# Analysing Inflation by the ARFIMA Model with Markov-Switching Fractional Differencing Parameter

Wen-Jen Tsay *

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The Institute of Economics, Academia Sinica, Taiwan

**Abstract**

Using an ARFIMA model made of an Markov-switching fractional differencing parameter, we find that sudden oil price shocks are important in shaping the paths of U.S. inflation. The estimation results also support that U.S. inflation is a mean-reverting long memory process.

*Key words*: Markov chain; MS-ARFIMA process; Long memory

*JEL classification*: E31

## 1 Introduction

This paper considers the time series properties of the aggregate price level which is well known as one of the most important variables in explaining the macroeconomy. Many studies have been devoted to investigate how aggregate prices respond to shocks. Nelson and Schwert (1977), Barsky (1987), and Ball and Cecchetti (1990) find that inflation contains a unit root. Hassler and Wolters (1995) and Baillie et al. (1996) by contrast show that there exists a mean-reverting long memory in the inflation rates of G7 countries. Because level shifts are likely for inflation, Bos et al. (1999) further examine whether evidence for long memory in

*Corresponding author. The Institute of Economics, Academia Sinica, Taipei, Taiwan, R.O.C. Tel.: (886-2) 2782-2791 ext. 296. Fax: (886-2) 2785-3946. E-Mail: wtsay@econ.sinica.edu.tw

the inflation of G7 countries is spurious or exaggerated due to the presence of level shifts. Their testing results reveal that allowing for level shifts has a huge effect on the degrees of fractional integration, because a considerable lower degree of fractional integration is found as compared to the no break case.

Bos et al. (1999) point out that a drawback inherent in their methodology is that the break points are exogenously fixed. We aim to remedy this shortcoming by allowing the number and timing of the break points of inflation to be determined endogenously. Another notable contribution of this paper is to address the possibility that fractional integration of inflation is likely to change under different regimes, too. For example, sudden oil price shocks might change the persistency of inflation, because the public might possess different inflation expectation under a higher oil price regime, or the central bank of each country might try very hard to control the potential negative impacts of oil price shocks on the macroeconomy and thus affect the time series properties of the inflation rates. We thus apply the Markov-switching autoregressive fractionally integrated moving average (MS-ARFIMA) model of Tsay and Härdle (2008) to re-examine the long memory properties of the U.S. price deflator, because this MS-ARFIMA model can estimate the order of fractional integration of inflation which might be subject to level shifts and persistency changes simultaneously.

# 2 MS-ARFIMA model

Before presenting the MS-ARFIMA model, let us illustrate the definition of an Markov chain and that of an ARFIMA process. Let $\{s_t\}_{t=1}^T$ be the latent sample path of an $N$-state Markov chain. Each time $s_t$ can assume only an integer value of $1, 2, \cdots, N$, and its transition probability matrix is:

$$\mathcal{P} \equiv \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{N1} \\ p_{12} & p_{22} & \cdots & p_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1N} & p_{2N} & \cdots & p_{NN} \end{bmatrix},$$

where $p_{ij} = P(s_t = j \mid s_{t-1} = i)$ and $\sum_{j=1}^N p_{ij} = 1$ for all $i$.

An $I(d)$ process, $x_t$, is defined as:

$$(1 - L)^d x_t = h_t,$$

where $L$ is the lag operator ($Lk_t = k_{t-1}$) and $h_t$ is a short memory process. When $d > 0$, the $I(d)$ process is often called the long memory process, because its autocovariance function is not summable so as to capture the long range dependence of a time series. In addition, the $I(d)$ process is nonstationary when $d \geq 1/2$, otherwise, it is covariance stationary. Note that the $I(d)$ process is mean-reverting when $d < 1$.

The MS-ARFIMA model of Tsay and Härdle (2008) offers a rich dynamic mixture of an Markov chain and an $I(d)$ process and is expressed as:

$$w_t = \mu_{s_t} I\{t \geq 1\} + (1-L)^{-d_{s_t}} \sigma_{s_t} z_t I\{t \geq 1\} = \mu_{s_t} I\{t \geq 1\} + y_t, \tag{1}$$

where $w_t$ is the observed inflation rate at time $t$, $I\{.\}$ is the indicator function, and $z_t$ is a stationary ARMA process with mean zero and bounded positive spectral density $f_z(\lambda) \sim G_z$ as $\lambda \to 0$ at each possible regime. The role of $I\{.\}$ is to truncate the influence of the infinite past observations of $z_t$ on $w_t$, because we allow $d_{s_t}$ to be greater than or equal to 1/2. The most innovative characteristic of the process in (1) is that the fractional differencing parameter $d_{s_t}$ well known in the long memory literature is allowed to be a Markov chain satisfying the following Assumption A.

**Assumption A**. *$s_t$ is independent of $z_\tau$ for all $t$ and $\tau$.*

When $N = 1$, the model in (1) reduces to be the ARFIMA process introduced in Granger (1980), Granger and Joyeux (1980), and Hosking (1981). The model in (1) cannot be estimated with the recursive algorithm in Hamilton (1989), because the possible routes of states running from time 1 to time $T$ expand exponentially to be $N^T$ if we want to extract $z_t$ to conduct the maximum likelihood estimation (MLE). In addition, this model cannot be written in a state-space form due to the presence of a fractional differencing parameter, implying that we cannot apply the EM algorithm considered in Hamilton (1990) for the model in (1), because the non-Markovian nature of the model prevents us from using the results in (4.2) of Hamilton (1990).

It is not difficult to write down the conditional likelihood function of the mixture model in (1) in terms of the ARMA process $z_t$, provided that we can exactly identify the true path of $s_t$ along with some suitable assumptions about the initial values, distribution, and model specification of $z_t$ . We surely do not in reality observe the true path of the latent state variable. Nevertheless, we can implement the Viterbi (1967) algorithm well-known in the digital communication literature to identify the most likely path of states among the $N^T$ possible routes within the inflation data. Essentially, the original Viterbi algorithm is the standard forward dynamic programming solution to maximum-likelihood decoding of a discrete-time, finite-state dynamic system observed in white noise as documented in Omura (1969). In this paper we apply the Viterbi algorithm to the finite-state dynamic system observed in a more general ARFIMA noise with a Markov-switching fractional differencing parameter. See Forney (1973) or Tsay and Härdle (2008) about the implementations of the Viterbi algorithm.

# 3 Inflation with Markov-switching persistency

We estimate the U.S. inflation rates with the following 2-state MS-ARFIMA$(1, d, 1)$ model:

$$w_t = \mu_{s_t} I\{t \geq 1\} + (1 - L)^{-d_{s_t}} \sigma_{s_t} z_t I\{t \geq 1\}, \quad (1 - \phi_1 L) z_t = (1 + \theta_1 L)\varepsilon_t, \tag{2}$$

where $\phi_1$ or $\theta_1$ is assumed to be zero depending on the noise specification, and $\varepsilon_t$ is standard normally distributed. The case of $\phi_1 = \theta_1 = 0$ has been adopted by Tsay and Härdle (2008) in characterizing Nile River data. The quarterly U.S. inflation data are calculated from the seasonally-adjusted implicit price deflator of the nonfarm business sector for the period 1947 I until 2007 IV. Following Hamilton (1989), asymptotic standard errors are calculated numerically. These calculated standard errors may not be valid, because the Viterbi algorithm is not equivalent to the usual full complete-data log-likelihood estimation. Nevertheless, the Viterbi algorithm provides a convenient way to identify the most likely regime path beneath the observed data under the model in (2).

Table 1 shows that the estimates of $\mu_1$, $\mu_2$, $p_{11}$, $p_{22}$, $\sigma_1$, and $\sigma_2$ are quite robust across all four different specifications. Interestingly, we identify 5 *identical* breakpoints at the time of 1951 II, 1973 IV, 1975 II, 1979 II, and 1981 II from both the MS-ARFIMA$(0, d, 1)$ and the MS-ARFIMA$(1, d, 1)$ models. These two specifications actually are the ones producing the highest likelihood functions among the four specifications considered in Table 1. Figure 1 illustrates the fitting performance of the MS-ARFIMA$(1, d, 1)$ model and clearly shows the great ability of the MS-ARFIMA model in capturing the historical record of the U.S. inflation rates.

The estimates of $\sigma_1$ and $\sigma_2$ in Table 1 show that the volatility of inflation is higher in the higher inflation regime, revealing that the uncertainty of inflation is relatively larger when the economy is facing a higher inflation regime. This result correctly reflects the changing pattern of observed inflation in Figure 1. For all configurations considered in Table 1, we also observe that the values of $p_{11}$ and $d_1$ are larger than those of $p_{22}$ and $d_2$, respectively. This finding reveals the probability that the state will move to the other regime is higher, and the weight of current and previous shocks, $\varepsilon_{t-j}$ $(j = 0, 1, \ldots)$, on current inflation is lower, when the current state is in the high inflation regime as compared to the lower inflation counterpart. These observations are reasonable, because the occurrence of oil shocks are not regular, though perhaps they are closely related to the historical events of the 1973-74 OPEC embargo and the 1979 Iranian revolution, respectively, as clearly demonstrated in Figure 2 concerning the estimated path of $\mu_{s_t}$ from the MS-ARFIMA$(1, d, 1)$ model. Therefore, our findings might contribute to the literature concerning the relationship between oil shocks and recession considered in Rasche and Tatom (1981), Hamilton (1983, 1985), Burbidge and Harrison (1984), among others.

Another important finding of this paper is that the fractional integration of U.S. inflation is within the range of 0.3307-0.6350 based on the eight estimates in Table 1. This indicates that the U.S. inflation rate is a mean-reverting long memory process as documented in Hassler and Wolters (1995), Baillie et al. (1996), and Bos et al. (1999), even though the the data-generating process is relaxed to allow for level shifts or persistency changes simultaneously.

# References


Baillie R.T., Chung, C.-F., Tieslau, M., 1996. Analysing inflation by the fractionally integrated ARFIMA-GARCH model. Journal of Applied Econometrics 11, 23-40.

Ball, L., Cecchetti, S.G., 1990. Inflation and uncertainty at short and long horizons, Brookings Papers on Economic Activity, 215-254.

Barsky, R.B., 1987. The Fisher hypothesis and the forecastability and persistency of inflation. Journal of Monetary Economics 19, 3-24.

Bos, C.S., Franses, P.H., Ooms, M., 1999. Long memory and level shifts: Re-analyzing inflation rates. Empirical Economics 24, 427-449.

Burbidge, J., Harrison, A., 1984. Testing for the effects of oil-price rises using vector autoregressions. International Economic Review 25, 459-484.

Forney, Jr. G.D., 1973. The Viterbi algorithm. Proceedings of the IEEE 61, 268-278.

Granger, C.W.J., 1980. Long memory relationships and the aggregation of dynamic models. Journal of Econometrics 14, 227-238.

Granger, C.W.J., Joyeux, R., 1980. An introduction to long-memory time series models and fractional differencing. Journal of Time Series Analysis 1, 15-29.

Hamilton, J.D., 1983. Oil and the macroeconomy since World War II. Journal of Political Economy 91, 228-248.

Hamilton, J.D., 1985. Historical causes of postwar oil shocks and recessions. Energy Journal 6, 97-116.

Hamilton, J.D., 1989. A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica 57, 357-384.

Hamilton, J.D., 1990. Analysis of time series subject to changes in regime. Journal of Econometrics 45, 39-70.

Hassler, U., Wolters, J., 1995. Long memory in inflation rates: International evidence. Journal of Business & Economic Statistics 13, 37-45.

Hosking, J.R.M., 1981. Fractional differencing. Biometrika 68, 165-176.

Nelson, C.R., Schwert, G.W., 1977. Short-term interest rates as predictors of inflation: On testing the hypothesis that the real rate of interest is constant. American Economic Review 67, 478-486.

Omura, J.K., 1969. On the Viterbi decoding algorithm. IEEE Transactions on Information Theory IT-15, 177-179.

Rasche, R., Tatom, J., 1981. Energy price shocks, aggregate supply and monetary policy: The theory and the international evidence. Carnegie-Rochester Conference Series on Public Policy 14, 9-93.

Tsay, W.J., Härdle, W.K., 2008. A generalized ARFIMA process with Markov-switching fractional differencing parameter. Journal of Statistical Computation and Simulation, forthcoming.

Viterbi, A.J., 1967. Error bounds for convolutional codes and an asymptotic optimum decoding algorithm. IEEE Transactions on Signal Processing IT-13, 260-269.

**Table 1. Estimates of Parameters based on Data for U.S. Quarterly Inflation Rate**

| | ARFIMA$(0, d, 0)$ | | ARFIMA$(0, d, 1)$ | | ARFIMA$(1, d, 0)$ | | ARFIMA$(1, d, 1)$ | |
|---|---|---|---|---|---|---|---|---|
| | Estimate | S.E. | Estimate | S.E. | Estimate | S.E. | Estimate | S.E. |
| $\mu_1$ | 0.6302 | 0.2531 | 0.5082 | 0.4740 | 0.7818 | 0.3991 | 0.5423 | 0.4668 |
| $\mu_2$ | 1.8784 | 0.2526 | 1.6541 | 0.5103 | 1.9834 | 0.3294 | 1.6778 | 0.4946 |
| $p_{11}$ | 0.9822 | 0.0088 | 0.9866 | 0.0077 | 0.9822 | 0.0088 | 0.9866 | 0.0077 |
| $p_{22}$ | 0.8707 | 0.0673 | 0.9261 | 0.0472 | 0.8706 | 0.0674 | 0.9261 | 0.0472 |
| $\sigma_1$ | 0.2884 | 0.0138 | 0.2825 | 0.0137 | 0.2848 | 0.0137 | 0.2819 | 0.0137 |
| $\sigma_2$ | 1.0838 | 0.1552 | 1.1095 | 0.1463 | 1.0834 | 0.1589 | 1.1006 | 0.1452 |
| $d_1$ | 0.4687 | 0.0424 | 0.6350 | 0.0939 | 0.5808 | 0.0599 | 0.6261 | 0.0851 |
| $d_2$ | 0.3307 | 0.1075 | 0.6231 | 0.0990 | 0.4754 | 0.1069 | 0.6092 | 0.0912 |
| $\phi_1$ | - | - | - | - | -0.2257 | 0.0816 | -0.2522 | 0.1791 |
| $\theta_1$ | - | - | -0.2481 | 0.1052 | - | - | -0.0129 | 0.2075 |
| $L^*$ | -104.9116 | | -101.3111 | | -101.5249 | | -100.5656 | |

*Notes*: The results are based on the MS-ARFIMA model defined in (2). S.E. stands for the standard error of the estimate. $L^*$ represents the log-likelihood function of the switching model.

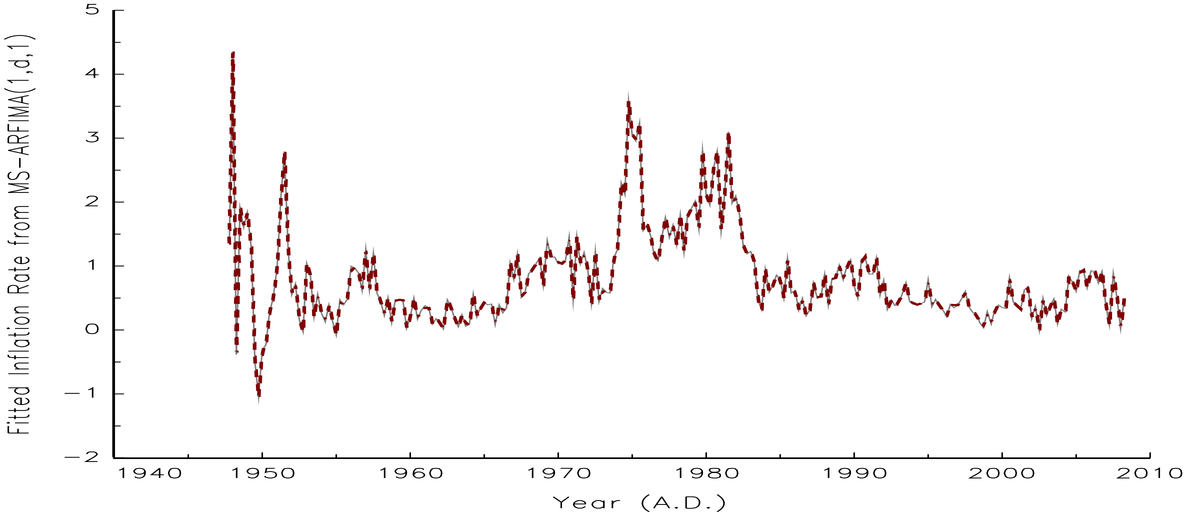


Figure 1: Solid line denotes the quarterly U.S. inflation rate, while dotted line represents the corresponding fitted values from the MS-ARFIMA$(1, d, 1)$ model presented in Table 1.

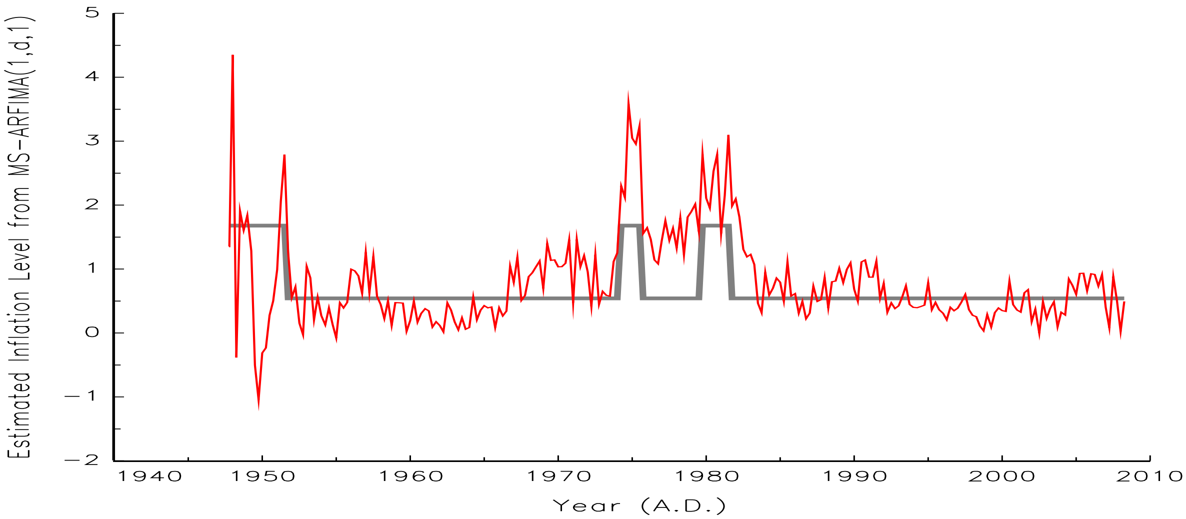


Figure 2: The observed quarterly U.S. inflation rates and the corresponding estimated inflation level $\widehat{\mu}_{s_t}$ from the MS-ARFIMA$(1, d, 1)$ model as presented in Table 1.