if we find the ratio in (19) is greater than 1. Similarly, we compare the finite sample relative efficiency of the FD estimator to its GMM counterpart as:

$$\text{Relative efficiency of GMM to FD estimator in estimating } \beta = \frac{\text{RMSE}_{\text{FD}}}{\text{RMSE}_{\widehat{\beta}_{\text{GMM}}}}. \tag{20}$$

It follows that the GMM estimator is more efficient than the FD counterpart in estimating β if we find the ratio in (20) is greater than 1.

For clarity of exposition, Table 2 shows the relative performance of the GMM estimator as compared to that of the OLS estimator and that of the FD one. When the values in (19) and (20) are both greater than or equal to 1, we plot the corresponding entry in Table 2 with a value of 1, indicating that both OLS and FD estimators cannot beat the GMM approach. Otherwise, we plot a value of 0. Table 2 clearly reveals that the finite sample performance of the proposed GMM estimator is very satisfactory in that it is at least as efficient as both OLS and FD approaches in most cases, even though the sample size is 100. Moreover, as expected from the asymptotic properties of the GMM estimator, we find the relative performance of the GMM approach as compared to that of the OLS estimator and that of the FD estimator increases with the increasing sample size. For example, when $n = 300$, there are only 11 out of 81 cases where the GMM estimator is defeated by either OLS or FD method. This phenomenon thus confirms the GMM literature that more moments basically increase the efficiency of the GMM estimator.

The sampling properties of the t-ratio statistic generated from the GMM estimator are important, because inference plays an inevitable role in empirical applications. Table 3 displays the empirical size of the GMM t-ratio statistic in a two-tailed test at the 5% level of significance and shows that the size control of the t statistic for testing the value of β_1 is very promising even though the sample size is moderately small. This also illustrates the power of Robinson's (1998) long-run variance estimator in controlling the effects of nuisance parameters on the inference performance of the GMM estimator. It also indicates that the empirical power analysis based on Robinson's (1998) approach is reliable.

We now consider the power performance of the GMM estimator. The strategy is to investigate the rejection percentages of t test when the simulated value of β_1 is 0.9, while the null hypothesis tested is $\beta_1 = 1$. By the regression direction analysis of Davidson and MacKinnon (1985), the GMM estimator will have a better local power than both OLS and FD counterparts do provided that the GMM estimator is more efficient than both OLS and FD methods. For expositional purposes, following the design in Table 2, we plot the corresponding entry in Table 4 with a value of 1 if the power of the GMM estimator is both greater than or equal to that of the OLS method and that of the FD estimator. Otherwise, we plot a value of 0.

Table 4 shows that the relative performance of the GMM estimator as compared to those of OLS and FD methods indeed improves with the sample size. Even under the moderately small sample size as $n = 100$, we find 70 out of 81 cases where both OLS and FD methods cannot beat the GMM estimator in Table 4. For the 81 configurations considered in Table 4, we only find 3 out of 81 cases where the GMM estimator is defeated by either OLS or FD estimator when $n = 300$. These 3 cases actually occur when ϕ_ε is 0.1 or 0.8 which correspond to the extreme situations where either OLS or FD estimator is close to achieve Gauss-Markov bound. On the contrary, the GMM estimator outperforms both OLS and FD competitors if ϕ_ε lies in the middle range of the parameters considered in Table 4. This pattern is what we find in Table 1 concerning the RMSE of the OLS, FD, and GMM estimators. By the simulation results not reported in the paper, in terms of power performance, we find 70 of 81 configurations where the GMM estimator outperforms both OLS and FD estimators when $n = 300$. All these findings strongly support that the GMM estimator based on the MD and the LD operators is a powerful tool to increase the efficiency of the time series regression coefficient estimates.

4 Conclusion

A general class of easily implemented GMM estimators built on the LD operator of Griliches and Hausman (1986) and the MD operator of Tsay (2007) is shown to be useful for dealing with a general class of time series regression model where the error term and regressors are allowed to have nonparametric autocorrelation. Since the proposed GMM estimator merges the information contained in the moment conditions inherent in both OLS and FD estimators, theoretically, $\widehat{\beta}_{\text{GMM}}$ is at least as efficient as both OLS and FD counterparts. This also implies that the GMM estimator is likely to greatly outperform both OLS and FD estimators when the order of differecing M adopted in (10) is larger than 1.

The GMM estimator is proven to be consistent and asymptotically normally distributed under suitable regularity conditions. The Monte Carlo simulation reveals that the size control of the t test associated with the GMM estimator is very satisfactory under various AR(1) regressors and AR(1) errors considered in this paper even though the sample size is moderately small as 100. Moreover, the GMM estimator is found to be more efficient than both OLS and FD estimators when the persistency parameter of AR(1) error term, ϕ_ε, is in the middle range of the parameters considered in Table 1. This confirms that the GMM estimators is more efficient than both OLS and FD approaches if neither OLS nor FD estimator attains Gauss-Markov bound.

We do not address the issues surrounding the choice of an optimal number of M or the moments from the LD and the MD operators. The seminal works in Andrews (1999) and Donald and Newey (2001) are the benchmarks to develop a consistent moment selection procedure for our GMM estimator. This task is important but out of the scope of this paper. Furthermore, the methodology developed in this paper can be used to combine the within estimator and the FD estimator popularly adopted in the panel data analysis to construct a more efficient coefficient estimates. That will be left for future studies.

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Table 1. **RMSE from Estimating the Regression Coefficient β_1: $n = 100$**

		ϕ_Z								
ϕ_ε	Estimator	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	OLS	0.102	0.108	0.108	0.105	0.104	0.113	0.111	0.112	0.129
	FD	0.118	0.129	0.141	0.142	0.154	0.165	0.194	0.216	0.311
	GMM	0.103	0.110	0.112	0.107	0.107	0.114	0.112	0.114	0.132
0.2	OLS	0.102	0.107	0.111	0.108	0.116	0.115	0.123	0.133	0.143
	FD	0.115	0.115	0.121	0.132	0.140	0.159	0.181	0.209	0.283
	GMM	0.101	0.103	0.107	0.107	0.114	0.112	0.123	0.131	0.145
0.3	OLS	0.106	0.106	0.112	0.114	0.120	0.119	0.127	0.142	0.148
	FD	0.106	0.108	0.116	0.126	0.134	0.148	0.163	0.206	0.270
	GMM	0.100	0.099	0.106	0.110	0.113	0.116	0.121	0.139	0.147
0.4	OLS	0.105	0.107	0.111	0.119	0.125	0.127	0.136	0.145	0.163
	FD	0.094	0.095	0.102	0.112	0.118	0.137	0.152	0.177	0.262
	GMM	0.090	0.091	0.096	0.105	0.108	0.114	0.123	0.132	0.158
0.5	OLS	0.108	0.113	0.118	0.124	0.128	0.135	0.147	0.158	0.188
	FD	0.084	0.092	0.095	0.101	0.113	0.122	0.135	0.167	0.236
	GMM	0.084	0.090	0.093	0.097	0.106	0.111	0.120	0.138	0.168
0.6	OLS	0.110	0.113	0.115	0.125	0.136	0.137	0.160	0.175	0.207
	FD	0.074	0.080	0.085	0.090	0.099	0.103	0.125	0.153	0.206
	GMM	0.074	0.078	0.084	0.087	0.097	0.097	0.120	0.138	0.176
0.7	OLS	0.107	0.114	0.124	0.130	0.148	0.154	0.171	0.193	0.223
	FD	0.062	0.066	0.072	0.073	0.084	0.089	0.107	0.130	0.182
	GMM	0.062	0.066	0.073	0.073	0.084	0.088	0.105	0.125	0.164
0.8	OLS	0.101	0.118	0.118	0.135	0.148	0.168	0.181	0.209	0.260
	FD	0.051	0.052	0.057	0.060	0.066	0.076	0.084	0.101	0.141
	GMM	0.051	0.052	0.057	0.060	0.067	0.075	0.083	0.101	0.139
0.9	OLS	0.098	0.115	0.119	0.137	0.151	0.166	0.186	0.227	0.298
	FD	0.034	0.037	0.039	0.042	0.048	0.053	0.059	0.072	0.103
	GMM	0.034	0.037	0.039	0.042	0.049	0.053	0.060	0.073	0.105

Notes: All the results are based on 1,000 replications of the simulated data defined in (15), (16), (17), and $\beta_1 = 1$.

Table 2. Relative Efficiency of the GMM Estimator to both OLS and FD Counterparts

					ϕ_Z				
ϕ_ε	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
					$n = 100$				
0.1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000
0.3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.6	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.7	0.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.8	0.0000	0.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000
0.9	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
					$n = 200$				
0.1	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
0.2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.7	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.8	0.0000	1.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.9	0.0000	1.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000
					$n = 300$				
0.1	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000	1.0000	0.0000
0.2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000
0.3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.8	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.9	0.0000	1.0000	0.0000	0.0000	1.0000	0.0000	0.0000	1.0000	0.0000

Notes: The value of each entry equals 1 when the RMSE of the GMM estimator is not less than that of the OLS mehtod and that of the FD one. Otherwise, it equals 0.

Table 3. Rejection Percentages of the GMM Estimator under the Null

		ϕ_Z								
ϕ_ε	n	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	100	6.40	7.60	8.20	6.70	5.70	7.10	8.00	6.40	9.90
	200	4.80	5.40	6.50	6.20	4.40	5.30	6.20	7.20	8.00
	300	5.30	6.60	5.80	6.80	5.80	4.70	7.00	6.20	9.00
0.2	100	7.20	6.00	7.30	7.00	7.70	6.70	7.00	9.70	10.40
	200	4.50	5.40	4.90	6.00	5.40	5.90	7.20	6.50	6.90
	300	5.70	6.80	6.10	5.80	5.60	6.80	6.50	6.60	7.50
0.3	100	7.60	6.70	7.40	7.10	6.70	7.60	8.00	9.50	10.50
	200	5.00	5.30	5.30	6.20	6.20	5.50	4.50	7.00	6.90
	300	6.00	6.80	5.50	5.40	5.20	5.10	4.70	6.70	6.60
0.4	100	5.50	5.20	5.70	6.90	6.60	7.20	6.10	8.20	9.40
	200	6.40	5.60	5.40	6.70	6.80	5.20	6.30	8.30	8.00
	300	7.00	4.70	5.40	5.30	6.10	6.30	6.00	6.50	6.70
0.5	100	6.70	7.50	6.90	6.40	7.20	6.50	7.10	8.10	10.20
	200	7.60	5.00	5.50	6.80	6.30	5.30	7.00	6.80	8.50
	300	5.70	6.10	5.00	7.00	5.10	5.70	6.50	7.10	7.10
0.6	100	6.30	6.10	7.40	5.60	6.70	5.50	8.20	8.20	9.90
	200	6.00	6.10	5.30	6.50	5.70	4.90	6.20	5.70	8.90
	300	5.20	4.70	5.60	6.40	5.00	5.40	5.90	5.70	8.40
0.7	100	6.50	6.30	7.30	4.70	6.10	4.90	7.50	7.10	9.90
	200	5.00	6.00	5.40	6.70	5.80	6.30	5.00	6.70	8.70
	300	4.20	4.70	6.00	5.50	6.20	4.80	5.00	6.10	6.60
0.8	100	6.20	5.60	6.20	5.20	5.80	6.10	5.30	5.30	7.00
	200	5.10	5.50	6.90	7.00	5.70	5.30	6.10	5.40	5.30
	300	4.80	5.60	4.70	4.70	5.60	5.70	5.00	5.50	5.80
0.9	100	5.80	5.10	6.20	5.40	7.60	5.70	5.90	5.70	6.90
	200	5.90	6.00	5.20	6.20	5.30	4.30	4.60	6.90	6.60
	300	4.90	5.30	5.40	4.50	5.90	3.70	5.30	5.50	4.70

Notes: All the results are based on 1,000 replications of the simulated data defined in (15), (16), (17), and $\beta_1 = 1$.

Table 4. Empirical Power Performance of the GMM Estimator Relative to both OLS and FD Counterparts

	ϕ_Z								
ϕ_ε	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
					$n = 100$				
0.1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	1.0000
0.3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000
0.4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	1.0000
0.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000
0.6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000
0.7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000
0.8	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000	1.0000	0.0000
0.9	1.0000	1.0000	0.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000
					$n = 200$				
0.1	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000
0.2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000
0.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.8	0.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.9	0.0000	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000	1.0000
					$n = 300$				
0.1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000
0.2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.8	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000
0.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Notes: The simulated value of β_1 in (17) is 0.9, while the null hypothesis tested is $\beta_1 = 1$. The value of each entry equals 1 when the power of the GMM estimator cannot be defeated by either OLS or FD estimator. Otherwise, it equals 0.

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