

Analysing Inflation by the ARFIMA Model with Markov-Switching Fractional Differencing Parameter

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Abstract

Using an ARFIMA model made of an Markov-switching fractional differencing parameter, we find that sudden oil price shocks are important in shaping the paths of U.S. inflation. The estimation results also support that U.S. inflation is a mean-reverting long memory process.

Key words: Markov chain; MS-ARFIMA process; Long memory

JEL classification: E31

1 Introduction

This paper considers the time series properties of the aggregate price level which is well known as one of the most important variables in explaining the macroeconomy. Many studies have been devoted to investigate how aggregate prices respond to shocks. Nelson and Schwert (1977), Barsky (1987), and Ball and Cecchetti (1990) find that inflation contains a unit root. Hassler and Wolters (1995) and Baillie et al. (1996) by contrast show that there exists a mean-reverting long memory in the inflation rates of G7 countries. Because level shifts are likely for inflation, Bos et al. (1999) further examine whether evidence for long memory in

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the inflation of G7 countries is spurious or exaggerated due to the presence of level shifts. Their testing results reveal that allowing for level shifts has a huge effect on the degrees of fractional integration, because a considerable lower degree of fractional integration is found as compared to the no break case.

Bos et al. (1999) point out that a drawback inherent in their methodology is that the break points are exogenously fixed. We aim to remedy this shortcoming by allowing the number and timing of the break points of inflation to be determined endogenously. Another notable contribution of this paper is to address the possibility that fractional integration of inflation is likely to change under different regimes, too. For example, sudden oil price shocks might change the persistency of inflation, because the public might possess different inflation expectation under a higher oil price regime, or the central bank of each country might try very hard to control the potential negative impacts of oil price shocks on the macroeconomy and thus affect the time series properties of the inflation rates. We thus apply the Markov-switching autoregressive fractionally integrated moving average (MS-ARFIMA) model of Tsay and Härdle (2008) to re-examine the long memory properties of the U.S. price deflator, because this MS-ARFIMA model can estimate the order of fractional integration of inflation which might be subject to level shifts and persistency changes simultaneously.

2 MS-ARFIMA model

Before presenting the MS-ARFIMA model, let us illustrate the definition of an Markov chain and that of an ARFIMA process. Let $\{s_t\}_{t=1}^T$ be the latent sample path of an N -state Markov chain. Each time s_t can assume only an integer value of $1, 2, \dots, N$, and its transition probability matrix is:

$$\mathcal{P} \equiv \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{N1} \\ p_{12} & p_{22} & \cdots & p_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1N} & p_{2N} & \cdots & p_{NN} \end{bmatrix},$$

where $p_{ij} = P(s_t = j \mid s_{t-1} = i)$ and $\sum_{j=1}^N p_{ij} = 1$ for all i .

An $I(d)$ process, x_t , is defined as:

$$(1 - L)^d x_t = h_t,$$

where L is the lag operator ($Lk_t = k_{t-1}$) and h_t is a short memory process. When $d > 0$, the $I(d)$ process is often called the long memory process, because its autocovariance function is not summable so as to capture the long range dependence of a time series. In addition, the $I(d)$ process is nonstationary when $d \geq 1/2$, otherwise, it is covariance stationary. Note that the $I(d)$ process is mean-reverting when $d < 1$.

The MS-ARFIMA model of Tsay and Härdle (2008) offers a rich dynamic mixture of an Markov chain and an $I(d)$ process and is expressed as:

$$w_t = \mu_{s_t} I\{t \geq 1\} + (1 - L)^{-d_{s_t}} \sigma_{s_t} z_t I\{t \geq 1\} = \mu_{s_t} I\{t \geq 1\} + y_t, \quad (1)$$

where w_t is the observed inflation rate at time t , $I\{\cdot\}$ is the indicator function, and z_t is a stationary ARMA process with mean zero and bounded positive spectral density $f_z(\lambda) \sim G_z$ as $\lambda \rightarrow 0$ at each possible regime. The role of $I\{\cdot\}$ is to truncate the influence of the infinite past observations of z_t on w_t , because we allow d_{s_t} to be greater than or equal to $1/2$. The most innovative characteristic of the process in (1) is that the fractional differencing parameter d_{s_t} well known in the long memory literature is allowed to be a Markov chain satisfying the following Assumption A.

Assumption A. s_t is independent of z_τ for all t and τ .

When $N = 1$, the model in (1) reduces to be the ARFIMA process introduced in Granger (1980), Granger and Joyeux (1980), and Hosking (1981). The model in (1) cannot be estimated with the recursive algorithm in Hamilton (1989), because the possible routes of states running from time 1 to time T expand exponentially to be N^T if we want to extract z_t to conduct the maximum likelihood estimation (MLE). In addition, this model cannot be written in a state-space form due to the presence of a fractional differencing parameter, implying that we cannot apply the EM algorithm considered in Hamilton (1990) for the model in (1), because the non-Markovian nature of the model prevents us from using the results in (4.2) of Hamilton (1990).

It is not difficult to write down the conditional likelihood function of the mixture model in (1) in terms of the ARMA process z_t , provided that we can exactly identify the true path of s_t along with some suitable assumptions about the initial values, distribution, and model specification of z_t . We surely do not in reality observe the true path of the latent state variable. Nevertheless, we can implement the Viterbi (1967) algorithm well-known in

the digital communication literature to identify the most likely path of states among the N^T possible routes within the inflation data. Essentially, the original Viterbi algorithm is the standard forward dynamic programming solution to maximum-likelihood decoding of a discrete-time, finite-state dynamic system observed in white noise as documented in Omura (1969). In this paper we apply the Viterbi algorithm to the finite-state dynamic system observed in a more general ARFIMA noise with a Markov-switching fractional differencing parameter. See Forney (1973) or Tsay and Härdle (2008) about the implementations of the Viterbi algorithm.

3 Inflation with Markov-switching persistency

We estimate the U.S. inflation rates with the following 2-state MS-ARFIMA(1, d , 1) model:

$$w_t = \mu_{s_t} I\{t \geq 1\} + (1 - L)^{-d_{s_t}} \sigma_{s_t} z_t I\{t \geq 1\}, \quad (1 - \phi_1 L) z_t = (1 + \theta_1 L) \varepsilon_t, \quad (2)$$

where ϕ_1 or θ_1 is assumed to be zero depending on the noise specification, and ε_t is standard normally distributed. The case of $\phi_1 = \theta_1 = 0$ has been adopted by Tsay and Härdle (2008) in characterizing Nile River data. The quarterly U.S. inflation data are calculated from the seasonally-adjusted implicit price deflator of the nonfarm business sector for the period 1947 I until 2007 IV. Following Hamilton (1989), asymptotic standard errors are calculated numerically. These calculated standard errors may not be valid, because the Viterbi algorithm is not equivalent to the usual full complete-data log-likelihood estimation. Nevertheless, the Viterbi algorithm provides a convenient way to identify the most likely regime path beneath the observed data under the model in (2).

Table 1 shows that the estimates of μ_1 , μ_2 , p_{11} , p_{22} , σ_1 , and σ_2 are quite robust across all four different specifications. Interestingly, we identify 5 *identical* breakpoints at the time of 1951 II, 1973 IV, 1975 II, 1979 II, and 1981 II from both the MS-ARFIMA(0, d , 1) and the MS-ARFIMA(1, d , 1) models. These two specifications actually are the ones producing the highest likelihood functions among the four specifications considered in Table 1. Figure 1 illustrates the fitting performance of the MS-ARFIMA(1, d , 1) model and clearly shows the great ability of the MS-ARFIMA model in capturing the historical record of the U.S. inflation rates.

The estimates of σ_1 and σ_2 in Table 1 show that the volatility of inflation is higher in the

higher inflation regime, revealing that the uncertainty of inflation is relatively larger when the economy is facing a higher inflation regime. This result correctly reflects the changing pattern of observed inflation in Figure 1. For all configurations considered in Table 1, we also observe that the values of p_{11} and d_1 are larger than those of p_{22} and d_2 , respectively. This finding reveals the probability that the state will move to the other regime is higher, and the weight of current and previous shocks, ε_{t-j} ($j = 0, 1, \dots$), on current inflation is lower, when the current state is in the high inflation regime as compared to the lower inflation counterpart. These observations are reasonable, because the occurrence of oil shocks are not regular, though perhaps they are closely related to the historical events of the 1973-74 OPEC embargo and the 1979 Iranian revolution, respectively, as clearly demonstrated in Figure 2 concerning the estimated path of μ_{st} from the MS-ARFIMA(1, d , 1) model. Therefore, our findings might contribute to the literature concerning the relationship between oil shocks and recession considered in Rasche and Tatom (1981), Hamilton (1983, 1985), Burbidge and Harrison (1984), among others.

Another important finding of this paper is that the fractional integration of U.S. inflation is within the range of 0.3307-0.6350 based on the eight estimates in Table 1. This indicates that the U.S. inflation rate is a mean-reverting long memory process as documented in Hassler and Wolters (1995), Baillie et al. (1996), and Bos et al. (1999), even though the the data-generating process is relaxed to allow for level shifts or persistency changes simultaneously.

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**Table 1. Estimates of Parameters based on Data for
U.S. Quarterly Inflation Rate**

	ARFIMA(0, d , 0)		ARFIMA(0, d , 1)		ARFIMA(1, d , 0)		ARFIMA(1, d , 1)	
	Estimate	S.E.	Estimate	S.E.	Estimate	S.E.	Estimate	S.E.
μ_1	0.6302	0.2531	0.5082	0.4740	0.7818	0.3991	0.5423	0.4668
μ_2	1.8784	0.2526	1.6541	0.5103	1.9834	0.3294	1.6778	0.4946
p_{11}	0.9822	0.0088	0.9866	0.0077	0.9822	0.0088	0.9866	0.0077
p_{22}	0.8707	0.0673	0.9261	0.0472	0.8706	0.0674	0.9261	0.0472
σ_1	0.2884	0.0138	0.2825	0.0137	0.2848	0.0137	0.2819	0.0137
σ_2	1.0838	0.1552	1.1095	0.1463	1.0834	0.1589	1.1006	0.1452
d_1	0.4687	0.0424	0.6350	0.0939	0.5808	0.0599	0.6261	0.0851
d_2	0.3307	0.1075	0.6231	0.0990	0.4754	0.1069	0.6092	0.0912
ϕ_1	-	-	-	-	-0.2257	0.0816	-0.2522	0.1791
θ_1	-	-	-0.2481	0.1052	-	-	-0.0129	0.2075
L^*	-104.9116		-101.3111		-101.5249		-100.5656	

Notes: The results are based on the MS-ARFIMA model defined in (2). S.E. stands for the standard error of the estimate. L^* represents the log-likelihood function of the switching model.

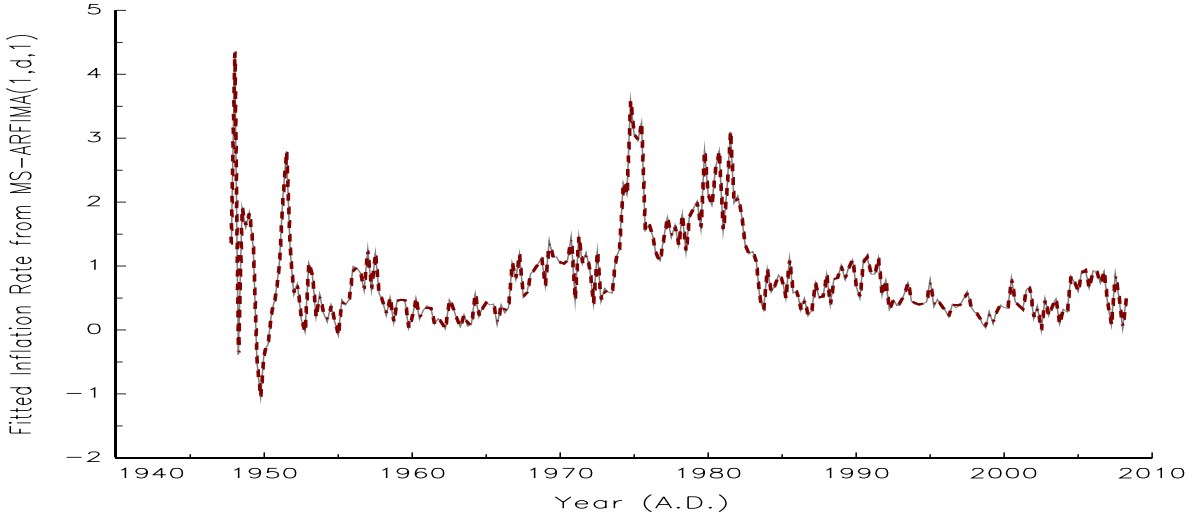


Figure 1: Solid line denotes the quarterly U.S. inflation rate, while dotted line represents the corresponding fitted values from the MS-ARFIMA(1, d , 1) model presented in Table 1.

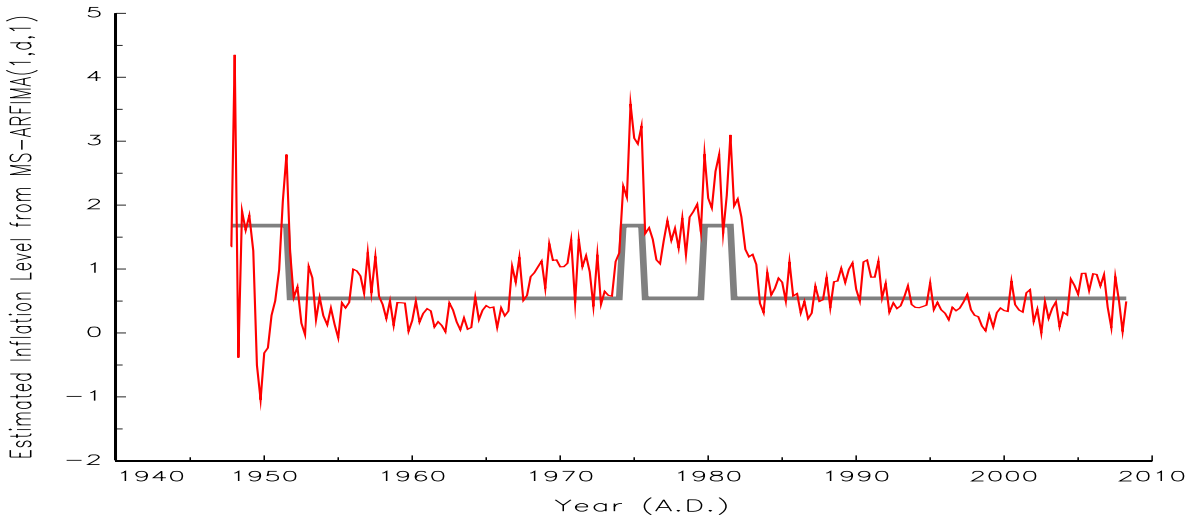


Figure 2: The observed quarterly U.S. inflation rates and the corresponding estimated inflation level $\hat{\mu}_{st}$ from the MS-ARFIMA(1, d , 1) model as presented in Table 1.