Coresidence With Husband’s Parents, Labor Supply, and Duration to First Birth

C. Y. Cyrus Chu · Seik Kim · Wen-Jen Tsay

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Abstract This article investigates the time to first birth, treating co-residence with husband’s parents and labor supply as endogenous and using representative data on Taiwanese married women born during 1933–1968. We use a full-information maximum likelihood estimator for a duration model with endogenous binary variables. Results controlling for endogeneity suggest that both co-residence and working are associated with a delay in childbearing, reversing the effect of co-residence on the timing of first birth but not that of working. Women in earlier cohorts tend to choose co-residency and not working, and an increasing number of women from later cohorts choose to do both or to work only.

Keywords Coresidence · Female labor supply · First birth

Introduction

A key factor that facilitates childbearing and the labor supply of married women is the availability of childcare. Working mothers rely on formal childcare institutions as well as informal private childcare, such as help from extended family members or nonrelatives. In this article, we focus on family-provided childcare, especially that from the child’s grandparents. Married women’s childbearing and labor supply are reported to be positively associated with the geographic proximity of their parents and in-laws. For example, Hank and Kreyenfeld (2004) showed that women in Germany who have parents living in the same town have an increased probability of giving birth. Compton and Pollak (2011) documented that U.S. married women with young children are more likely to work if they live close to their mothers or their mothers-in-law compared with those who live farther away.

In developing countries, the role of married women’s parents or parents-in-law is more important because institutional childcare opportunities are limited. In general, the availability of family childcare can provide the attractive means for women to

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pursue both childbearing and labor force participation. In a Chinese society, however, the proximity of a married woman’s parents and parents-in-law may also have other effects. Living close by often means coresidence, and the coresiding grandparents are, in most cases, the husband’s parents. With coresidence, a woman’s parents-in-law can provide help with housework and childcare, but they exert more control over her behavior. Thus, women confront important decisions at the time of marriage regarding coresidence with their parents-in-law, participation in the labor force, and childbearing and child-rearing.

Previous studies have only partially explored the relationships among these three choices. For instance, Rosenbaum and Gilbertson (1995) explored how coresidence affects labor force participation and concluded that coresidence with other adults increases the labor supply of certain groups of immigrant women in New York City. Recognizing that coresidence is not exogenously assigned, Sasaki (2002) reevaluated the effect of endogenous coresidence choice on married women’s work decisions in Japan; he found a positive impact of coresidence on married women’s labor supply. Using Chinese data for 1982–2000, Maurer-Fazio et al. (2011) noted that coresidence with parents or parents-in-law increases the labor supply of married women. Similar to Sasaki (2002), their results were robust to controlling for potential endogeneity of coresidency.

Studies have also explored the effect of coresidence or work status on the timing of childbearing. Using Taiwanese data, Tsay and Chu (2005) found that coresidence with parents-in-law can shorten a woman’s duration to first birth early in a marriage. Heckman et al. (1985) found that working women substantially delay childbearing. Heckman and Walker (1990) found a strong negative impact of females’ wage on the time to first birth in Sweden, and Merrigan and St.-Pierre (1998) presented similar results for Canadian women. These studies, however, did not account for the endogeneity of the coresidence or work decision.

This article investigates the effects of coresidence with husband’s parents and labor supply on married women’s duration to first birth. The analysis examines the choices of Taiwanese married women born during the period 1933–1968 and treats the coresidence and work decisions as endogenous. We specifically focus on the duration to first birth because, as documented in Schultz (1997), the timing of the first birth may be a critical threshold for predicting the pace of subsequent childbearing in many parts of the world. For example, Marini and Hodsdon (1981) found that a short first-birth interval from marriage increases the probability of having a second birth in a short interval. Another reason for analyzing the effects on the time to first birth is to make our results comparable with the findings of other studies. A large body of literature that explores the duration to birth or the effect of childbearing on labor market outcomes has focused mostly on the time to first birth.

By treating the coresidence and labor supply decisions as endogenous, we can also learn about unobserved attitudes about coresidence and working. It is important to note that marriage in a Chinese society is often an agreement between the husband’s family and the wife rather than between the husband and wife. For example, parents-in-law in a traditional Chinese society often emphasize the notion of lineage preservation and influence young coresiding couples’ childbearing decisions (Freedman et al. 1982). In modern society, however, career-driven young married women may choose not to coreside with parents-in-law and seek success in their careers. It is also possible that a third group of women may exist who care about their career success.
while also trying to accommodate traditionalism of the family. We call these three attitude types “traditional,” “career-driven,” and “have-it-all,” respectively.

Methodologically, we develop a full-information maximum likelihood (FIML) estimator to evaluate the endogenous treatment effects of coresidence and labor market activity on the timing of first birth. This method helps us to achieve two main objectives. First, by estimating the error covariance structure of the model, we can learn about how the distribution of attitudes of women toward coresidence and labor force participation changes across cohorts. Second, by controlling for the endogeneity induced by these different attitude types, we can consistently estimate the effects of coresidency and working on the duration to first birth.

Our results controlling for the endogeneity suggest that both coresidence and working lead to a delay in childbearing, reversing the effect of coresidence on the timing of first birth but not that of the labor supply. Thus, an analysis that does not control for the endogeneity would find that the duration to first birth is shortened by coresidence, simply because women who tend to coreside with their husbands’ parents are women who also tend to give birth earlier. Our findings suggest that there may be other sources—such as pressure, stress, or bargaining with husbands’ parents—that affect the duration to birth for women coresiding with in-laws. We also find that coresidency and working are negatively correlated for older cohorts, but this correlation disappears for younger cohorts. Accompanied by other evidence, these observations suggest that the majority of older-cohort women are the traditional type, whereas the share of career-driven and have-it-all type women has increased among the younger cohorts.

The remainder of this article is organized as follows. The following section provides background for this study by showing the trends of first marriage in modern Taiwan and suggesting three possible attitudes toward coresidence with in-laws and labor force participation. Then, we introduce the data and discuss findings from summary statistics. Following that, we specify the model including the three different attitudes; this section develops the FIML estimator for the duration model with endogenous dummy variables as an estimation device. We then present the empirical results concerning the impact of coresidence with husband’s parents and labor supply on the childbearing behavior of Taiwanese married women, contrasting the results from models that control for and those that ignore the endogeneity of coresidence and work decisions.

**Background**

**Trends in First Marriage Among Taiwanese Women**

We use Taiwanese data in this article because Taiwan is a well-known developing country and has experienced significant socioeconomic changes during recent decades. Here, we discuss the changes in the timing of first marriage for Taiwanese women: we confine our sample to ever-married women in our main analysis. Chang and Li (2011) documented the trends in the age at first marriage among Taiwanese women. Overall, their estimates suggested that women born in more recent cohorts

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1 See Schultz (1973) and Thornton et al. (1986) for a summary of socioeconomic changes in Taiwan in the recent past. For an analysis of the changes in married women’s employment in Taiwan, see Yu (2005).
married later and less often than those born in earlier cohorts. About 98% of the women born between 1930 and 1939 were married by age 30. The share drops to 96% for the 1940–1949 cohort of women, 91% for the 1950–1959 cohort, and 83% for the 1960–1969 cohort. By age 40, 99%, 98%, 95%, and 89% of women in each cohort (respectively) had married at least once. For our analysis, there might be some concern that married women are self-selected. The share of women who married by their mid-30s, however, is sufficiently high in every cohort, suggesting that this selection problem is not severe.

Further breaking down this trend of first marriage, Chang and Li (2011) found that more-educated women tend to marry later and less often than their less-educated counterparts. This pattern is more pronounced for younger cohorts of women. For example, among female college graduates born between 1960 and 1969, 62% married by age 30, and 75% marry by age 40; among female college graduates born between 1930 and 1939, the figures are 91% and 94%, respectively. This would be a concern for our analysis if more-educated unmarried women in the younger cohort retain some unobservable characteristics—such as attitudes, preferences, or ability—that correlate with their decisions of coresidence, working, or childbearing. For example, by limiting our analysis to married women, we may disproportionately exclude women who are more productive in the labor market. Even if this were the case, however, our results do not change qualitatively. As we show later, an increasing number of women from later cohorts choose to do both or to work only; therefore, this selection, if it exists, will strengthen the significance of our findings.

Attitudes Toward Coresidency and Working

In a Chinese society, we can immediately think of two possible scenarios that may be correlated with the unobservable factors behind the choice of coresidence, labor force participation, and duration to birth:

- Some married women may value traditionalism. In this article, a traditional woman is characterized by two factors: (1) she lives with her husband’s parents in order to take care of them (filial piety) and do household chores, and (2) she gives birth as early as possible to preserve the family lineage (see, e.g., Chu et al. 2011). These two factors make it harder for her to participate in the labor force. Even if she participates, she lives with her parents-in-law and gives birth earlier. We name individuals belonging to this scenario as traditional.

- Some married women may possess the desire or ability for career success. A woman of this type can be characterized by the following two factors: (1) she participates in the labor force, and (2) she is willing to delay childbearing because her career is her first priority and having children would limit her labor market activities. As a consequence of these two factors, she is likely not to live with her parents-in-law. Even if she lives with her parents-in-law, she cares about her career and delays the timing of childbearing. We name individuals belonging to this scenario as career-driven.

The role of a married woman and her parents-in-law is very different in each of the two aforementioned cases. In the first scenario, parents set the norms. In the second scenario, the married woman is the key decision maker in the family. However, there
may exist a third group of women who care about their career success but also try to accommodate traditionalism of the family. In this case, parents are actually an element of the household division of labor, sharing the burden of housework and childcare, but may also influence the sample of families living in Taiwan. This scenario can be described as follows:

- Consider a woman who balances traditionalism and career success at the same time. We characterize this type of woman using three factors: (1) she does not delay the time of childbearing, (2) she chooses to work in the labor market, and (3) as a compromise between her first two choices, she lives with her husband’s parents so as to allow the latter to share the household work and the childrearing cost. We name this group of women as have-it-all.

These three scenarios, of course, do not exhaust all possible unobservable factors, but we expect that they are consistent with practices in many developing countries. The proportion of women in each group likely differs over location and time of the society considered, and changes over time cannot occur abruptly. Which type of woman coresides with her parents-in-law is an empirical question, and this will affect the interpretation of estimation results.

Data and Descriptive Statistics

Our sample is drawn from the Panel Study of Family Dynamics (PSFD), a representative panel of respondents. We use the first waves of the 1999, 2000, and 2003 panel samples of the PSFD because the first waves include information on past fertility history. Each sample consists of adult males and females born during 1953–1963, 1934–1954, and 1964–1976, respectively. If an individual is selected, his or her families are included in the sample. As a result, the oldest woman in the sample was born in 1933. We exclude individuals born outside of Taiwan to minimize variation in the childhood background of respondents. In addition, we drop interviewees with missing or incomplete records, and accordingly, the number of observations decreases from 4,105 to 2,256. We then further restrict our sample to married women aged 35 or older at the time of the survey. After applying these criteria, our sample consists of 1,814 married women born in 1933–1968.

To proceed with an accurate analysis of fertility transition, we adopt a strategy proposed by Tsay and Chu (2005) to compute the duration between marriage and first birth, $T$, as $T = T_1 - T_0 + 1$, where $T_0$ and $T_1$ represent the recorded time of marriage and first birth, respectively. The rationale is that the time span for pregnancy takes nearly one year. We assume that there is no childbearing after age 45, but changing this age to 50 does not qualitatively alter our results because 98.6% of married women in our sample have had at least one birth by age 45.\(^2\)

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\(^2\) Tsay and Chu (2005) used these survey data to investigate the fertility behavior of married Taiwanese women. Details are available online (http://psfd.sinica.edu.tw).

\(^3\) Our estimates of fertility rates are consistent with the findings of previous research. Feeney (1991) showed that levels of marriage and motherhood in Taiwan were high and almost constant from the 1950s through the late 1980s. He used period parity progression ratios for 1979–1988 to find that 99% of all women married and that 98% of married women gave birth at least once.
Table 1 presents summary statistics of the variables used in this analysis separately for the two birth cohorts: 1933–1954 and 1955–1968. This grouping helps to evaluate the relative importance of the various scenarios across cohorts. A comparison of older (1933–1954) and younger (1955–1968) cohorts of women in Table 1 shows that women’s labor supply at the time of marriage rose substantially from 42% to 54%, while coresidence with parents-in-law at the time of marriage declined modestly from 48% to 43%. The coresidence rate did not change much between cohort groups, even after women are classified into two groups by work status. Further tabulation (not reported in Table 1) reveals that the coresidence rate dropped by 3 percentage points (from 43% to 40%) among those who work and by 6 percentage points (from 53% to 47%) among those who do not work. Later, however, we show that the attitudes of women who choose to coreside with their parents-in-law have changed distinctively.

Table 2 presents average duration to first birth by coresidence and work status. Coresidence does not seem to have any relationship with the timing of first birth for women in the older cohort but seems to shorten the duration to first birth for the younger cohort. Among working women in the younger cohort, coresiding women are likely to give birth 8.9 months (= (3.05 – 2.31) × 12) earlier than women who do not coreside. Among nonworking women in the younger cohort, coresidence is associated with a duration to first birth that is 1.6 months (= (2.21 – 2.08) × 12) shorter. This negative association, however, does not necessarily imply causality because it is also possible that traditional type women simultaneously choose to coreside and have children. One has to account for potential endogeneity, and we later show that coresidence actually lengthens the duration to first birth. Working has a small effect on the duration to first birth for women in the older cohort, but working women in the younger cohort have durations to first birth that are 10.1 months (= (3.05 – 2.21) × 12) longer if they do not live with their parents-in-law. Working in the labor market seems to delay childbearing, and later we find that this observation is not affected by potential endogeneity.

The variables listed in Table 1 are, by and large, the same as those used in the previous birth duration literature. In general, however, coresidence depends on the characteristics of both married children and their parents. Another weakness of the data is that they do not contain information on job or earnings at the time of marriage or first birth. Chu et al. (2011) reported that low-earning couples tend to coreside with parents. Moreover, low-earning couples may also pursue early childbearing. Therefore, it is possible that the observed association between coresidence and early childbearing is caused jointly by low socio-economic status.

Overall, the summary statistics suggest interesting relationships, but as we discussed previously, the effects of coresidence and working on the duration to childbearing may be confounded by other observable variables and unobservable interactions among family members. Therefore, in the next section, we develop a

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4 The data are divided to make the size of each subsample roughly equal.
5 See, for example, Takagi and Silverstein (2011). We do not use parents’ information because of a lack of data.
FIML estimator to shed more light on such an interrelationship to identify the relative importance of these factors behind an individual’s fertility behavior. We compare the FIML results with estimation results that are based on models that do not control for endogeneity.

**Empirical Strategy**

In this section, we develop an estimation strategy for the duration to first birth from first marriage, treating coresidence with husband’s parents and labor supply as endogenous.

### Table 1 Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Birth Cohort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration to First Birth</td>
<td>2.35 (1.70)</td>
</tr>
<tr>
<td>Coreside (= 1 if lived with parents-in-law right after marriage)</td>
<td>0.48 (0.50)</td>
</tr>
<tr>
<td>Work (= 1 if worked in the labor market right after marriage)</td>
<td>0.42 (0.49)</td>
</tr>
<tr>
<td>Wife’s Years of Education</td>
<td></td>
</tr>
<tr>
<td>0–9</td>
<td>0.76 (0.43)</td>
</tr>
<tr>
<td>10–12</td>
<td>0.13 (0.34)</td>
</tr>
<tr>
<td>13+</td>
<td>0.10 (0.30)</td>
</tr>
<tr>
<td>Wife’s Age at Marriage</td>
<td></td>
</tr>
<tr>
<td></td>
<td>22.27 (3.52)</td>
</tr>
<tr>
<td>Husband’s Years of Education</td>
<td></td>
</tr>
<tr>
<td>0–9</td>
<td>0.66 (0.48)</td>
</tr>
<tr>
<td>10–12</td>
<td>0.15 (0.36)</td>
</tr>
<tr>
<td>13+</td>
<td>0.18 (0.38)</td>
</tr>
<tr>
<td>Husband’s Age at Marriage</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25.88 (4.78)</td>
</tr>
<tr>
<td>Observations</td>
<td>905</td>
</tr>
</tbody>
</table>

*Note: Numbers in parentheses are standard deviations.*

### Table 2 Average duration to first birth, by coresidence with parents-in-law and labor supply status

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coreside</td>
</tr>
<tr>
<td>Work</td>
<td>2.42</td>
</tr>
<tr>
<td></td>
<td>[164]</td>
</tr>
<tr>
<td>No Work</td>
<td>2.29</td>
</tr>
<tr>
<td></td>
<td>[275]</td>
</tr>
</tbody>
</table>

*Notes: Numbers in brackets are the number of observations in each category. The total number of observations is 1,814.*
Modeling the Duration to First Birth

We employ a random variable $T$ to denote the duration to first birth from first marriage and let a scalar $t$ be the realization of $T$. The duration is determined by a set of observed economic and demographic variables, represented by a vector $\mathbf{x}_d$. These variables include endogenous binary variables, such as coresidence with parents-in-law and labor market status. The duration is also affected by unobserved heterogeneity, denoted as a scalar $\varepsilon$. If all the elements of vector $\mathbf{x}_d$ are exogenous, one may use standard parametric or nonparametric methods to estimate the duration model with heterogeneity.\(^6\) If some elements of vector $\mathbf{x}_d$ are correlated with $\varepsilon$, however, these methods result in inconsistent estimation. To highlight the distinction between exogenous and endogenous variables, we use $f(t|\mathbf{x}, d_1, d_2, \varepsilon)$ to represent the conditional probability density function of the duration dependent variable, $t$, given exogenous variables, $\mathbf{x}$ and two endogenous binary variables: coresidence, $d_1$, and labor supply, $d_2$.\(^7\)

To estimate the model consistently, we propose a FIML estimator for a duration model with multiple endogenous binary explanatory variables.\(^8\) We use a Weibull distribution, following the literature that models fertility duration. The associated hazard function is then given by

$$
\lambda(t|\mathbf{x}, d_1, d_2, \varepsilon) = p \left\{ \exp\left(-x^T \beta - c_1 d_1 - c_2 d_2 + \varepsilon\right) \right\}^p t^{p-1},
$$

where $p$ denotes the duration dependence of the Weibull distribution, and $c_1$ and $c_2$ represent the coefficients of the endogenous dummy variables, $d_1$ and $d_2$, respectively.\(^9\) We provide a detailed presentation of the FIML estimator in the appendix.

We specify the coresidence, $d_1$, and work, $d_2$, decisions by

$$
d_1 = \begin{cases} 1 & \text{if } z_1^T \alpha_1 + u_1 > 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad d_2 = \begin{cases} 1 & \text{if } z_2^T \alpha_2 + u_2 > 0 \\ 0 & \text{otherwise} \end{cases},
$$

where $z_1$ and $z_2$ are vectors of exogenous variables in determining the values of $d_1$ and $d_2$, respectively. In general, the $z$ variables include exogenous variables in $\mathbf{x}$ and other additional exogenous variables.\(^10\) An important assumption we make is that coresidence and work decisions are made at the time of marriage. Therefore, the childbearing decision does not appear in Eq. (2).\(^11\)

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\(^6\) See Hausman et al. (1984) and Heckman and Singer (1984) for details of these two estimation methods.
\(^7\) Our method can be easily extended to estimate a duration model with more than two endogenous dummy variables.
\(^8\) This FIML estimator is designed under the frequently adopted hazard framework and thus permits the occurrence of right-censoring. Olsen and Farkas (1989) considered the endogenous covariates in duration models via the regression methodology.
\(^9\) The notation used here follows exactly the one adopted in Greene (2000:941). If $p > 1$, then the hazard is monotonically increasing, and it is monotonically decreasing when $p < 1$. If $p = 1$, the hazard is a constant.
\(^10\) The model is identified by the Gaussian distributional assumption used for our FIML estimator, and exclusion restrictions based on the proposition in Heckman (1978:936) are not necessary.
\(^11\) In fact, this is the typical modeling framework adopted in Terza (1998) for other models of endogenous treatment effects.
Conditional on all the exogenous variables contained in $x$, $z_1$, and $z_2$, we assume that the joint distribution of $\varepsilon$, $u_1$, and $u_2$ is normal with a mean of zero and covariance matrix $\Sigma$; that is,

$$\begin{align*}
N(0, \Sigma) \equiv N \left( \begin{bmatrix}
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\sigma^2 & \rho_1 & \rho_2 \\
\rho_1 & 1 & \gamma \\
\rho_2 & \gamma & 1
\end{bmatrix} \right), \text{ and } -1 < \rho_1, \rho_2, \gamma < 1.
\end{align*} \tag{3}$$

To predict the signs of $\rho_1$, $\rho_2$, and $\gamma$, we need to understand what $\varepsilon$, $u_1$, and $u_2$ represent. First, $\varepsilon$ reflects a married woman’s preference for having a baby conditional on all exogenous variables in $x$, $z_1$, and $z_2$. Second, $u_1$ measures the degree of traditionalism conditional on observable demographic controls. Finally, $u_2$ reflects the heterogeneity in labor market productivity conditional on observable human capital variables.

Covariance Structure and Attitude Types

Here we discuss how the parameters in the error covariance structure can be linked to three attitude types regarding coresidence and working. The signs of $\rho_1$, $\rho_2$, and $\gamma$ in Eq. (3) are indicative of the attitudes of women in the sample. We use the estimation results to identify the three different attitudes of women in terms of coresidence with parents-in-law, labor supply, and duration to first birth. Table 3 summarizes the relationship between the attitude types and the correlation structure.

The sign of $\gamma$ is the key factor that identifies women’s attitudes toward coresidence and work choices. For example, a negative $\gamma$ implies that married women choose between coresiding and working. A positive $\gamma$ implies that they coreside and work at the same time or that they neither coreside nor work. Therefore, a negative $\gamma$ indicates traditional and career-driven types, and a positive $\gamma$ indicates the have-it-all type.

The correlation between the unobserved heterogeneity in the coresidence equation and the birth hazard equation is measured by $\rho_1$. A positive $\rho_1$ is consistent with traditional and have-it-all type women because the former reside with parents-in-law and give birth to preserve the family lineage, and the latter live with their husbands’ parents so as to share household work and childcare. If $d_1$ is instead assumed to be exogenous, then a positive $\rho_1$ (and a zero $\gamma$, for simplicity) implies that by not allowing for the endogeneity, the estimated effects of coresidence on birth duration will be biased toward increasing the hazard or shortening the time to first birth.

**Table 3** Interactions among the error components and the three preference types

<table>
<thead>
<tr>
<th>Type</th>
<th>Traditional</th>
<th>Have-It-All</th>
<th>Career-Driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$</td>
<td>+</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_2$</td>
<td></td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td></td>
<td></td>
<td>+</td>
</tr>
</tbody>
</table>
Similarly, $\rho_2$ measures the correlation between the unobserved heterogeneity in labor market status and the birth hazard. A negative $\rho_2$ is expected for the career-driven type because more-productive women in the labor market would also prefer a smaller birth hazard (or longer duration to first birth). A positive $\rho_2$ is associated with the have-it-all type given that these women tend to work in the labor market and prefer having children earlier. If $\rho_2$ is positive (and $\gamma$ is zero, again for simplicity), not accounting for the endogeneity of participation would overstate the hazard or understate the duration to first birth.

Conditional on observable demographic controls, traditional women tend to live with husbands’ parents and give birth earlier ($\rho_1 > 0$). They also prefer not working ($\gamma < 0$). The sign of $\rho_2$ is uncertain because labor market productivity is not a defining characteristic for this type of women. For example, a traditional woman with a high $u_2$ may participate in the labor force, but she still tries to have a baby as early as possible.

Career-driven women aim for career success. It is reasonable to assume that the larger the error term is in the labor supply decision, the more likely a woman is to pursue career success and therefore the more likely she is to delay the timing of first birth ($\rho_2 < 0$). These women also prefer living apart from their husbands’ parents ($\gamma < 0$). The sign of $\rho_1$ is uncertain. Even if a career woman lives with her husband’s parents because of a high $u_1$, her first priority is always career success, and she will delay childbearing.

Women of the have-it-all type aim to balance traditionalism and career success. Therefore, the higher her degree of traditionalism, the earlier a have-it-all type of woman gives birth ($\rho_1 > 0$). Moreover, the higher her labor market productivity, the earlier she gives birth ($\rho_2 > 0$) because she can reduce her input into her career and spend more time in childrearing. Finally, $\gamma$ is positive because these women both coreside and work.

**Coresidence, Work, and the Duration to First Birth**

In this section, we evaluate the effects of coresidence with parents-in-law and labor supply status on the timing of first birth using the preceding FIML estimator. For comparison purposes, we first consider the case in which coresidence and work decisions are assumed to be exogenous, as in previous studies, and then the case in which those decisions are treated as being made endogenously.

**Estimation of the Duration Model, Treating Coresidence and Work Decisions as Exogenous**

In Table 4, we first present the estimation results for older and younger cohorts of women, not accounting for the endogeneity of coresidency and working. Each equation in (2) is estimated independently using a probit model. The probit results for the older cohort are in the first two columns, and those for the younger cohort are in the fourth and fifth columns. We investigate the duration to first birth in Eq. (1) under the Weibull distribution assumption, treating the coresidence and work decisions as exogenous. We exclude the unobserved heterogeneity, $\varepsilon$, to make Eq. (1) a standard duration model. The hazard model estimates for the older cohort of women are in the third column, and those for the younger cohort are in the last column.
In the first and fourth columns of Table 4, we find that couples with higher levels of human capital, as measured by age and education at the time of marriage, are less likely to live with the husband’s parents. Specifically, the husband’s age at the time of marriage is negatively associated with coresidence with his parents for both older and the younger cohort of women. We also find that the older cohort of women married to highly educated men are less likely to coreside with their husband’s parents, but own education level is more important for the younger cohort of women for not living with parents-in-law. This pattern seems quite reasonable given that the wife’s role as a major decision maker in the household has been expanding for younger generations.

The second and fifth columns of Table 4 display the probit results concerning the choice of labor supply. As is predicted by theory, labor supply is an increasing
function of human capital. For both cohorts, we find that the higher her educational attainment or her age at the time of marriage, the more likely she is to be in the labor market. We also find that a married woman is more likely to participate in the market if her husband attains the highest level of education. Moreover, the effect of the husband’s educational attainment in increasing his wife’s labor force participation is more pronounced for younger women. Because education may also enlighten the egalitarian perception of the husband, the positive coefficient and its changing pattern may represent this attitude change.

The third and last columns of Table 4 indicate that married women participating in the labor market significantly delay their timing of first birth. Because of the Weibull distribution assumption, we can interpret the coefficient estimates as semi-elasticities of the covariates on the expected duration. Working in the labor market seems to lengthen the duration to first birth. The duration for working women compared with nonworking women is 12.0 % ($= e^{0.113} - 1$) longer for the older cohort and 23.6 % ($= e^{0.212} - 1$) longer for the younger cohort. If these estimates are applied to the average duration to first birth, then working women in the older cohort delay the first birth by around 3.4 months ($= 12 \% \times 2.353 \times 12$), and working women in the younger cohort do so by around 7.0 months ($= 23.6 \% \times 2.484 \times 12$). These results are consistent with the findings of Heckman et al. (1985), who did not take into account the endogeneity of co-residence and labor supply.

Coresidence does not seem to have a statistically significant effect on the duration to first birth for the older cohort of women, but for the younger cohort, coresiding women have a shorter duration by 18.9 % ($= e^{-0.209} - 1$), or about 5.6 months ($= 18.9 \% \times 2.484 \times 12$). This phenomenon is consistent with the findings in the literature (see, e.g., Tsay and Chu 2005). However, as discussed earlier, the causal effect of coresidence and working on fertility behavior cannot be understood clearly without considering the endogeneity of these two binary variables in the duration model. We thus postpone all discussions about the influence of endogenous and exogenous variables on the timing of first birth until we control the endogeneity of these two binary variables.

Endogenous Treatment Effects of Coresidence and Work on the Timing of First Birth

We next consider the case in which both coresidence with husband’s parents and labor supply decisions are allowed to be endogenous. For these purposes, we employ the FIML estimator discussed earlier. Table 5 presents the results. The chi-square statistic for the test of $\rho_1 = \rho_2 = \gamma = \sigma = 0$ suggests that the FIML estimation is the preferred method for both cohorts.

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12 The chi-square statistic for the test of $\rho_1 = \rho_2 = \gamma = \sigma = 0$ suggests that the FIML estimation is the preferred method for both cohorts.
<table>
<thead>
<tr>
<th>Variables</th>
<th>Older Cohort</th>
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<th>Younger Cohort</th>
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<tr>
<td></td>
<td>Coreside</td>
<td>Work Duration</td>
<td>Coreside</td>
<td>Work Duration</td>
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<td>0.083**</td>
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<td>(0.74)</td>
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<td>Completed Spell</td>
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</table>

Notes: Older cohort women are born during 1933–1954. Younger cohort women are born during 1955–1968. Numbers in parentheses are the absolute value of t statistics. The chi-square statistic is for the likelihood ratio test of $\rho_1 = \rho_2 = \gamma = \sigma = 0$, and the corresponding p value is 0 for both cohorts.

† p < .10; *p < .05; **p < .01 (two-tailed tests)
coresidence indeed postpones a woman’s first birth is in sharp contrast with results presented in Table 4. This suggests that in Table 4, coresidence mistakenly reflects the behavioral pattern of traditional women because of the endogeneity of coresidence in the duration model.\textsuperscript{13} It also implies that the seemingly positive effect of coresidence on accelerating childbearing is explained by traditional women. Specifically, the majority of women who prefer coresiding also happen to prefer early childbearing.

The unobservables behind the remaining heterogeneity $\varepsilon$ in Eq. (1) and $u_1$ and $u_2$ in Eq. (2) are correlated, supporting the approach that coresidence with parents-in-law and labor supply decisions cannot be taken as exogenous variables in the analysis of birth duration.\textsuperscript{14} In the left panel of Table 5, a positive $\rho_1$ and a negative $\gamma$ indicate that the majority of women in the older cohort are the traditional type. The estimates of $\rho_1$, $\rho_2$, and $\gamma$ suggest a substantial change in the proportion of the three types of women. A positive and significant $\rho_1$ becomes even larger for the younger cohort of women. The sign of $\rho_2$ changes from positive to negative, although neither estimate is statistically significant. A negative and significant $\gamma$ becomes positive and insignificant for the younger cohort. These changes can be explained by the increasing share of women of the career-driven and have-it-all types in recent years. An increasing share of women who are career-driven contributes to negative $\rho_2$ and $\gamma$. These negative effects are offset by positive $\rho_2$ and $\gamma$ from women of the have-it-all type, making the estimate of $\gamma$ not statistically different from 0. Women of the have-it-all type also contribute to a more positive $\rho_1$, making the estimate of $\rho_1$ more positive and significant. This result is also consistent with the common observation that the ratio of women with a strong career attachment should not be widespread for older Taiwanese women, who faced an economy only just beginning to modernize at that time.\textsuperscript{15}

The answer to why coresidence delays childbearing is beyond the scope of this study, but we provide some conjecture. Previous research by Freedman et al. (1982) explains the possible influence of parents’ coresidence on a married woman’s fertility by some unobservable family interactions, such as parents’ pressure related to lineage preservation. Moreover, coresidence itself can increase the mental stress for a married woman because she must coordinate with her parents-in-law in the arrangement of daily family life, even though they may help her with the household work. We also note that mental stress is often proposed as a cause of unexplained infertility and has been investigated in experiments.\textsuperscript{16} Therefore, we suggest that the effect of

\textsuperscript{13} The changing pattern of the sign of coresidence in Tables 4 and 5 can also be explained by the arguments in the previous section, where we show that the effects of coresidence on birth duration are biased toward shortening the time to first birth given that $\rho_1 > 0$ and $\gamma = 0$ are imposed.

\textsuperscript{14} We conduct likelihood ratio tests of $\rho_1 = \rho_2 = 0$ to justify that coresidence and work are endogeneously determined. The resulting chi-square statistic is 3.548 ($p$ value = .169) for older cohorts and 49.614 ($p$ value = 0) for younger cohorts. We also test whether $\gamma = 0$, and the two correlation coefficients are jointly zero. The chi-square statistic is 7.294 ($p$ value = .063) for the older cohort of women and 50.122 ($p$ value = 0) for the younger cohort of women. These results suggest that coresidence and work are endogeneously determined, especially for the younger cohort. Detailed estimation results under these restrictions are available upon request.

\textsuperscript{15} Similar findings have been reported in studies of other countries. Ruggles (2007), for example, documented that the percentage of persons age 65 or older residing with their adult children dropped from almost 70 % in the 1850s to fewer than 15 % in the 2000s in the United States. He concluded that more education, greater earning power, and declining parental control over children are the main reasons for this decline.

\textsuperscript{16} Among them, Sanders and Bruce (1997) found that psychosocial stress influences fertility in females, and Hjollund et al. (1999) conducted a follow-up study of time to first pregnancy and documented that psychological distress may be a risk factor for reduced fertility.
coresidence on the timing of birth is through the increase of mental stress, subsequently reducing the woman’s chance of conception.

Another possible explanation is that younger married women have more bargaining power in their fertility decisions. The intrahousehold bargaining literature often uses the level of education or the education ratio between husbands and wives as a proxy variable for bargaining power. This explanation is consistent with the fact that women in the younger cohort are more educated than those in the older cohort as well as our findings on the changing proportions of the three types of women between cohorts.

Table 5 contains other interesting results for the duration model estimates. We first observe that the hazard of giving birth is increasing monotonically because the estimate of $p$ is greater than unity. Educational attainment of wife and husband does not play any significant role in determining the duration of the first birth for the older cohort, but it increases the duration for the younger cohort. Table 5 also shows that among the older cohort of women, age at the time of marriage significantly accelerates the timing of birth, but this biological characteristic does not significantly affect the duration to birth for younger women. The results concerning the choices of coresidence and working are more or less qualitatively similar to those found in Table 4.

Our results in Table 5 are robust. In this table, we exclude husband’s age at the time of marriage from the duration equation, but our equations without this exclusion restriction are still identified given that $\varepsilon$, $u_1$, and $u_2$ in Eq. (3) have a trivariate normal distribution as suggested by Heckman (1978:936). To demonstrate this fact, we adjust the duration model to include husband’s age at marriage so that the exclusion restriction is not imposed. The results are robust, and the only qualitative change is that the estimates of $\rho$ in both estimations become insignificant. We also add controls for birthplace into the model because the transmission of human capital is especially important during early childhood. The results are again robust. We do not report these estimates, but details can be provided by the authors upon request.

Conclusion

This article integrates three interrelated strands of literature by introducing a FIML estimator to evaluate the endogenous treatment effects of coresidence with husband’s parents and female labor supply on the timing of first birth. Our first set of results, which does not account for endogeneity of coresidence and work decisions, shows

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17 See Behrman (1997) or Fafchamps and Quisumbing (2007) for an overview of the literature on household bargaining.
18 When exclusionary restrictions have little explanatory power, the information from distributional or functional form assumptions may still drive the results. However, we find that the excluded variable possesses substantial explanatory power. The coefficient on husband’s age at the time of marriage is significant in the coresidence equations for both older and younger cohorts, further supporting our findings of coresidence delaying the duration to first birth.
19 A woman’s birth neighborhood characteristics are included according to the 2003 population for each of Taiwan’s 366 zip codes. The data about the population of each zip code are from the Department of Household Registration Affairs, Ministry of the Interior, 2004 (http://www.moi.gov.tw).
that working delays the timing of first birth significantly. The results are robust to controlling for the endogeneity, and these findings are consistent with those in former studies. Our second set of results, which controls for the endogeneity, reverses the findings of previous literature: coresiding with the husband’s parents postpones the timing of first birth after we control the endogeneity of coresidence status. Because individual characteristics determining whether to coreside with parents-in-law are correlated with those determining when to have a first birth, failing to control for such an endogeneity may distort the regression coefficient, as we show in our analysis.

We also identify three possible preferences behind the aforementioned decision correlations: a married woman may live according to traditional values, may be career-driven and care for career success, or may aim to “have it all” such that she tries to balance family and a career. Each attitude identifies a distinct pattern of correlation among error terms. Our hypothesis is that along with Taiwan’s economic development, if women’s attitudes have changed from the traditional type to the career-driven and have-it-all types between the older and younger cohorts, this transition should be reflected in the correlation among error terms from regressions for each cohort. By separating our samples into younger and older cohorts, we find that the data do support our hypothesis. We also try various sensitivity analyses by changing the threshold age of fertility, removing exclusion conditions, and controlling for the wife’s birth neighborhood effects, and we find that all results hold.

Acknowledgment We thank Joelle Abramowitz, Shelly Lundberg, and Edward Vytlacil for constructive comments on an earlier stage of this article. We also thank three anonymous referees and seminar participants at the annual meeting of the Population Association of America, University of Seoul, and University of Washington for helpful discussions. Research assistance from Shih-Tang Hwu and Yi-Fen Yang is gratefully appreciated.

Appendix

For ease of exposition, hereafter we represent all the exogenous variables contained in \( \mathbf{x} \), \( \mathbf{z}_1 \), and \( \mathbf{z}_2 \) as \( \mathbf{w} \), where \( \mathbf{z}_1 \) and \( \mathbf{z}_2 \) are \( (k_1 \times 1) \) and \( (k_2 \times 1) \) vectors of exogenous variables in determining the values of \( d_1 \) and \( d_2 \), respectively. With the independently and identically distributed observations \( (t_i, d_{1i}, d_{2i} | w_i), i = 1, \ldots, N \), the log-likelihood function consists of four parts:

\[
\ln L = \sum_{i=1}^{N} d_{1i} d_{2i} \ln f \left( t_i, d_{1i} = 1, d_{2i} = 1 | w_i \right) \\
+ \sum_{i=1}^{N} d_{1i} (1 - d_{2i}) \ln f \left( t_i, d_{1i} = 1, d_{2i} = 0 | w_i \right) \\
+ \sum_{i=1}^{N} (1 - d_{1i}) d_{2i} \ln f \left( t_i, d_{1i} = 0, d_{2i} = 1 | w_i \right) \\
+ \sum_{i=1}^{N} (1 - d_{1i})(1 - d_{2i}) \ln f \left( t_i, d_{1i} = 0, d_{2i} = 0 | w_i \right). 
\]
Given that \( f(t \mid x, d_1, d_2, \varepsilon) \) is a Weibull distribution, we can show that Eq. (4) implies the following log-likelihood function

\[
\ln L = \sum_{i=1}^{N} d_{i1} d_{2i} \ln \left\{ \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(t \mid w_{i1}, d_{i1} = 1, d_{i2} = 1, \sqrt{2\sigma^2}\xi) \Phi_2^*(h_i, k_i, \rho) \exp(-\xi^2) d\xi \right\} \\
+ \sum_{i=1}^{N} d_{i1} (1 - d_{i2}) \\
\times \ln \left\{ \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(t \mid w_{i1}, d_{i1} = 0, d_{i2} = 1, \sqrt{2\sigma^2}\xi) \Phi_2^*(-h_i, -k_i, -\rho) \exp(-\xi^2) d\xi \right\} \\
+ \sum_{i=1}^{N} (1 - d_{i1}) (1 - d_{i2}) \\
\times \ln \left\{ \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(t \mid w_{i1}, d_{i1} = 0, d_{i2} = 0, \sqrt{2\sigma^2}\xi) \Phi_2^*(-h_i, -k_i, \rho) \exp(-\xi^2) d\xi \right\},
\]

where

\[
f(t \mid x, d_1, d_2, \sqrt{2\sigma^2}\xi) = \exp(-x_t^T \beta + \sqrt{2\sigma^2}\xi) p \left[ \exp(-x_t^T \beta + \sqrt{2\sigma^2}\xi) t \right]^{p-1} \\
\times \exp\left( -\left[ \exp(-x_t^T \beta + \sqrt{2\sigma^2}\xi) t \right] \right)^p,
\]

and \( \Phi_2^* \) denotes the bivariate standard normal cumulative distribution function with correlation coefficient \( \rho = \left( \gamma - \rho_1 \rho_2 \right) / \sqrt{(1 - \rho_1^2)(1 - \rho_2^2)} \).

After applying the Hermite quadrature to the results in Eqs. (5), (6), and (7), we can evaluate the log-likelihood function to estimate the parameters of interest.\(^{20}\) Accordingly, the Gaussian error distribution assumption allows us to parametrically solve the estimation problems of the duration model with endogenous binary switching variables. Using the “textbook” selection model framework adopted by Heckman and Vytlacil (2000), we consider our FIML estimator as the starting point to evaluate the endogenous treatment effects under a hazard framework.

\(^{20}\) See Butler and Moffitt (1982) about the implementation of the Hermite quadrature.
To evaluate the first term of the right side of Eq. (4), first note that it can be represented as

\[ f(t_i, d_{i1} = 1, d_{i2} = 1|w_i) = \int_{-\infty}^{\infty} f(t_i, d_{i1} = 1, d_{i2} = 1|w_i, \varepsilon_i) f(\varepsilon_i|w_i) d\varepsilon_i, \]  

where \( f(\varepsilon_i|w_i) \) is the conditional density function of \( \varepsilon_i \) given the exogenous variables. By Eq. (3), conditional on the exogenous variables \( w_i \), we also note that the conditional distribution of \((u_{1i}, u_{2i})^T\), given \( \varepsilon_i \) is normally distributed with mean \( \mu^*_i \) and variance matrix \( \Sigma^* \), is

\[ N(\mu^*_i, \Sigma^*) \equiv N \begin{bmatrix} \frac{\rho_{1\varepsilon_i}}{\sigma} \\ \frac{\rho_{2\varepsilon_i}}{\sigma} \end{bmatrix}, \begin{bmatrix} 1 - \rho^2_{1} & \gamma - \rho_1\rho_2 \\ \gamma - \rho_1\rho_2 & 1 - \rho^2_2 \end{bmatrix}. \]

This implies that

\[ f(t_i, d_{i1} = 1, d_{i2} = 1|w_i, \varepsilon_i) = \text{Prob}\left( z_{i1}^T \alpha_1 + u_{i1} > 0, z_{i2}^T \alpha_2 + u_{i2} > 0|w_i, \varepsilon_i \right) \]

\[ = \Phi_2 \left( \frac{z_{i1}^T \alpha_1 + \frac{\rho_{1\varepsilon_i}}{\sigma}}{\sqrt{1 - \rho^2_{1}}}, \frac{z_{i2}^T \alpha_2 + \frac{\rho_{2\varepsilon_i}}{\sigma}}{\sqrt{1 - \rho^2_{2}}}, \frac{\gamma - \rho_1\rho_2}{\sqrt{(1 - \rho^2_{1})(1 - \rho^2_{2})}} \right) \equiv \Phi_2 \left( h_i, k_i, \rho \right), \]

where \( \Phi_2 \) denotes the bivariate standard normal cumulative distribution function with correlation coefficient \( \rho = \sqrt{(1 - \rho^2_{1})(1 - \rho^2_{2})}. \)

Because

\[ f(t_i, d_{i1} = 1, d_{i2} = 1|w_i, \varepsilon_i) \]

\[ = f(t_i|w_i, d_{i1} = 1, d_{i2} = 1, \varepsilon_i) \times f(d_{i1} = 1, d_{i2} = 1|w_i, \varepsilon_i), \]

using Eqs. (8), (10), and (11), we can show that

\[ f(t_i, d_{i1} = 1, d_{i2} = 1|w_i) \]

\[ = \int_{-\infty}^{\infty} f(t_i|w_i, d_{i1} = 1, d_{i2} = 1, \varepsilon_i) \Phi_2 \left( h_i, k_i, \rho \right) f(\varepsilon_i|w_i) d\varepsilon_i. \]
Following the preceding methodology, we evaluate the other three terms of the right side of Eq. (4) and obtain

\[
\ln L = \sum_{i=1}^{N} d_i d_{2i} \ln \left\{ \int_{-\infty}^{\infty} f(t_i | w_i, d_{1i}, 1, d_{2i}, 1, \varepsilon_i) \Phi_2 \left( h_i, k_i, \rho \right) f(\varepsilon_i | w_i) d\varepsilon_i \right\} \\
+ \sum_{i=1}^{N} d_i \left( 1 - d_{2i} \right) \ln \left\{ \int_{-\infty}^{\infty} f(t_i | w_i, d_{1i}, 1, d_{2i}, 0, \varepsilon_i) \Phi_2 \left( h_i, -k_i, -\rho \right) f(\varepsilon_i | w_i) d\varepsilon_i \right\} \\
+ \sum_{i=1}^{N} \left( 1 - d_{i} \right) d_{2i} \ln \left\{ \int_{-\infty}^{\infty} f(t_i | w_i, d_{1i}, 0, d_{2i}, 1, \varepsilon_i) \Phi_2 \left( -h_i, k_i, -\rho \right) f(\varepsilon_i | w_i) d\varepsilon_i \right\} \\
+ \sum_{i=1}^{N} \left( 1 - d_{i} \right) \left( 1 - d_{2i} \right) \times \ln \left\{ \int_{-\infty}^{\infty} f(t_i | w_i, d_{1i}, 0, d_{2i}, 0, \varepsilon_i) \Phi_2 \left( -h_i, -k_i, \rho \right) f(\varepsilon_i | w_i) d\varepsilon_i \right\}.
\]

(13)

Based on the independent and identically distributed (iid) assumption of \(\varepsilon_i\), we apply the change of variable, \(\varepsilon_i = \sqrt{2\sigma}\xi\), to derive the results in Eqs. (5) and (7).

Finally, combining the assumption that \(f(t_i | w_i, d_{1i}, d_{2i}, \varepsilon_i)\) is a Weibull distribution

\[
f(t_i | w_i, d_{1i}, d_{2i}, \varepsilon_i) = \lambda_i \rho \left( \lambda t_i \right)^{\rho - 1} \exp \left( -\lambda t_i^\rho \right) \\
= \exp \left( -x_i^T \beta + \varepsilon_i \right) \left[ \exp \left( -x_i^T \beta + \varepsilon_i \right) \right]^{\rho - 1} \exp \left( - \left[ \exp \left( -x_i^T \beta + \varepsilon_i \right) \right]^\rho \right),
\]

(14)

and using the change of variable again, we obtain the value of \(f(t_i \mid x_i, d_{1i}, d_{2i}, \sqrt{2\sigma}\xi)\) in Eq. (6).

References


