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Forecasting Commodity Prices with Mixed-Frequency Data: An OLS-Based Generalized ADL Approach*

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Abstract

This paper presents a generalized autoregressive distributed lag (GADL) model for conducting regression estimations that involve mixed-frequency data. As an example, we show that daily asset market information - currency and equity market movements - can produce forecasts of quarterly commodity price changes that are superior to those in the previous research. Following the traditional ADL literature, our estimation strategy relies on a Vandermonde matrix to parameterize the weighting functions for higher-frequency observations. Accordingly, inferences can be obtained using ordinary least squares principles without Kalman filtering, non-linear optimizations, or additional restrictions on the parameters. Our findings provide an easy-to-use method for conducting mixed data-sampling analysis as well as for forecasting world commodity price movements.

Key words: Mixed frequency data; autoregressive distributed lag, commodity prices, forecasting

JEL classifications: C22, C53, F31, F47, Q02

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1 Introduction

This paper proposes a generalized autoregressive distributed lag (GADL) model for estimating and testing regressions that involve data of different frequencies (MIxed DAta Sampling or "MIDAS" regressions).[1] Using this methodology, we revisit the conclusion in Chen, Rossi, and Rogoff (2010a, hereinafter CRR) where they show that lagged quarterly exchange rate returns from key commodity-exporting countries can predict subsequent quarterly movements of world commodity prices. While their result is generally confirmed in Groen and Pesenti (2011) for example, the predictive power of quarterly exchange rate returns is shown to have declined over the mid-2008 market crisis.[2] One previously unexplored question is whether higher-frequency information from the markets can be incorporated to improve quarterly forecasts, especially over crisis periods where each day unveils significant new information. Using the GADL model, we show that the answer is affirmative.

When one forecasts using predictors of matched frequencies, such as predicting quarterly returns with lagged quarterly variables, one implicitly assumes that higher frequency fluctuations within each quarter receive equal weights in delivering the forecast. The equal-weight assumption may be innocuous during tranquil periods, but, intuitively, one can easily imagine that in general, more recent data reflect a larger information set and thus should be

[1] After we circulated the first draft of this manuscript, we were notified that the Almon (1965) lag weighting function proposed in our GADL model is identical to eq.(2.17) in the "MATLAB Toolbox for MIDAS" by Sinko, Sockin, and Ghysels (2011). As will be made clear, GADL is a one-step closed-form procedure for estimating aggregate impact parameters in the mixed frequency context; it does not require any restrictions on the weighting coefficients for higher-frequency data, nor does it require re-scaling of the weighting parameters (as stated on p.7 of Sinko et al. 2011).

[2] See CRR Appendix III Sec. D. In addition, Groen and Pesenti (2010) use data up to 2009Q2 and find that the forecast superiority of exchange-rate based models vary over either the random walk or autoregressive benchmarks depending on the forecast horizons.

more useful in predicting the future. Our paper explores this possibility. We adopt an easy-to-implement method based on ordinary least squares (OLS) that allows higher frequency information in the predictors - certain daily exchange rates and equity prices in this case - to have differential impact on subsequent lower frequency (here quarterly) commodity price movements. Our results show that while the forecast performance of the same-frequency approach declines after the 2008 crisis, the mixed-frequency GADL model continues to deliver superior forecasts over the standard benchmarks.

Estimation using mixed-data sampling is recently popularized by the influential MIDAS literature, pioneered by Ghysels, Santa-Clara, and Valkanov (2004, 2006). This rapidly expanding literature typically assumes a Beta distribution or exponential Almon lag polynomial to model the weighting structure for higher frequency data; the estimation is then carried out using non-linear least squares (NLS).[3] As discussed extensively in Bai, Ghysels, and Wright (2010), under certain conditions, NLS-MIDAS regressions can be viewed as a reduced-form alternative to the Kalman filter state space approach for mixed frequency data estimations.[4] Our GADL framework is motivated by the vast body of NLS-MIDAS research, yet our approach follows more closely the classical ADL literature of Almon (1965). We approximate the distributed lag coefficients on the higher-frequency data with simple low-order

[3] As noted in footnote 1, Sinko et al. (2011) mentions Almon (1965) lag polynomials as an option, which would allow the MIDAS models to be estimated via OLS. However, the literature so far has not discussed the relative estimation advantages of the OLS-based approach over the NLS-MIDAS models; rather, most papers adopt the NLS approach directly in various applications. It is thus one of the objectives of this paper to highlight this comparison.

[4] The state space models typically involve a system of equations, and treat the lower frequency data as having missing values. NLS-MIDAS regressions on the other hand rely on a single equation. As a consequence, NLS-MIDAS regressions may be less efficient but they are also less prone to specification errors. In cases where the MIDAS regression is only an approximation, Bai et al. (2010) show that the approximation errors tend to be small.

polynomials (hence the name "Generalized" ADL), allowing the mixed-frequency regressions to be estimated by OLS. One of the goals of this paper is to provide a comparison between the OLS-based GADL and NLS-based MIDAS approaches in the context of mixed-frequency data regressions.

Merging ADL into the MIDAS literature, GADL offers two main advantages over previous approaches. First, standard NLS-MIDAS typically imposes a positive restriction on the weights for the higher frequency data and requires that they sum up to one.[5] The weights are then commonly parameterized as an order-2 exponential polynomial.[6] GADL, on the other hand, parameterizes the coefficients on higher-frequency data with Almon's (1965) polynomial distributed lags and does not impose *any* restrictions on them. Since the positive weights imposed in NLS-MIDAS may not always be appropriate, we view the Vandermonde matrix in our GADL setup as an efficient and more robust instrument for extracting information content from higher frequency data. This generality can be especially valuable in empirical studies where theory does not offer much guidance on the shape of weights.[7]

A second major advantage of GADL over NLS-MIDAS is its simplicity - both in establishing identification and in computation. Conceptually, GADL constitutes a one-step procedure for obtaining the "aggregate impact" slope parameters, which come out straight

[5] Under the OLS option mentioned in Sinko et al. (2011), the positive restriction on the weights is relaxed but they then require the sum of the weights to be non-zero: "Once the weights are estimated via OLS, one can always rescale them to obtain a slope coeffcient (assuming the weights do not sum up to zero)"(p.7). GADL does not need this assumption.

[6] In Section 3, we present the popular two-parameter exponential polynomial setup; see also Ghysels, Sinko, and Valkanov (2007) for a detailed discussion on the flexibility of this functional form. Alternative weights are discussed in the literature as well, such as in Sinko et al. (2011).

[7] While imposing restrictions that are wrong can sometimes deliver better out-of-sample forecast performance via increased efficiency, in general, there is no a priori reason to favor such restrictions.

from the OLS estimations, as in the classical ADL models. The MIDAS literature, on the other hand, builds upon a setup that delineates between aggregate impact parameters from individual weights on higher frequency data. To achieve identification, it thus needs additional assumptions, such as that the weights sum up to one. The simplicity of the GADL framework stems from the fact that it does not require any of these assumptions; one simply sums up the estimated OLS coefficients, transformed using the Vandermonde matrix, to obtain the "aggregate impact" slope parameters (see Sec.2 for details). Computationally, GADL offers significant advantage over NLS-MIDAS, and also over the state space approach. By approximating the weights with a simple polynomial, estimations and inferences under GADL can be carried out using OLS. In higher dimensional estimations involving multiple sets of high-frequency data, OLS clearly dominates approaches that entail non-linear optimizations over a large set of parameters. An OLS-framework such as GADL can also incorporate higher order polynomials to allow for more flexibility without adding computational complexity.[8] We thus view GADL as a simple and robust complementary methodology to the existing NLS-MIDAS and state space Kalman filter procedures for conducting mixed-frequency estimations.

The next section describes GADL and its asymptotic properties. Section 3 presents Monte Carlo simulation results for GADL and shows that it delivers excellent and superior results even when the true data-generating process (DGP) is NLS-MIDAS. In Section 4, we apply the GADL method to forecast quarterly aggregate world commodity prices using daily

[8]We recognize that the practical gain of this flexibility isn't always relevant under small data samples and limited degrees of freedom, but in principle, higher order polynomials can improve estimation efficiency. While the non-linear exponential framework can also include higher orders, the estimation would be considerably more cumbersome.

exchange rates (as in CRR 2010a) and daily stock market returns (as in CRR 2010b) from the commodity currency economies. We first confirm the CRR (2010a) finding that the same-frequency model using quarterly data suffers a deterioration of its forecasting power after the structural break of 2008Q2. We then show that information from the daily movements of exchange rates and equity prices delivers better forecasts even through the crisis period.

2 Mixed Data Sampling (MIDAS) Regressions

Mixed frequency sampling models aim to extract information content from high frequency indicators to help forecast target variables observed at lower frequency. The chapter in Oxford Handbook on Economic Forecasting by Andreou et al. (2010a) provides a good survey on how these models have been used extensively to forecast various macroeconomic indicators as well as financial series. As mentioned earlier, two approaches have been popular in the literature. The state space approach is adopted in Harvey and Pierse (1984), Bernanke, Gertler, and Watson (1997), Mariano and Murasawa (2003), Proietti and Moauro (2006), Aruoba, Diebold, and Scotti (2009), for example. These models have a measurement equation that links observed series to a latent state process, and a state equation that describes the dynamics of the state variables. By treating low-frequency series as having missing observations, the system can be estimated with the Kalman filter. A second approach is represented by the recent literature on NLS-MIDAS proposed by Ghysels, Santa-Clara and Valkanov (2002, 2006). NLS-MIDAS differ from mixed frequency state space models as they typically use a smaller set of predicting indicators. Using exponential polynomial lags or a Beta function, NLS-MIDAS regressions combine high frequency indicators with the low frequency target variable. We refer readers to Bai, Ghysels, and Wright (2010) for a comparison between the two approaches, and note that one can view NLS-MIDAS regressions as a reduced-form approximation for the larger Kalman filter state space systems. We present below the OLS-based GADL model, which can be viewed a simpler alternative to NLS-MIDAS. (As noted in the earlier footnotes, Sinko et al. (2011) also mentions conducting MIDAS regressions using Almon weights and OLS. There are nevertheless some conceptual differences between GADL and their "OLS-MIDAS", as we will make clear in Section 2.3 below. In addition, we also provide a comparison between the OLS- and the NLS-based approaches in Section 3, which to our knowledge has not been done previously.)

2.1 Identifying the Aggregate Impact Parameters under MIDAS

A key set of variables of interest in the MIDAS regression framework is the "aggregate impact" parameters that measure the total contribution from each set of high frequency data; here we define what they are. For ease of exposition, we first consider a mixed-frequency model with only one predictor x_1. We follow the notation in Ghysels et al. (2007) and consider the following h-period ahead predictive regression:

$$y_t = \beta_0 + \beta_1 W(L^{1/m}, \theta) x_{1,t-h}^{(m)} + \varepsilon_t, \text{ where} \tag{1}$$

$$W(L^{1/m}, \theta) = \sum_{k=1}^{K} b(k;\theta) L^{(k-1)/m}, \qquad L^{s/m} x_{1,t}^{(m)} = x_{1,t-s/m}^{(m)} \tag{2}$$

Here t denotes the basic time unit for the lower frequency data (from 1 to T), m and $x^{(m)}$ indicate higher sampling frequency and observations, which we index from 1 to K (where K is finite).[9] $L^{1/m}$ is the lag operator in frequency-m space, and $b(k;\theta)$ is the weight on each of the K lagged higher frequency predictors. ε_t is a white noise process. (All of the parameters of the MIDAS model depend on the predictive horizon h, even though we suppress it in the notation.)

In line with the notation above, a traditional ADL model with matched-frequency data can be expressed as follows:

$$y_t = \beta_0 + W(L^1, \theta)x_{1,t-h} + \varepsilon_t, \tag{3}$$

We note that when the explanatory variable and the dependent variable are sampled at the same frequency, the parameter β_1 in eq.(1) is not needed. If one were to express eq.(3) as a MIDAS-like representation with a β_1, one would then need to impose additional restrictions to identify it:

$$y_t = \beta_0 + \beta_1 W(L^1, \theta)x_{1,t-h} + \varepsilon_t. \tag{4}$$

As done in Ghysels et al. (2002) and the subsequent NLS-MIDAS literature, one intuitive way to identify β_1 is to restrict the sum of weights, $b(k;\theta)$, in the lag polynomial $W(.)$ to be 1. Under this restriction, parameter β_1 can then be interpreted as a measure for the "aggregate impact" of current and lagged x_1 on y. As we demonstrate below, this

[9] This paper does not address the optimal selection of K, though in practice one may be able to adopt testing procedures similar to Ng and Perron's (1995) general-to-specific method to pick K.

identification condition is actually not needed under the GADL framework. However, to keep the comparisons clear, we first present GADL using the same MIDAS notations in the next section. In Section 2.3, we show that GADL offers a more general framework for specifying and estimating mixed-frequency data regressions.

2.2 Generalized ADL (GADL) Model using MIDAS Notations

Generalizing eq.(1), we express a regression with q sets of mixed-frequency predictors as follows:

$$\begin{aligned} y_t &= \beta_0 + \beta_1 W_1(L^{1/m}, \theta_1) x^m_{1,t-h} + \beta_2 W_2(L^{1/m}, \theta_2) x^m_{2,t-h} + \\ &\quad \cdots + \beta_q W_q(L^{1/m}, \theta_q) x^m_{q,t-h} + \varepsilon_t \quad \text{where} \qquad (5) \\ W_i(L^{1/m}, \theta_i) &= \sum_{k=1}^{K} b_i(k; \theta_i) L^{(k-1)/m}, \qquad L^{s/m} x^{(m)}_{i,t} = x^{(m)}_{i,t-s/m}, \forall i = 1..q \qquad (6) \end{aligned}$$

where parameters β_1, β_2,. . . , β_q measure the aggregate impact of predictors $x_{1,t-h}$, $x_{2,t-h}$,. . . , $x_{q,t-h}$ on y_t, respectively, provided that the sum of the weighting polynomial in $W_1(L^{1/m}, \theta_1)$, $W_2(L^{1/m}, \theta_2)$, . . . , $W_q(L^{1/m}, \theta_q)$, are all normalized to 1.

Following the rationale in the traditional ADL literature, GADL characterizes the weighting coefficients $b_i(k; \theta_i)$ (or $b_i(k; \alpha_i)$ in our notation below) with a $K \times n$ Vandermonde matrix of Almon (1965):

$$V = \begin{bmatrix} 1 & 1^1 & 1^2 & \cdots & 1^{n-1} \\ 1 & 2^1 & 2^2 & \cdots & 2^{n-1} \\ 1 & 3^1 & 3^2 & \cdots & 3^{n-1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & K^1 & K^2 & \cdots & K^{n-1} \end{bmatrix}. \tag{7}$$

The procedure assumes that each lag coefficients can be approximated by a polynomial of degree $n-1 < K$,thereby reducing the number of parameters to be estimated from $1 + Kq$ to $1 + nq$. To illustrate this, we note that the GADL model, eq.(5), can be represented succinctly as follows:

$$\begin{aligned} Y &= \beta_0 + \beta_1 X_1 V \alpha_1 + \beta_2 X_2 V \alpha_2 + \cdots + \beta_q X_q V \alpha_q + \varepsilon \qquad (8) \\ &= \gamma_0 + Z_1 \gamma_1 + \cdots + Z_q \gamma_q + \varepsilon \\ &= Z\gamma + \varepsilon, \qquad \text{where} \end{aligned}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_T \end{bmatrix}, \qquad X_i = \begin{bmatrix} x_{i,1-h} & x_{i,1-h-1/K} & x_{i,1-h-2/K} & \cdots & x_{i,1-h-(K-1)/K} \\ x_{i,2-h} & x_{i,2-h-1/K} & x_{i,2-h-2/K} & \cdots & x_{i,2-h-(K-1)/K} \\ x_{i,3-h} & x_{i,3-h-1/K} & x_{i,3-h-2/K} & \cdots & x_{i,3-h-(K-1)/K} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ x_{i,T-h} & x_{i,T-h-1/K} & x_{i,T-h-2/K} & \cdots & x_{i,T-h-(K-1)/K} \end{bmatrix}. \tag{9}$$

Here $Z = XV$ is a $T \times (nq+1)$ matrix, and $\gamma = \left(\beta_0, \gamma_1^\top, \ldots, \gamma_q^\top\right)^\top$ is a $(nq+1) \times 1$ vector of parameters to be estimated. Note also that each α_i in eq.(8) is an $n \times 1$ vector where $n \leq K$.

The use of a simple low order $(n-1)$ polynomial (of k) to approximate the K distributed lag coefficients $b_i(k; \alpha_i)$ is the key characteristics of the Almon (1965) method. Estimating GADL can now be carried out using OLS on the transformed variable Z, instead of the original X, significantly reducing the dimensions. We note that the same dimension-reduction strategy is implemented in the NLS-MIDAS literature of Ghysels et al. (2004, 2006), but they employ a two-parameter *exponential* Almon lag polynomial instead, as described in Section 3 below, which require their estimation to be carried out using NLS.

The OLS estimators for GADL, as represented in eq.(8), have been extensively used in the estimations of distributed lag models (see Amemiya and Morimune 1974). In particularly, for given n and K, γ can be consistently and efficiently estimated as:

$$\widehat{\gamma} = (Z^\top Z)^{-1} Z^\top Y. \tag{10}$$

Provided that the regularity conditions in Theorem 5.17 of White (2001) are satisfied in the GADL model and that ε_t is a white noise process, we know that as $T \to \infty$,

$$\widehat{D}^{-1/2} T^{1/2} (\widehat{\gamma} - \gamma) \xrightarrow{d} N(0, I_{nq+1}), \tag{11}$$

where $\xrightarrow{d}$ denotes convergence in distribution, $\widehat{D} = \widehat{\sigma}^2 (T^{-1} Z^\top Z)^{-1}$, $\widehat{\sigma}^2 = (T - nq - 1)^{-1} \sum_{t=1}^{T} e_t^2$ with e_t being the OLS residuals, and I_{nq+1} is an $(nq+1) \times (nq+1)$ identity matrix. The GADL model can be estimated with OLS under a wide range of $n \leq K$. Its computational simplicity offers a major advantage over NLS-MIDAS with exponential Almon weights, which can become computationally-demanding when the set of predictors q are large.

The GADL framework allows us to test the significance of the aggregate impact parameter β_i in a straight-forward fashion. As discussed in Section 2.1 above, by restricting the weights on the higher frequency data to sum up to 1, we can identify $\beta_1, \ldots, \beta_q$. In GADL, this means letting $\mathbf{1}^\top V\alpha_i = 1$, $i = 1, \ldots, q$, and we obtain:

$$\widehat{\beta}_i = \mathbf{1}^\top (V\widehat{\gamma}_i), \quad i = 1, \ldots, q. \tag{12}$$

$$var(\widehat{\beta}_i) = \mathbf{1}^\top V var(\widehat{\gamma}_i) V^\top \mathbf{1} \tag{13}$$

2.3 Comparing GADL and MIDAS

The previous section derives the properties of the GADL estimators using the MIDAS setup where we separate out the aggregate impact parameters $\beta_i's$ from the weighting coefficients $b_i's$ for higher frequency data. This section clarifies a fundamental difference between the MIDAS approach and the GADL approach: under GADL, this separation is unnecessary.

From the perspective of the classical ADL literature, each $\beta_i V\alpha_i$ ($i = 1$ to q) in eq.(8) represents the impact of different lags of X_i on Y. The sum of $\beta_i V\alpha_i$, expressed as $\mathbf{1}^\top \beta_i V\alpha_i$ or $\mathbf{1}^\top V\beta_i\alpha_i$, then measures the aggregate impact of all the lagged X_i on Y. To estimate it, we note that eq.(8) can be re-written as follows:

$$\begin{aligned} Y &= \beta_0 + X_1 V \beta_1 \alpha_1 + X_2 V \beta_2 \alpha_2 + \cdots + X_q V \beta_q \alpha_q + \varepsilon \\ &= \beta_0 + X_1 V \alpha_1^* + X_2 V \alpha_2^* + \cdots + X_q V \alpha_q^* + \varepsilon \\ &= Z\gamma + \varepsilon \end{aligned} \tag{14}$$

where $\gamma = \left(\beta_0, \alpha_1^{*\top}, \ldots, \alpha_q^{*\top}\right)^\top$ so $\gamma_i = \alpha_i^*$ for $i \neq 0$. One can then use eqs.(10) and (11) for the estimation of γ. Since $\gamma_i = \beta_i \alpha_i$, the aggregate impact parameter for X_i on Y is simply $\mathbf{1}^\top V \gamma_i$. This demonstrates that eq.(12) can be applied directly to obtain the aggregate impacts of lagged X_i on Y without the need for separating out $\beta_i' s$ versus $b_i' s$ in the MIDAS-style specifications (eqs.5 & 6). In other words, under GADL, any restrictions imposed on $b_i' s$ are extraneous.

To generalize this further, rather than eq.(5), the GADL model can be more naturally expressed as:

$$\begin{aligned} y_t = \; & \beta_0 + \sum_{k=1}^{K} b_1^*(k; \alpha_1) L^{(k-1)/m} x_{1,t-h}^m + \sum_{k=1}^{K} b_2^*(k; \alpha_2) L^{(k-1)/m} x_{2,t-h}^m + \\ & \cdots + \sum_{k=1}^{K} b_q^*(k; \alpha_q) L^{(k-1)/m} x_{q,t-h}^m + \varepsilon_t, \end{aligned} \tag{15}$$

where the $\beta_i' s$ and the $b_i' s$ in the MIDAS specification are combined into free parameters $b_i^{*\prime} s$. These free coefficients can then be parameterized with the Vandermonde matrix as in eq.(14) to deliver OLS-estimates of the aggregate impact parameters. Thus, GADL constitutes a one-step procedure that automatically embeds the identification condition for β_i; the typical weight restrictions or rescaling procedure required in the MIDAS models - linear or non-linear - is not needed. This is why we name our framework, "Generalized ADL", to distinguish it from (OLS) MIDAS; its idea and derivations follow more naturally the traditional ADL literature. The straight-forward structure of eq.(12) also allows us to conduct inferences on β_i using the asymptotic distribution in eq.(11). The Monte Carlo experiments in the next section confirm these theoretical properties.

In the traditional ADL literature, the Almon lag polynomial is well-known to offer useful approximations for a variety of weighting functions, provided that n is large enough. This indicates that eq.(12) can deliver consistent estimates of the aggregate impact parameters under a wide range of true underlying weighting functions. The Monte Carlo experiments in Section 3 demonstrate that even when the true weighting function is an *exponential* Almon lag polynomial, the GADL models produce accurately estimated aggregate impact parameters.

As a parallel to the discussion in Ghysels et al. (2006b) and Clements and Galvão (2008), we further note that GADL can also be extended easily, such as to include general AR(p) dynamics. As an illustration, we generalize eq.(8) in Section 4 to the following AR(1)-GADL model:

$$Y = \rho Y_{-1} + Z\gamma + \varepsilon, \tag{16}$$

where Y_{-1} is the one-period lagged dependent variable. We then follow the modeling strategy proposed in Clements and Galvão (2008) and use:

$$Y = \rho Y_{-1} + Z\gamma - \rho Z_{-1}\gamma + \varepsilon, \tag{17}$$

so that the response of the dependent variable to the regressors remains nonseasonal. We note that the inclusion of AR dynamic in either of these two ways can both be estimated easily with the proposed OLS-based method.

3 Monte Carlo Experiments

3.1 Aggregate Impact Parameter Estimation

This section examines the properties of the GADL estimators, eq.(12), for different low frequency sample sizes, T, and aggregation horizons, K. We assume the underlying DGP is based on the NLS-MIDAS process of Ghysels et al. (2002, 2006), and show that the GADL estimators deliver excellent performance in accordance with eq.(11). We note that while the idea of an OLS-based MIDAS model is mentioned previously in Sinko et al. (2011), its relative performance, compared to the NLS-MIDAS models that dominate the literature, has not been examined.

Following Andreou et al. (2010) Section 6, we set up the following DGP for our Monte Carlo experiments:

$$y_t = \beta_0 + \beta_1 x_t(\theta) + u_t, t = 1, 2, ..., T \tag{18}$$

where $x_t(\theta) = W(L^{1/m}, \theta)x^{(m)}_{1,t/m}$ with $m = 1...K$,and $u_t \sim N.i.i.d.(0, 0.125)$. Note that u_t is sampled at low frequency with sample size T, whereas the regressor $x^{(m)}_{1,t/m}$ is sampled K times between t and $t-1$ from $N.i.i.d.(0, 1)$ such that the higher frequency sample size is KT. The high frequency data $\{x^{(m)}_{1,t/m}\}^K_{m=1}$ are projected onto the low frequency data $x_t(\theta)$, using a two-parameter *exponential* Almon lag polynomial:

$$w_m(\theta_1, \theta_2) = \frac{\exp\{\theta_1 m + \theta_2 m^2\}}{\sum_{m=1}^{K} \exp\{\theta_1 m + \theta_2 m^2\}}. \tag{19}$$

Parameter values are set to be $v = [\beta_0, \beta_1, \theta_1, \theta_2] = [0.5, 1.5, 7 \times 10^{-4}, -5 \times 10^{-2}]$, as in Andreou et al. (2010), for our simulations. In the next sub-section, we provide a robustness check using $\beta_1 = 0.6$.as well. All the programs are written in GAUSS. Two hundred additional values are generated in order to obtain random starting values. The optimization algorithm used to implement the NLS-MIDAS estimation is the quasi-Newton algorithm of Broyden, Fletcher, Goldfarb, and Shanno (BFGS) contained in the GAUSS MAXLIK library. The maximum number of iterations for each replication is 100. The first 1000 replication of normal convergence are recorded for numerical analysis.

Table I shows our simulation results for $T = 100, 300, 500$, $K = 14, 34, 54$,and $n = 3$ and 4. We report both the mean and the root mean squared errors (RMSE) of the estimated coefficients, $\widehat{\beta}_0$ and $\widehat{\beta}_1$, over 1000 replications. We make the following observations:

- Even though we are using GADL - a *linear* polynomial distributed lag model - to estimate data generated by a *non-linear* exponential Almon lag model, the mean estimates show very little bias even under low T and high aggregation K.

- As sample size T increases, RMSE declines in accordance to asymptotic theory (eq. 11).

- While RMSEs increase with higher K, the degree of Almon lag polynomial ($n = 3$

versus $n = 4$) makes little difference to the estimations.[10]

- Overall, GADL works well.

INSERT TABLE I

3.2 Comparing GADL with a Non-Linear Approach

We next compare estimations results obtained under GADL and NLS-MIDAS. We again use simulated data generated by the NLS-MIDAS model described above, with the exception that we now set $\beta_1 = 0.6$ instead of 1.5. We initialize the NLS estimation using values that are a small deviation from the true values. Specifically, we set:

$$v^0 = v + \epsilon, \text{ where } \epsilon \sim N(0, 1)/\sqrt{20}.$$

Table II reports estimation results for β_1 over 1000 simulations: the mean estimates, biases, and the RMSEs under GADL (both $n = 3$ and $n = 4$) and NLS-MIDAS. The last two rows also report the relative RMSEs of GADL over NLS-MIDAS. We note the following:

- Even though the true DGP is NLS-MIDAS, we observe the non-linear estimation can show significant bias when the initial values used for NLS involve a small random deviation from the true values.[11] As T increases, the RMSEs in the estimations do not

[10] As K gets larger, the extra flexibility offered by $n = 4$ may provide slightly better fit, but it would depend on the underlying DGP. The NLS-MIDAS literature reports that the Beta function can be more appropriate for aggregating time lags larger than 20 ($K > 20$) while the expotential Almon weights are more appropriate for small number of lags.

[11] Seperate unreported results show that if we start the NLS at exactly the true values, the estimation works very well showing less bias than GADL. Of course, in actuality, one never knows what the true values are, and the devations we consider here are very small.

decline very much.

- GADL continues to works well: the biases are at least an order of magnitude smaller than those obtained under NLS, and the RMSEs are also significantly lower than those for the NLS estimates. For larger T, we see RMSEs declining. Again, given this particular DGP, we observe little differences in estimation results using $n = 3$ vs. 4.

These observations suggest that in more general setups where the true DGP differs from the exponential Almon lag NLS-MIDAS model (eqs.18 and 19), one may need additional refinements in order to obtain good estimates under NLS-MIDAS.[12] While there are methods to raise estimation speed and efficiency in NLS optimization, it should be apparent that the OLS-based GADL is a viable alternative for its ease of implementation. This advantage is especially relevant in estimations involving multiple sets of high-frequency predictors, as in the application we explore below. We thus conduct the rest of our empirical analyses using GADL (with $n = 3$).

INSERT TABLE II

4 Forecasting Aggregate Commodity Price Index

Building upon the concept of commodity currencies developed in Chen and Rogoff (2003) and the forward-looking nature of nominal exchange rates, CRR (2010a) demonstrate that quarterly changes in a few key exchange rates - the Australian dollar, the Canadian dollar, the

[12] For example, the actual weighting function may involve some negative numbers.

Chilean peso, the New Zealand dollar, and the South African rand - can predict subsequent quarterly movements in aggregate world commodity prices. This conclusion is generally confirmed in Groen and Pesenti (2011), and CRR (2010b) extend the idea and use the equity indexes from these major commodity-exporting countries to predict world agricultural commodity price movements.[13] As discussed in more details in CRR (2010a), the mechanism for these predictive ability follows directly from the present value formulation of asset prices discussed in Campbell and Shiller (1987) and Engel and West (2005). For countries that rely heavily on primary commodity production, global commodity price movements affect the valuation of a substantial share of their productions and exports, and thereby influencing their currency and equity valuation. Knowing this connection, when market participants foresee a future commodity price shock, its anticipated impact on future asset values will be priced into the current asset prices, thus resulting in the predictability link. As each of these countries' currency and equity valuations embody information about the future price prospects of their relevant commodity exports, by combining them we can obtain forecasts for price movements in the *aggregate* commodity market.

One common issue concerning testing out-of-sample forecast performance is the possibility of structural breaks or parameter instability. As CRR (2010a) showed, the 2008 Financial Crisis led to a change in the predictive relationship between quarterly exchange rate returns and subsequent commodity price movements. We confirm this "forecast breakdown" below, and show that by using GADL and allowing more recent exchange rate and stock market

[13] Using data covering the financial crisis period, Groen and Pesenti (2011) note that the choice of commodity indices, forecast horizons, and comparison benchmarks can all play a role in determining the exact specifications that produce the best forecast.

data within a quarter to play a bigger role, we obtain more accurate forecasts than under the time-aggregated approaches. The reason is intuitive. When there is a break at time t, both exchange rates and commodity prices are affected, and the new development would only be reflected in the more recent data. If one uses exchange rates averaged over the full past three months, the relevant information would be washed out.[14]

The data we use for the empirical analysis below are obtained from the Global Financial Data and the IMF. The commodity price series is the IMF non-fuel commodity price index, and the daily exchange rates and equity prices are daily closing values covering 1984Q1 to the end of 2010Q3. We look at exchange rates from Australia, Chile, Canada, and New Zealand, and equity indexes from Australia, Canada, and New Zealand only.[15]

4.1 Matched-Frequency Forecasts and the 2008 Crisis

Here we follow the exact setup in CRR(2010a) and test whether exchange rate changes Δs_t can predict commodity price movements Δcp_{t+1}out of sample, using *matched frequency* quarterly data up to 2008Q2, and also up to 2010Q3. We adopt a rolling forecast scheme based on the following two commodity currency-based equations:

$$E_t \Delta cp_{t+1} = \beta_0 + \beta_1 \Delta s_t^{AUS} + \beta_2 \Delta s_t^{CAN} + \beta_3 \Delta s_t^{CHI} + \beta_4 \Delta s_t^{NZ} \tag{20}$$

$$E_t \Delta cp_{t+1} = \beta_0 + \rho \Delta cp_t + \beta_1 \Delta s_t^{AUS} + \beta_2 \Delta s_t^{CAN} + \beta_3 \Delta s_t^{CHI} + \beta_4 \Delta s_t^{NZ} \tag{21}$$

[14] Andreou et al.(2010c) use NLS-MIDAS regressions to predict quarterly real economic activity. They reach the same conclusion concerting model performance over the crisis period.

[15] The Chilean Equity Index is excluded due to many missing observations.

We refer to the first one as "CRR", and the second one as "AR-CRR" in our reporting in Table III. We compare their forecast performance relative to two time-series benchmarks: an AR(1) model and a driftless random walk (RW):[16]

$$E_t \Delta cp_{t+1} = \gamma_0 + \gamma \Delta cp_t \tag{22}$$

$$E_t \Delta cp_{t+1} = 0 \tag{23}$$

We generate a sequence of 1-step-ahead forecasts using the standard rolling window out-of-sample procedure for each of the above four models. Motivated by discussion in Rossi and Inoue (2011), we report forecast comparison results using several alternative rolling window sizes ($w = 60, 76$, and 92) for all our empirical analyses in order to check the robustness of our conclusions.[17] Table III Panel A reports the RMSEs produced under eqs. (20)-(23). Panel B shows two sets of forecast comparison results: RMSE ratios and the p-values for forecast comparison based on either the Diebold-Mariano (1995) or the Clark-West (2006) tests. (If the two models under comparison are nested, e.g. any models over the RW, we use the Clark-West statistics, and for comparing CRR and AR, which are not nested, we use the Diebold-Mariano statistics.[18])

[16] The order of the benchmark autoregressive model is selected by the Bayesian information criterion in CRR (2010a).

[17] The first regression uses the first w quarterly observations and makes a forecast for the commodity price change at $w + 1$, where w is the rollwing window size. The second regression moves forward over time by one quarter and make another forecast, and so on. At the end of the rolling process, we calculate the RMSE for our model, and compare it with the RMSE produced by a drift-less random walk and by the AR model. Additional results are available for window sizes of 68 and 84.

[18] Under the null of equal predictability, the sample RMSE of the larger model is expected to be greater than those of the more restricted models (AR or RW). The Clark and West (2006) test statistic adjusts for this upward shift in the sample RMSEs.

For forecast comparisons covering the pre-crisis period (1984Q1 to 2008Q2), we see in Table III Panel B that the exchange-rate based CRR and AR-CRR models overall generate smaller RMSEs than the two benchmarks, and the Clark-West statistics reject the null hypothesis of equal predictability in favor of the CRR and AR-CRR specifications. These findings are consistent with the results reported in CRR (2010a). The right-hand side columns of Table III reports results using a longer sample period, up to 2010Q3. We see that the superior forecast performance of the exchange rate-based specifications are no longer as robust. In fact, most RMSE ratios now around 1. As discussed in CRR (2010a) Appendix III, this forecast breakdown is most likely attributable to the 2008 Financial Crisis.

INSERT TABLE III

4.2 GADL Forecasts

We next implement the same set of forecast comparisons with daily exchange rates instead, using the GADL and AR-GADL specifications described in eqs.(5) and (17) to replace the matched-frequency regressions eqs.(20) and (21). Table III report results based on sample period $1984Q1 - 2010Q3$. Panel A shows the RMSEs for GADL and AR-GADL under different aggregations K and rolling window sizes. Panel B provides RMSE ratio comparisons with benchmark models as well as the p-values for testing the null hypothesis of equal predictability. We note the following results:

- For K small, GADL produces superior forecasts (smaller RMSEs) relative to RW, AR, as well as to the matched-frequency forecast of CRR, suggesting that using newer information from within a quarter can deliver sharper forecast results.

- For $K = 54$,we don't see much advantage of using GADL over the other models. This result may not be surprising, as the number of (trading) days within a quarter is slightly over 60 in our data set, using $K = 54$, we are essentially including the whole quarter of data into the regressions. Under the view that more recent data contain stronger signal for forecasting, as demonstrated by the $K = 14$ and 34 results, using a full quarter of data may be diluting the signal. Relative to CRR (which weight all daily information equally over the full quarter), GADL with $K = 54$ can introduce additional estimation noise as well.

- Adding the AR term does not improve the pure exchange-rate based GADL forecasts. Even though the Clark-West (2006) test of equal predictability mostly favors AR-GADL over RW and AR, the RMSEs produced under AR-GADL do not show a consistent pattern in out-performing the benchmarks.

Overall, GADL regressions using a small set of recent daily exchange rates, such as a few weeks to over a month of data, can consistently forecast better than the benchmarks even over the sample period that includes the 2008 Financial Crisis. This finding supports the view that in forecasting the future, more up-to-date data should be allowed to play a more important role.

INSERT TABLE IV

4.3 Forecast Combination

This section considers forecast combination under GADL, which is an alternative way to exploit the information content in the daily exchange rates while avoiding running multivariate regressions in our small samples. The approach involves computing a weighted average of different forecasts, each obtained from a single exchange rate-based GADL regression. Specifically, we estimate the following four GADL regressions and generate one-step ahead world commodity price forecasts:

$$E_t \Delta cp^i_{t+1} = \beta_{0,i} + \beta_{1,i} W_i(L^{1/m}, \theta_i) \Delta s^{(m)}_{i,t} \quad \text{where } i = AUS, CAN, CHI, NZ. \tag{24}$$

Similarly, we produce forecasts using single-exchange rate-based AR-GADL regressions for each of the four countries. The forecast combination literature has proposed various methods to weigh individual forecasts, yet it is well known that simple combination schemes tend to work best (Stock and Watson 2004 and Timmermann 2006.) We thus consider equal weighting here, and compare our combined forecasts with the RW and AR benchmark forecasts.

Table V reports results for three sets of combined forecasts:

- FC1 is the equal-weighted average of the four individual GADL forecasts: $(\Delta \widehat{cp}^{AUS}_{t+1} + \Delta \widehat{cp}^{CAN}_{t+1} + \Delta \widehat{cp}^{CHI}_{t+1} + \Delta \widehat{cp}^{NZ}_{t+1})/4$.
- FC2 is the average of the above four individual forecasts as well as the AR forecasts.
- FC3 is the equal-weighted average of the four individual AR-GADL forecasts.

We observe in Table V patterns similar to what we found in the Table IV.[19] The GADL approach in forecast combination comparisons outperforms the benchmarks for $K = 14$ and 34. But when we include almost a full-quarter of daily information ($K = 54$), GADL shows similar conclusions as we saw in Table III under matched-frequency analyses: using data up to 2010Q3, exchange-rate based forecasts perform similarly to the benchmarks. To summarize, forecast combinations support the conclusion that newer data within a quarter are more relevant for forecasting, but when they are averaged out with older data, their predictive content gets diluted.

INSERT TABLE V

4.4 Forecasts using Equity Indexes

As discussed in CRR (2010b), the net-present-value relationship with commodity prices should extend to other assets whose valuations depend on world commodity prices. The equity market indexes in major commodity-producing economies thus constitute another set of candidate predictors with daily observations. We replace the daily exchange rates in the GADL models above with equity prices from Australia, Canada, and New Zealand, and compare their forecast performance with AR and RW. Table VI report the results for a multivariate GADL estimation using all three sets of daily equity indexes, and for two forecast combination schemes involving: 1) (equal-weighted) combined forecasts of the three single-country GADL regressions as in eq.(24); 2) (equal-weighted) combined forecasts of the three

[19] To judge the significance of forecast combinations, we used critical values based on Diebold and Mariano (1995).

single-country GADL regressions and the AR prediction. We see that the equity-index based forecasts produce smaller RMSEs than the two statistical benchmarks very consistently. In addition, we note that unlike the exchange rate-based results, forecast superiority may be increasing when more daily data is used (K is large). While the differences may not be statistically significant, the pattern may also be due to the fact that equity prices are overall more noisy in reflecting the commodity price signal, and as such, one needs a larger set of data to reflect the underlying linkage with commodity prices. Overall, we note that GADL using daily equity index movements can also forecast commodity prices.

INSERT TABLE VI

5 Conclusion

This paper presents a generalized autoregressive distributed lag model for conducting regression estimations that involve mixed-frequency data. The motivation and setup of GADL merge the pioneering work on NLS-MIDAS by Ghysels et al. (2002) and the classic work of Almon (1965) of approximating distributed lag coefficients with simple low-order polynomials. GADL inherits the ease of estimation from the ADL literature; in one step, it delivers estimates for the "aggregate impact" parameters emphasized in the MIDAS literature and their asymptotic properties.

Although the idea of using Vandermonde matrix and the associated OLS estimation has been independently mentioned in the *Matlab Toolbox for MIDAS* by Sinko et al. (2011), this paper demonstrates that the GADL framework is conceptually more general; it does not impose any restrictions on, nor require any re-scaling of, the weights on the higher frequency data. This paper also offers a comparison of the relative performance between the OLS-based GADL specifications and the NLS-MIDAS models. The Monte Carlo simulations reveal that the GADL approach delivers good finite-sample performance and is much easier to implement than methods proposed in previous research. We view our results as indicative of the great potential for using OLS-based setups to conduct empirical studies involving mixed frequency data estimations.

As an application, we show that daily asset market information - currency and equity market movements - can produce forecasts of quarterly commodity price changes that are superior to those shown in the previous literature. Specifically, the superior forecasting ability relative to standard benchmarks is robust to the 2008 financial crisis.

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TABLES

Table I. GADL Estimations for a NLS-MIDAS Model

$\beta_0 = 0.5, \beta_1 = 1.5$, $\sigma^2 = 0.125$

Time span: T	100		300		500	
True β_i	0.5	1.5	0.5	1.5	0.5	1.5
			Aggregation $K = 14$			
Mean $\widehat{\beta}_i^{GADL,n=3}$	0.500	1.501	0.500	1.502	0.500	1.503
Bias$^{GADL,n=3}$	0.000	−0.001	0.000	−0.002	0.000	−0.003
RMSE$^{GADL,n=3}$	0.037	0.138	0.021	0.076	0.016	0.059
Mean $\widehat{\beta}_i^{GADL,n=4}$	0.500	1.502	0.500	1.502	0.500	1.503
Bias$^{GADL,n=4}$	0.000	−0.002	0.000	−0.002	0.000	−0.003
RMSE$^{GADL,n=4}$	0.037	0.138	0.021	0.076	0.016	0.058
			Aggregation $K = 34$			
Mean $\widehat{\beta}_i^{GADL,n=3}$	0.499	1.492	0.501	1.504	0.501	1.503
Bias$^{GADL,n=3}$	0.001	0.008	−0.001	−0.004	−0.001	−0.003
RMSE$^{GADL,n=3}$	0.047	0.272	0.026	0.157	0.020	0.121
Mean $\widehat{\beta}_i^{GADL,n=4}$	0.500	1.492	0.501	1.500	0.501	1.501
Bias$^{GADL,n=4}$	0.000	0.008	−0.001	0.000	−0.001	−0.001
RMSE$^{GADL,n=4}$	0.040	0.229	0.022	0.132	0.018	0.101
			Aggregation $K = 54$			
Mean $\widehat{\beta}_i^{GADL,n=3}$	0.501	1.506	0.499	1.502	0.500	1.507
Bias$^{GADL,n=3}$	−0.001	−0.006	0.001	−0.002	0.001	−0.007
RMSE$^{GADL,n=3}$	0.054	0.418	0.032	0.233	0.024	0.185
Mean $\widehat{\beta}_i^{GADL,n=4}$	0.501	1.506	0.499	1.506	0.500	1.507
Bias$^{GADL,n=4}$	−0.001	−0.006	0.001	−0.006	0.000	−0.007
RMSE$^{GADL,n=4}$	0.044	0.335	0.026	0.191	0.020	0.150

Note: The DGP for the Monte Carlo exercise is the NLS-MIDAS model described in Sec. 3. All the results are based on 1000 replications. Bias is computed as the true parameter values minus the average estimated values.

Table II: NLS vs. OLS-GADL Estimates of the Aggregate Impact Parameter
$\beta_1 = 0.6$, $\sigma^2 = 0.125$

Aggregation K		14			34			54	
Time span: T	100	300	500	100	300	500	100	300	500
Mean $\widehat{\beta_1}^{NLS}$	0.486	0.470	0.466	0.423	0.403	0.405	0.439	0.428	0.419
Bias^{NLS}	0.114	0.130	0.134	0.177	0.197	0.195	0.161	0.172	0.181
RMSE^{NLS}	0.302	0.287	0.284	0.359	0.348	0.343	0.344	0.328	0.335
Mean $\widehat{\beta_1}^{GADL,n=3}$	0.615	0.605	0.602	0.602	0.598	0.602	0.601	0.600	0.598
Bias$^{GADL,n=3}$	−0.015	−0.005	−0.002	−0.002	0.002	−0.002	−0.001	0.000	0.002
RMSE$^{GADL,n=3}$	0.134	0.076	0.058	0.229	0.129	0.096	0.295	0.171	0.135
Mean $\widehat{\beta_1}^{GADL,n=4}$	0.615	0.604	0.602	0.607	0.600	0.602	0.598	0.600	0.601
Bias$^{GADL,n=4}$	−0.015	−0.004	−0.002	−0.007	0.000	−0.002	0.002	0.000	−0.001
RMSE$^{GADL,n=4}$	0.135	0.076	0.058	0.220	0.126	0.091	0.281	0.161	0.127
$\frac{RMSE^{GADL,n=3}}{RMSE^{NLS}}$	0.44	0.26	0.20	0.64	0.37	0.28	0.86	0.52	0.40
$\frac{RMSE^{GADL,n=4}}{RMSE^{NLS}}$	0.45	0.26	0.21	0.61	0.36	0.27	0.82	0.49	0.38

Note: Simulated data is generated using the NLS-MIDAS model described in Sec. 3. OLS estimates are obtained using Generalized ADL estimations. All the results are based on 1000 replications. Bias is computed as the true parameter values minus the average estimated values. Each NLS estimations are initiated at the true values + ϵ, where ϵ is drawn from $N(0,1)/\sqrt{20}$, and reported numbers are computed using only iterations with normal convergence.

Table III. Out-of-Sample Forecast Performance Using Matched Frequency Data

	Pre-Crisis Sample: 1984Q1 to 2008Q2			Full-Sample: 1984Q1 to 2010Q3		
Window Size:	60	76	92	60	76	92
# of Forecasts:	37	21	5	46	30	14
A) RMSEs						
RW	0.0512	0.0616	0.0758	0.0742	0.0886	0.1149
AR	0.0501	0.0595	0.0736	0.0756	0.0887	0.1159
CRR*	0.0492	0.0557	0.0677	0.0752	0.0878	0.1168
AR-CRR	0.0485	0.0561	0.0686	0.0769	0.0890	0.1186
B) RMSE Ratios						
CRR/RW	0.95	0.90	0.89	1.01	0.99	1.02
p-value**	0.00	0.00	0.01	0.04	0.04	
AR-CRR/ RW	0.96	0.91	0.91	1.04	1.00	1.03
p-value	0.00	0.00	0.01	0.03	0.03	
CRR/AR	0.97	0.94	0.92	0.99	0.99	1.01
p-value			0.04			
AR-CRR/AR	0.98	0.94	0.93	1.02	1.00	1.02
p-value	0.03	0.02	0.01			

Note: CRR refers to matched frequency (quarterly) forecasts based on: $E_t\Delta cp_{t+1}=\beta_0+\sum_i \beta_i \Delta s_t^i$ and AR- CRR: $E_t\Delta cp_{t+1}=\beta_0+\rho\Delta cp_t+\sum_i \beta_i \Delta s_t^i$ where i = AUS, CAN, CHI, and NZ. The p-values are for tests of equal predictability based on Diebold-Mariano (1995) statistics (for non-nested models) or the Clark-West (2006) test if the models are nested. P-values $>$ 0.10 are not reported. See text for details

Table IV: Out-of-Sample Performance: Generalized ADL & Mixed Frequency Data

(1984Q1 - 2010Q3)

	$K = 14$			$K = 34$			$K = 54$		
Window Size:	60	76	92	60	76	92	60	76	92
# of Forecasts:	46	30	14	46	30	14	46	30	14
					A) RMSEs				
GADL*	0.0683	0.0806	0.0983	0.0737	0.0819	0.1043	0.0817	0.0919	0.1208
AR-GADL*	0.0778	0.0865	0.1063	0.0796	0.0867	0.1092	0.0834	0.0932	0.1232
					B) RMSE Ratios				
GADL/RW	0.92	0.91	0.86	0.99	0.92	0.91	1.10	1.04	1.05
p-value*	0.03	0.03	0.00	0.05	0.05	0.02		0.10	
AR-GADL/ RW	1.05	.98	.93	1.07	0.98	0.95	1.13	1.05	1.07
p-value	0.03	0.03	0.01	0.05	0.05	0.03		0.06	0.1
GADL/AR	0.90	0.91	0.86	0.98	0.92	0.90	1.08	1.04	1.05
p-value			0.02						
AR-GADL/AR	1.03	0.98	0.92	1.05	0.98	0.94	1.10	1.05	1.06
p-value	0.10	0.07	0.02			0.09	0.08		
GADL/CRR	0.91	0.92	0.84	0.98	0.93	0.89	1.09	1.05	1.03
p-value			0.03				0.07		
AR-(GADL/CRR)	1.01	0.97	0.90	1.04	0.97	0.92	1.09	1.05	1.03
p-value									

Note: GADL and AR-GADL are mixed frequency versions of CRR and AR-CRR from Table III, using daily exchange rates. The order of polynomial in the Almon weight (n) is set to 3. The p-values are for tests of equal predictability based on Diebold-Mariano (1995) statistics (for non-nested models) or the Clark-West (2006) test (for nested models). P-values > 0.10 are not reported. See text for details.

Table V: Out-of-Sample Performance: Forecast Combinations

(1984Q1 - 2010Q3)

	$K = 14$			$K = 34$			$K = 54$		
Window Size:	60	76	92	60	76	92	60	76	92
# of Forecasts:	46	30	14	46	30	14	46	30	14
				A) RMSEs					
FC1*	0.0698	0.0830	0.1078	0.0702	0.0821	0.1082	0.0753	0.0880	0.1152
FC2	0.0706	0.0838	0.1092	0.0707	0.0830	0.1095	0.0750	0.0880	0.1151
FC3	0.0733	0.0854	0.1111	0.0740	0.0852	0.1114	0.0775	0.0895	0.1175
				B) RMSE Ratios					
FC1/ RW	0.94	0.94	0.94	0.95	0.93	0.94	1.02	0.99	1.00
p-value	0.01	0.00	0.00	0.01	0.01	0.00			
FC2/AR	0.93	0.95	0.94	0.94	0.94	0.95	0.99	0.99	0.99
p-value		0.07	0.01			0.04			
FC3/AR	0.97	0.96	0.96	0.98	0.96	0.96	1.03	1.01	1.01
p-value									

Note: FC1 is the average of single-country GADL forecasts; FC2 is the average of single-country GADL and AR forecasts; FC3 is the average of single country AR-GADL forecasts. The p-values are for tests of equal predictability based on Diebold-Mariano (1995) statistics. P-values > 0.10 are not reported.

Table VI: Out-of-Sample Performance Using Equity Indexes
Generalized ADL & Mixed Frequency Data
(1984Q1 - 2010Q4)

	$K = 14$			$K = 34$			$K = 54$		
Window Size:	60	76	92	60	76	92	60	76	92
# of Forecasts:	47	31	15	47	31	15	47	31	15
				A) RMSEs					
RW	0.0752	0.0895	0.1147	0.0752	0.0895	0.1147	0.0752	0.0895	0.1147
AR	0.0755	0.0882	0.1138	0.0755	0.0883	0.1138	0.0755	0.0882	0.1138
GADL*	0.0759	0.0880	0.1130	0.0740	0.0848	0.1029	0.0737	0.0853	0.1033
FC1*	0.0737	0.0858	0.1112	0.0718	0.0833	0.1072	0.0728	0.0842	0.1078
FC2*	0.0737	0.0861	0.1116	0.0721	0.0841	0.1085	0.0728	0.0847	0.1089
				B) RMSE Ratios					
GADL/RW	1.01	0.98	0.98	0.98	0.94	0.88	0.97	0.94	0.89
p-value*	0.06	0.03	0.02	0.07	0.03	0.00	0.04	0.02	0.02
FC1/RW	0.97	0.96	0.96	0.95	0.92	0.92	0.96	0.93	0.93
p-value		0.00	0.03	0.07	0.01	0.00	0.07	0.01	0.02
FC2/ RW	0.98	0.96	0.97	0.96	0.94	0.94	0.97	0.94	0.94
p-value		0.02	0.09	0.07	0.01	0.00	0.10	0.02	0.02
FC1/AR	0.96	0.95	0.96	0.93	0.92	0.91	0.94	0.93	0.92
p-value			0.07		0.08	0.02		0.08	0.03
FC2/AR	0.96	0.96	0.97	0.94	0.94	0.93	0.95	0.94	0.94
p-value			0.07		0.08	0.02		0.08	0.03

Note: GADL is the mixed frequency model: $E_t \Delta cp_{t+1} = \beta_0 + \sum_i \beta_i W_i(L^{1/m}, \theta_i) \Delta Equity\ Index^i_{t-m}$ where $i =$ AUS, CAN, and NZ. FC1 is the average of single country forecasts; FC2 is the average of single country and AR forecasts. The p-values are for tests of equal predictability based on Diebold-Mariano (1995) statistics (for non-nested models) or the Clark-West (1996) test if the models are nested. P-values > 0.10 are not reported. See text for details.

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