Table 11: Results of probability of working for the female sample

	Pooled	Analytic PLE	Quadrature
Constant	−2.087 (0.22)**	−2.167 (0.33)**	−2.210 (0.30)**
Personal characteristics:			
Ethnic group: black - Caribbean	−0.283 (0.18)	−0.295 (0.34)	−0.340 (0.33)
Ethnic group: black - African	−0.525 (0.34)	−0.513 (0.85)	−0.522 (0.60)
Ethnic group: Indian	−0.567 (0.11)**	−0.580 (0.20)**	−0.602 (0.19)**
Ethnic group: Pakistani, Bangladesh	−0.918 (0.27)**	−0.886 (0.47)*	−0.975 (0.44)**
Ethnic group: Chinese	−1.614 (0.41)**	−1.524 (0.10)**	−1.428 (0.69)**
Married	0.193 (0.04)**	0.157 (0.05)**	0.117 (0.04)**
Age / 10	1.569 (0.12)**	1.557 (0.18)**	1.525 (0.16)**
Age squared / 100	−0.222 (0.02)**	−0.216 (0.02)**	−0.206 (0.02)**
Number of children = 1	−0.414 (0.04)**	−0.402 (0.05)**	−0.344 (0.04)**
Number of children = 2	−0.648 (0.04)**	−0.574 (0.06)**	−0.441 (0.05)**
Number of children ≥ 3	−0.853 (0.06)**	−0.804 (0.08)**	−0.671 (0.07)**
Youngest child < 5 years	−0.726 (0.04)**	−0.580 (0.04)**	−0.438 (0.04)**
Public school	−0.212 (0.07)**	−0.215 (0.12)*	−0.198 (0.11)*
Grammar school	0.001 (0.04)	0.012 (0.07)	0.025 (0.07)
Highest educational qualification:			
Sub-O-level	0.124 (0.05)**	0.117 (0.08)	0.098 (0.08)
O-levels	0.450 (0.04)**	0.428 (0.07)**	0.371 (0.07)**
A-levels	0.275 (0.06)**	0.263 (0.09)**	0.228 (0.08)**
Other higher	0.497 (0.05)**	0.459 (0.07)**	0.369 (0.07)**
Degree	0.675 (0.06)**	0.655 (0.10)**	0.640 (0.10)**
Region of residence:			
London & Southeast	0.071 (0.03)**	0.079 (0.05)	0.109 (0.05)**
Wales	−0.273 (0.06)**	−0.260 (0.10)**	−0.237 (0.10)**
Scotland	0.040 (0.05)	0.060 (0.09)	0.090 (0.08)
Rural location	−0.059 (0.04)	−0.036 (0.06)	0.007 (0.06)
Housing tenure:			
Owner-occupier	0.428 (0.05)**	0.392 (0.06)**	0.317 (0.05)**
Local authority renter	−0.108 (0.06)*	−0.127 (0.07)*	−0.145 (0.06)**
Non-labour income:			
Savings and investments / 1000	−0.478 (0.09)**	−0.320 (0.11)**	−0.211 (0.09)**
Health:			
Hospital overnight stay last year	−0.282 (0.05)**	−0.204 (0.04)**	−0.161 (0.04)**
Smokes ≥ 5 cigarettes per day	−0.047 (0.03)	−0.024 (0.05)	0.020 (0.04)
Labour market tightness: U-V ratio / 100	−0.315 (0.14)**	−0.254 (0.16)	−0.198 (0.14)
ρ		0.799 (0.01)**	0.807 (0.01)**

Notes: Data are taken from Booth et al. (1999). Dependent variable is the indicator of being in paid work. Total number of persons is 2,138, and total number of person-waves observations is 10,690. Please refer to Booth et al. (1999) for detailed definitions of explanatory variables. Regressions also include wave dummies. * and ** denote significance at the 10% and 5% levels, respectively. The numbers in parenthesis are standard errors. The standard error of the analytic approximation estimator is computed from (5) and (6) in Kuk and Nott (2000).

Mathematic Appendix

We define the following identities,

$$erf(v)=\frac{2}{\sqrt{\pi}}\int_0^v e^{-t^2}dt=2\int_0^{\sqrt{2}v}\varphi(t)\,dt\,, \tag{B.1}$$

$$\begin{cases} erf(v\geq 0)\approx 1-e^{c_1 v+c_2 v^2}\,, \\ erf(v<0)\approx -1+e^{-c_1 v+c_2 v^2}\,, \\ c_1=-1.0950081470333\,, \\ c_2=-0.75651138383854\,, \end{cases} \tag{B.2}$$

$$\phi(v)=(2\pi)^{(-1/2)}e^{-\left(v^2/2\right)}. \tag{B.3}$$

And we use (7.4.32) of Abramowitz and Stegun (1970),

$$\int e^{-\left(kv^2+2mv+n\right)}dx=\frac{1}{2}\sqrt{\frac{\pi}{k}}e^{\left(\frac{m^2-kn}{k}\right)}\left[erf\left(\sqrt{k}x+\frac{m}{\sqrt{k}}\right)\right]+C,\,k\neq 0\,. \tag{B.4}$$

We derive the pairwise likelihood function as following,

$$L_i=\int_{-\infty}^{\infty}\phi(v)\,\Phi(P_1v+Q_1)\,\Phi(P_2v+Q_2)\,dv$$
$$=\int_{-\infty}^{\infty}\left[\frac{1}{2}+\frac{1}{2}erf\left(\frac{P_1v+Q_1}{\sqrt{2}}\right)\right]\left[\frac{1}{2}+\frac{1}{2}erf\left(\frac{P_2v+Q_2}{\sqrt{2}}\right)\right]\phi(v)\,dv\,.$$

Set $-\frac{Q_2}{P_2}>-\frac{Q_1}{P_1}$, and v located in the three mutually exclusive segments,

$$v\in\left(-\infty,-\frac{Q_1}{P_1}\right),\ \text{ or }\ v\in\left[-\frac{Q_1}{P_1},-\frac{Q_2}{P_2}\right),\ \text{ or }\ v\in\left[-\frac{Q_2}{P_2},\infty\right),$$

such that the likelihood function becomes

$$L_i=\int_{-\infty}^{-\frac{Q_1}{P_1}}\left[\frac{1}{2}+\frac{1}{2}erf\left(\frac{P_1v+Q_1}{\sqrt{2}}\right)\right]\left[\frac{1}{2}+\frac{1}{2}erf\left(\frac{P_2v+Q_2}{\sqrt{2}}\right)\right]\phi(v)\,dv$$
$$+\int_{-\frac{Q_1}{P_1}}^{-\frac{Q_2}{P_2}}\left[\frac{1}{2}+\frac{1}{2}erf\left(\frac{P_1v+Q_1}{\sqrt{2}}\right)\right]\left[\frac{1}{2}+\frac{1}{2}erf\left(\frac{P_2v+Q_2}{\sqrt{2}}\right)\right]\phi(v)\,dv$$
$$+\int_{-\frac{Q_2}{P_2}}^{\infty}\left[\frac{1}{2}+\frac{1}{2}erf\left(\frac{P_1v+Q_1}{\sqrt{2}}\right)\right]\left[\frac{1}{2}+\frac{1}{2}erf\left(\frac{P_2v+Q_2}{\sqrt{2}}\right)\right]\phi(v)\,dv$$
$$=I_1+I_2+I_3\,,$$

where

We thank for preparing this mathematic appendix by the research assistance of Chuan-Kai Wang.

$$I_1 = \frac{1}{4}\int_{-\infty}^{-\frac{Q_1}{P_1}} \left[1 + erf\left(\frac{P_1 \nu + Q_1}{\sqrt{2}}\right)\right]\left[1 + erf\left(\frac{P_2 \nu + Q_2}{\sqrt{2}}\right)\right]\phi(\nu)\, d\nu \, ,$$

$$I_2 = \frac{1}{4}\int_{-\frac{Q_1}{P_1}}^{-\frac{Q_2}{P_2}} \left[1 + erf\left(\frac{P_1 \nu + Q_1}{\sqrt{2}}\right)\right]\left[1 + erf\left(\frac{P_2 \nu + Q_2}{\sqrt{2}}\right)\right]\phi(\nu)\, d\nu \, ,$$

$$I_3 = \frac{1}{4}\int_{-\frac{Q_2}{P_2}}^{\infty} \left[1 + erf\left(\frac{P_1 \nu + Q_1}{\sqrt{2}}\right)\right]\left[1 + erf\left(\frac{P_2 \nu + Q_2}{\sqrt{2}}\right)\right]\phi(\nu)\, d\nu \, .$$

And let

$$A = \frac{P_1 \nu + Q_1}{\sqrt{2}}, \quad B = \frac{P_2 \nu + Q_2}{\sqrt{2}}, \quad U = -\frac{Q_2}{P_2}, \quad L = -\frac{Q_1}{P_1},$$

We then have,

$$I_1 = \frac{1}{4}\int_{-\infty}^{L} [1 + erf(A)][1 + erf(B)]\phi(\nu)\, d\nu \, ,$$

$$I_2 = \frac{1}{4}\int_{L}^{U} [1 + erf(A)][1 + erf(B)]\phi(\nu)\, d\nu \, ,$$

$$I_3 = \frac{1}{4}\int_{U}^{\infty} [1 + erf(A)][1 + erf(B)]\phi(\nu)\, d\nu \, .$$

Depending on the sign of P_1 and that of P_2, we have four cases to consider:

Case 1: $P_1 > 0$, $P_2 > 0$.

We observe,

$\nu \in (-\infty, -\frac{Q_1}{P_1})$, $A < 0$, $B < 0$.

$\nu \in [-\frac{Q_1}{P_1}, -\frac{Q_2}{P_2})$, $A > 0$, $B < 0$.

$\nu \in [-\frac{Q_2}{P_2}, \infty)$, $A > 0$, $B > 0$.

Case 2: $P_1 > 0$, $P_2 < 0$.

We observe,

$\nu \in (-\infty, -\frac{Q_1}{P_1})$, $A < 0$, $B > 0$.

$\nu \in [-\frac{Q_1}{P_1}, -\frac{Q_2}{P_2})$, $A > 0$, $B > 0$.

$\nu \in [-\frac{Q_2}{P_2}, \infty)$, $A > 0$, $B < 0$.

Case 3: $P_1 < 0$, $P_2 > 0$.

We observe,

$\nu \in (-\infty, -\frac{Q_1}{P_1})$, $A > 0$, $B < 0$.

$\nu \in [-\frac{Q_1}{P_1}, -\frac{Q_2}{P_2})$, $A < 0$, $B < 0$.

$\nu \in [-\frac{Q_2}{P_2}, \infty)$, $A < 0$, $B > 0$.

Case 4: $P_1 < 0$, $P_2 < 0$.

We observe,

$\nu \in (-\infty, -\frac{Q_1}{P_1})$, $A > 0$, $B > 0$.

$\nu \in [-\frac{Q_1}{P_1}, -\frac{Q_2}{P_2})$, $A < 0$, $B > 0$.

$\nu \in [-\frac{Q_2}{P_2}, \infty)$, $A < 0$, $B < 0$.

Now, considering each case defined above, we derive the likelihood function separately:

Case 1:

For I_1, we first observe that

$$I_1 = \frac{1}{4}\int_{-\infty}^{L} [1 + erf(A)][1 + erf(B)]\phi(\nu)\,d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_{-\infty}^{L} \left[1 - 1 + e^{-c_1 A + c_2 A^2}\right]\left[1 - 1 + e^{-c_1 B + c_2 B^2}\right] e^{\left(-\nu^2/2\right)}\,d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_{-\infty}^{L} e^{\left[-c_1 A + c_2 A^2 - c_1 B + c_2 B^2 - \left(\nu^2/2\right)\right]} d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_{-\infty}^{L} e^{\left[\frac{-c_1(P_1\nu + Q_1)}{\sqrt{2}} + \frac{c_2(P_1\nu + Q_1)^2}{2} + \frac{-c_1(P_2\nu + Q_2)}{\sqrt{2}} + \frac{c_2(P_2\nu + Q_2)^2}{2} + \left(\frac{-\nu^2}{2}\right)\right]} d\nu$$

$$= \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} e^{-\left[\frac{-c_2\left(P_1^2+P_2^2\right)+1}{2}\nu^2+2\frac{-\sqrt{2}c_2\left(P_1Q_1+P_2Q_2\right)+c_1\left(P_1+P_2\right)}{2\sqrt{2}}\nu+\frac{\sqrt{2}c_1\left(Q_1+Q_2\right)-c_2\left(Q_1^2+Q_2^2\right)}{2}\right]} d\nu .$$

Let $g_1 = \frac{1-c_2\left(P_1^2+P_2^2\right)}{2}$, $g_2 = \frac{c_1\left(P_1+P_2\right)-\sqrt{2}c_2\left(P_1Q_1+P_2Q_2\right)}{2\sqrt{2}}$, $g_3 = \frac{\sqrt{2}c_1\left(Q_1+Q_2\right)-c_2\left(Q_1^2+Q_2^2\right)}{2}$,

and use (B.4), we obtain

$$I_1 = \frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_2^2-g_1g_3}{g_1}\right)}\left\{erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_2}{\sqrt{g_1}}\right]-erf\left[\sqrt{g_1}\left(-\infty\right)+\frac{g_2}{\sqrt{g_1}}\right]\right\}$$

$$= \frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_2^2-g_1g_3}{g_1}\right)}\left\{1+erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_2}{\sqrt{g_1}}\right]\right\}.$$

For I_2, we first observe that

$$I_2 = \frac{1}{4}\int_L^U \left[1+erf\left(A\right)\right]\left[1+erf\left(B\right)\right]\phi\left(\nu\right)d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_L^U \left(1+1-e^{c_1A+c_2A^2}\right)\left(1-1+e^{-c_1B+c_2B^2}\right)e^{\left(-\nu^2/2\right)}d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_L^U \left(2-e^{c_1A+c_2A^2}\right)\left(e^{-c_1B+c_2B^2}\right)e^{\left(-\nu^2/2\right)}d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_L^U \left[2e^{-c_1B+c_2B^2-\left(\nu^2/2\right)}-e^{c_1A+c_2A^2-c_1B+c_2B^2-\left(\nu^2/2\right)}\right]d\nu$$

$$= I_2(1)+I_2(2),$$

where

$$I_2(1) = \frac{1}{4\sqrt{2\pi}}\int_L^U \left[2e^{-c_1B+c_2B^2-\left(\nu^2/2\right)}\right]d\nu,$$

$$I_2(2) = \frac{1}{4\sqrt{2\pi}}\int_L^U \left[-e^{c_1A+c_2A^2-c_1B+c_2B^2-\left(\nu^2/2\right)}\right]d\nu.$$

For $I_2(1)$, we observe that

$$\frac{1}{4\sqrt{2\pi}}\int_L^U 2e^{-c_1B+c_2B^2-\left(\nu^2/2\right)}d\nu$$

$$= \frac{1}{2\sqrt{2\pi}}\int_L^U e^{\left[-c_1\left(\frac{P_2\nu+Q_2}{\sqrt{2}}\right)+c_2\left(\frac{P_2\nu+Q_2}{\sqrt{2}}\right)^2-\left(\nu^2/2\right)\right]}d\nu$$

$$= \frac{1}{2\sqrt{2\pi}}\int_L^U e^{-\left[\frac{-c_2P_2^2+1}{2}\nu^2+2\frac{P_2\left(c_1-\sqrt{2}c_2Q_2\right)}{2\sqrt{2}}\nu+\frac{Q_2\left(\sqrt{2}c_1-c_2Q_2\right)}{2}\right]}d\nu.$$

Let $g_4 = \dfrac{1-P_2^{\,2}c_2}{2}$, $g_5 = \dfrac{P_2\left(c_1-\sqrt{2}Q_2c_2\right)}{2\sqrt{2}}$, $g_6 = \dfrac{Q_2\left(\sqrt{2}c_1-Q_2c_2\right)}{2}$,

and use (B.4), we obtain

$$I_2(1) = \frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_5^{\,2}-g_4g_6}{g_4}\right)}\left\{erf\left[\sqrt{g_4}\left(\frac{-Q_2}{P_2}\right)+\frac{g_5}{\sqrt{g_4}}\right]-erf\left[\sqrt{g_4}\left(\frac{-Q_1}{P_1}\right)+\frac{g_5}{\sqrt{g_4}}\right]\right\}.$$

For $I_2(2)$, we observe that

$$\frac{1}{4\sqrt{2\pi}}\int_L^U -e^{c_1A+c_2A^2-c_1B+c_2B^2-\left(v^2/2\right)}dv$$

$$=\frac{-1}{4\sqrt{2\pi}}\int_L^U e^{c_1\left(\frac{P_1v+Q_1}{\sqrt{2}}\right)+c_2\left(\frac{P_1v+Q_1}{\sqrt{2}}\right)^2-c_1\left(\frac{P_2v+Q_2}{\sqrt{2}}\right)+c_2\left(\frac{P_2v+Q_2}{\sqrt{2}}\right)^2-\left(v^2/2\right)}dv$$

$$=\frac{-1}{4\sqrt{2\pi}}\int_L^U e^{-\left[\frac{-c_2\left(P_1^{\,2}+P_2^{\,2}\right)+1}{2}v^2+2\frac{-\sqrt{2}c_2\left(P_1Q_1+P_2Q_2\right)-c_1\left(P_1-P_2\right)}{2\sqrt{2}}v+\frac{-\sqrt{2}c_1\left(Q_1-Q_2\right)-c_2\left(Q_1^{\,2}+Q_2^{\,2}\right)}{2}\right]}dv.$$

Let $g_7 = \dfrac{-c_1\left(P_1-P_2\right)-\sqrt{2}c_2\left(P_1Q_1+P_2Q_2\right)}{2\sqrt{2}}$, $g_8 = \dfrac{-2c_1\left(Q_1-Q_2\right)-\sqrt{2}c_2\left(Q_1^{\,2}+Q_2^{\,2}\right)}{2\sqrt{2}}$,

and use (B.4), we obtain

$$I_2(2) = \frac{-1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_7^{\,2}-g_1g_8}{g_1}\right)}\left\{erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_7}{\sqrt{g_1}}\right]-erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_7}{\sqrt{g_1}}\right]\right\}.$$

Accordingly,

$$I_2 = \frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_5^{\,2}-g_4g_6}{g_4}\right)}\left\{erf\left[\sqrt{g_4}\left(\frac{-Q_2}{P_2}\right)+\frac{g_5}{\sqrt{g_4}}\right]-erf\left[\sqrt{g_4}\left(\frac{-Q_1}{P_1}\right)+\frac{g_5}{\sqrt{g_4}}\right]\right\}$$
$$-\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_7^{\,2}-g_1g_8}{g_1}\right)}\left\{erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_7}{\sqrt{g_1}}\right]-erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_7}{\sqrt{g_1}}\right]\right\}.$$

For I_3, we first observe that

$$I_3 = \frac{1}{4}\int_U^\infty\left[1+erf\left(A\right)\right]\left[1+erf\left(B\right)\right]\phi(v)\,dv$$

$$=\frac{1}{4\sqrt{2\pi}}\int_U^\infty\left(1+1-e^{c_1A+c_2A^2}\right)\left(1+1-e^{c_1B+c_2B^2}\right)e^{\left(-v^2/2\right)}dv$$

$$=\frac{1}{4\sqrt{2\pi}}\int_U^\infty\left(2-e^{c_1A+c_2A^2}\right)\left(2-e^{c_1B+c_2B^2}\right)e^{\left(-v^2/2\right)}dv$$

$$= \frac{1}{4\sqrt{2\pi}} \int_U^\infty \left[4e^{\left(-\nu^2/2\right)} - 2e^{c_1 A + c_2 A^2 + \left(-\nu^2/2\right)} - 2e^{c_1 B + c_2 B^2 + \left(-\nu^2/2\right)} + e^{c_1 A + c_2 A^2 + c_1 B + c_2 B^2 + \left(-\nu^2/2\right)} \right] d\nu$$

$$= I_3(1) + I_3(2) + I_3(3) + I_3(4),$$

where

$$I_3(1) = \frac{1}{4\sqrt{2\pi}} \int_U^\infty \left[4e^{\left(-\nu^2/2\right)} \right] d\nu,$$

$$I_3(2) = \frac{1}{4\sqrt{2\pi}} \int_U^\infty \left[-2e^{c_1 A + c_2 A^2 + \left(-\nu^2/2\right)} \right] d\nu,$$

$$I_3(3) = \frac{1}{4\sqrt{2\pi}} \int_U^\infty \left[-2e^{c_1 B + c_2 B^2 + \left(-\nu^2/2\right)} \right] d\nu,$$

$$I_3(4) = \frac{1}{4\sqrt{2\pi}} \int_U^\infty \left[e^{c_1 A + c_2 A^2 + c_1 B + c_2 B^2 + \left(-\nu^2/2\right)} \right] d\nu.$$

For $I_3(1)$, we use (B.4) to obtain

$$I_3(1) = \frac{1}{2} \left\{ erf\left[\sqrt{\frac{1}{2}} (\infty) \right] - erf\left[\sqrt{\frac{1}{2}} \left(\frac{-Q_2}{P_2} \right) \right] \right\} = \frac{1}{2} \left[1 - erf\left(\frac{-Q_2}{\sqrt{2} P_2} \right) \right].$$

For $I_3(2)$, we observe that

$$\frac{1}{4\sqrt{2\pi}} \int_U^\infty -2e^{c_1 A + c_2 A^2 + \left(-\nu^2/2\right)} d\nu$$

$$= \frac{-1}{2\sqrt{2\pi}} \int_U^\infty e^{c_1\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right) + c_2\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right)^2 + \left(-\nu^2/2\right)} d\nu$$

$$= \frac{-1}{2\sqrt{2\pi}} \int_U^\infty e^{-\left[\frac{-c_2 P_1^2 + 1}{2}\nu^2 + 2\frac{-P_1\left(c_1 + \sqrt{2} c_2 Q_1\right)}{2\sqrt{2}}\nu + \frac{-Q_1\left(\sqrt{2} c_1 + c_2 Q_1\right)}{2}\right]} d\nu.$$

Let $g_9 = \dfrac{1 - P_1^2 c_2}{2}$, $g_{10} = \dfrac{-P_1\left(c_1 + \sqrt{2} Q_1 c_2\right)}{2\sqrt{2}}$, $g_{11} = \dfrac{-Q_1\left(\sqrt{2} c_1 + Q_1 c_2\right)}{2}$,

and use (B.4), we obtain

$$I_3(2) = \frac{-1}{4\sqrt{2}} \sqrt{\frac{1}{g_9}} e^{\left(\frac{g_{10}{}^2 - g_9 g_{11}}{g_9}\right)} \left\{ erf\left[\sqrt{g_9}(\infty) + \frac{g_{10}}{\sqrt{g_9}} \right] - erf\left[\sqrt{g_9}\left(\frac{-Q_2}{P_2} \right) + \frac{g_{10}}{\sqrt{g_9}} \right] \right\}$$

$$= \frac{-1}{4\sqrt{2}} \sqrt{\frac{1}{g_9}} e^{\left(\frac{g_{10}{}^2 - g_9 g_{11}}{g_9}\right)} \left\{ 1 - erf\left[\sqrt{g_9}\left(\frac{-Q_2}{P_2} \right) + \frac{g_{10}}{\sqrt{g_9}} \right] \right\}.$$

For $I_3(3)$, we observe that

$$\frac{1}{4\sqrt{2\pi}} \int_U^\infty -2e^{c_1 B + c_2 B^2 + \left(-\nu^2/2\right)} d\nu$$

$$=\frac{-1}{2\sqrt{2\pi}}\int_U^{\infty} e^{c_1\left(\frac{P_2 v+Q_2}{\sqrt{2}}\right)+c_2\left(\frac{P_2 v+Q_2}{\sqrt{2}}\right)^2+\left(-v^2/2\right)} dv$$

$$=\frac{-1}{2\sqrt{2\pi}}\int_U^{\infty} e^{-\left[\frac{-c_2 P_2^2+1}{2}v^2+2\frac{-P_2\left(c_1+\sqrt{2}c_2 Q_2\right)}{2\sqrt{2}}v+\frac{-Q_2\left(\sqrt{2}c_1+c_2 Q_2\right)}{2}\right]} dv .$$

Let $g_{12}=\frac{-P_2\left(c_1+\sqrt{2}Q_2 c_2\right)}{2\sqrt{2}}$, $g_{13}=\frac{-Q_2\left(\sqrt{2}c_1+Q_2 c_2\right)}{2}$,

and use (B.4), we derive

$$I_3(3)=\frac{-1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_{12}^2-g_4 g_{13}}{g_4}\right)}\left\{erf\left[\sqrt{g_4}(\infty)+\frac{g_{12}}{\sqrt{g_4}}\right]-erf\left[\sqrt{g_4}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{12}}{\sqrt{g_4}}\right]\right\}$$

$$=\frac{-1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_{12}^2-g_4 g_{13}}{g_4}\right)}\left\{1-erf\left[\sqrt{g_4}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{12}}{\sqrt{g_4}}\right]\right\}.$$

For $I_3(4)$, we observe that

$$\frac{1}{4\sqrt{2\pi}}\int_U^{\infty} e^{c_1 A+c_2 A^2+c_1 B+c_2 B^2+\left(-v^2/2\right)} dv$$

$$=\frac{1}{4\sqrt{2\pi}}\int_U^{\infty} e^{c_1\left(\frac{P_1 v+Q_1}{\sqrt{2}}\right)+c_2\left(\frac{P_1 v+Q_1}{\sqrt{2}}\right)^2+c_1\left(\frac{P_2 v+Q_2}{\sqrt{2}}\right)+c_2\left(\frac{P_2 v+Q_2}{\sqrt{2}}\right)^2+\left(-v^2/2\right)} dv$$

$$=\frac{1}{4\sqrt{2\pi}}\int_U^{\infty} e^{-\left[\frac{-c_2\left(P_1^2+P_2^2\right)+1}{2}v^2+2\frac{-c_1(P_1+P_2)-\sqrt{2}c_2(P_1Q_1+P_2Q_2)}{2\sqrt{2}}v+\frac{-\sqrt{2}c_1(Q_1+Q_2)-c_2\left(Q_1^2+Q_2^2\right)}{2}\right]} dv .$$

Let $g_{14}=\frac{-c_1(P_1+P_2)-\sqrt{2}c_2(P_1Q_1+P_2Q_2)}{2\sqrt{2}}$, $g_{15}=\frac{-2c_1(Q_1+Q_2)-\sqrt{2}c_2\left(Q_1^2+Q_2^2\right)}{2\sqrt{2}}$,

and use (B.4), $I_3(4)$ becomes

$$I_3(4)=\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{14}^2-g_1 g_{15}}{g_1}\right)}\left\{erf\left[\sqrt{g_1}(\infty)+\frac{g_{14}}{\sqrt{g_1}}\right]-erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{14}}{\sqrt{g_1}}\right]\right\}$$

$$=\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{14}^2-g_1 g_{15}}{g_1}\right)}\left\{1-erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{14}}{\sqrt{g_1}}\right]\right\}.$$

Combining the above findings, we then prove

$$I_3=\frac{1}{2}\left[1-erf\left(\frac{-Q_2}{\sqrt{2}P_2}\right)\right]$$

$$-\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}e^{\left(\frac{g_{10}{}^2-g_9g_{11}}{g_9}\right)}\left\{1-erf\left[\sqrt{g_9}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{10}}{\sqrt{g_9}}\right]\right\}$$

$$-\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_{12}{}^2-g_4g_{13}}{g_4}\right)}\left\{1-erf\left[\sqrt{g_4}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{12}}{\sqrt{g_4}}\right]\right\}$$

$$+\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{14}{}^2-g_1g_{15}}{g_1}\right)}\left\{1-erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{14}}{\sqrt{g_1}}\right]\right\}.$$

Hence, for case 1, the pairwise likelihood function becomes

$$L_i=I_1+I_2+I_3=$$

$$\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_2{}^2-g_1g_3}{g_1}\right)}\left\{1+erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_2}{\sqrt{g_1}}\right]\right\}$$

$$+\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_5{}^2-g_4g_6}{g_4}\right)}\left\{erf\left[\sqrt{g_4}\left(\frac{-Q_2}{P_2}\right)+\frac{g_5}{\sqrt{g_4}}\right]-erf\left[\sqrt{g_4}\left(\frac{-Q_1}{P_1}\right)+\frac{g_5}{\sqrt{g_4}}\right]\right\}$$

$$-\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_7{}^2-g_1g_8}{g_1}\right)}\left\{erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_7}{\sqrt{g_1}}\right]-erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_7}{\sqrt{g_1}}\right]\right\}$$

$$+\frac{1}{2}\left[1-erf\left(\frac{-Q_2}{\sqrt{2}P_2}\right)\right]$$

$$-\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}e^{\left(\frac{g_{10}{}^2-g_9g_{11}}{g_9}\right)}\left\{1-erf\left[\sqrt{g_9}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{10}}{\sqrt{g_9}}\right]\right\}$$

$$-\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_{12}{}^2-g_4g_{13}}{g_4}\right)}\left\{1-erf\left[\sqrt{g_4}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{12}}{\sqrt{g_4}}\right]\right\}$$

$$+\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{14}{}^2-g_1g_{15}}{g_1}\right)}\left\{1-erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{14}}{\sqrt{g_1}}\right]\right\}.$$

Case 2:

For I_1, we first observe that

$$I_1=\frac{1}{4}\int_{-\infty}^{L}\left[1+erf(A)\right]\left[1+erf(B)\right]\phi(v)\,dv$$

$$= \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} \left[1 - 1 + e^{-c_1 A + c_2 A^2}\right]\left[1 + 1 - e^{c_1 B + c_2 B^2}\right] e^{\left(-\nu^2/2\right)}\, d\nu$$

$$= \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} \left(e^{-c_1 A + c_2 A^2}\right)\left(2 - e^{c_1 B + c_2 B^2}\right) e^{\left(-\nu^2/2\right)}\, d\nu$$

$$= \frac{1}{4\sqrt{2\pi}} L \int_{-\infty}^{L} \left[2e^{-c_1 A + c_2 A^2 + \left(-\nu^2/2\right)} - e^{-c_1 A + c_2 A^2 + c_1 B + c_2 B^2 + \left(-\nu^2/2\right)}\right] d\nu$$

$$= I_1(1) + I_1(2),$$

where

$$I_1(1) = \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} 2e^{-c_1 A + c_2 A^2 + \left(-\nu^2/2\right)}\, d\nu,$$

$$I_1(2) = \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} - e^{-c_1 A + c_2 A^2 + c_1 B + c_2 B^2 + \left(-\nu^2/2\right)} d\nu.$$

For $I_1(1)$, we observe that

$$\frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} 2e^{-c_1 A + c_2 A^2 - \left(\nu^2/2\right)}\, d\nu$$

$$= \frac{1}{2\sqrt{2\pi}} \int_{-\infty}^{L} e^{\left[-c_1\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right) + c_2\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right)^2 - \left(\nu^2/2\right)\right]} d\nu$$

$$= \frac{1}{2\sqrt{2\pi}} \int_{-\infty}^{L} e^{-\left[\frac{-c_2 P_1^2 + 1}{2}\nu^2 + 2\frac{-P_1\left(-c_1 + \sqrt{2}c_2 Q_1\right)}{2\sqrt{2}}\nu + \frac{-Q_1\left(-\sqrt{2}c_1 + c_2 Q_1\right)}{2}\right]} d\nu.$$

Let $g_{16} = \dfrac{P_1\left(c_1 - \sqrt{2}Q_1 c_2\right)}{2\sqrt{2}}$, $\quad g_{17} = \dfrac{Q_1\left(\sqrt{2}c_1 - Q_1 c_2\right)}{2}$,

and use (B.4), $I_1(1)$ becomes

$$I_1(1) = \frac{1}{4\sqrt{2}} \sqrt{\frac{1}{g_9}} e^{\left(\frac{g_{16}{}^2 - g_9 g_{17}}{g_9}\right)} \left\{ erf\left[\sqrt{g_9}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{16}}{\sqrt{g_9}}\right] - erf\left[\sqrt{g_9}\left(-\infty\right) + \frac{g_{16}}{\sqrt{g_9}}\right]\right\}$$

$$= \frac{1}{4\sqrt{2}} \sqrt{\frac{1}{g_9}} e^{\left(\frac{g_{16}{}^2 - g_9 g_{17}}{g_9}\right)} \left\{ 1 + erf\left[\sqrt{g_9}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{16}}{\sqrt{g_9}}\right]\right\}.$$

For $I_1(2)$, we observe that

$$\frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} - e^{-c_1 A + c_2 A^2 + c_1 B + c_2 B^2 - \left(\nu^2/2\right)} d\nu$$

$$= \frac{-1}{4\sqrt{2\pi}} \int_{-\infty}^{L} e^{-c_1\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right) + c_2\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right)^2 + c_1\left(\frac{P_2\nu + Q_2}{\sqrt{2}}\right) + c_2\left(\frac{P_2\nu + Q_2}{\sqrt{2}}\right)^2 - \left(\nu^2/2\right)} d\nu$$

$$= \frac{-1}{4\sqrt{2\pi}} \int_{-\infty}^{L} e^{-\left[\frac{-c_2\left(P_1^2+P_2^2\right)+1}{2}\nu^2+2\frac{-\sqrt{2}c_2\left(P_1Q_1+P_2Q_2\right)+c_1\left(P_1-P_2\right)}{2\sqrt{2}}\nu+\frac{\sqrt{2}c_1\left(Q_1-Q_2\right)-c_2\left(Q_1^2+Q_2^2\right)}{2}\right]} d\nu .$$

Let $g_{18} = \dfrac{c_1\left(P_1 - P_2\right) - \sqrt{2}c_2\left(P_1Q_1 + P_2Q_2\right)}{2\sqrt{2}}$, $g_{19} = \dfrac{2c_1\left(Q_1 - Q_2\right) - \sqrt{2}c_2\left(Q_1^2 + Q_2^2\right)}{2\sqrt{2}}$,

and use (B.4), we derive

$$I_1(2) = \frac{-1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{18}^2-g_1g_{19}}{g_1}\right)}\left\{erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{18}}{\sqrt{g_1}}\right]-erf\left[\sqrt{g_1}\left(-\infty\right)+\frac{g_{18}}{\sqrt{g_1}}\right]\right\}$$

$$= \frac{-1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{18}^2-g_1g_{19}}{g_1}\right)}\left\{1+erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{18}}{\sqrt{g_1}}\right]\right\}.$$

Accordingly, we have

$$I_1 = \frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}e^{\left(\frac{g_{16}^2-g_9g_{17}}{g_9}\right)}\left\{1+erf\left[\sqrt{g_9}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{16}}{\sqrt{g_9}}\right]\right\}$$

$$-\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{18}^2-g_1g_{19}}{g_1}\right)}\left\{1+erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{18}}{\sqrt{g_1}}\right]\right\}.$$

For I_2, we first observe that

$$I_2 = \frac{1}{4}\int_L^U \left[1+erf\left(A\right)\right]\left[1+erf\left(B\right)\right]\phi(\nu)\, d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_L^U \left(1+1-e^{c_1A+c_2A^2}\right)\left(1+1-e^{c_1B+c_2B^2}\right)e^{\left(-\nu^2/2\right)}d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_L^U \left(2-e^{c_1A+c_2A^2}\right)\left(2-e^{c_1B+c_2B^2}\right)e^{\left(-\nu^2/2\right)}d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_L^U \left[4e^{\left(-\nu^2/2\right)}-2e^{c_1A+c_2A^2+\left(-\nu^2/2\right)}-2e^{c_1B+c_2B^2+\left(-\nu^2/2\right)}+e^{c_1A+c_2A^2+c_1B+c_2B^2+\left(-\nu^2/2\right)}\right]d\nu$$

$$= I_2(1)+I_2(2)+I_2(3)+I_2(4),$$

where

$$I_2(1) = \frac{1}{4\sqrt{2\pi}} \int_L^U \left[4e^{\left(-\nu^2/2\right)}\right] d\nu\,,$$

$$I_2(2) = \frac{1}{4\sqrt{2\pi}} \int_L^U \left[-2e^{c_1 A + c_2 A^2 + \left(-\nu^2/2\right)}\right] d\nu\,,$$

$$I_2(3) = \frac{1}{4\sqrt{2\pi}} \int_L^U \left[e^{c_1 A + c_2 A^2 + c_1 B + c_2 B^2 + \left(-\nu^2/2\right)}\right] d\nu\,,$$

$$I_2(4) = \frac{1}{4\sqrt{2\pi}} \int_L^U \left[-2e^{c_1 B + c_2 B^2 + \left(-\nu^2/2\right)}\right] d\nu\,.$$

For $I_2(1)$, by using (B.4), we derive

$$I_2(1) = \frac{1}{2}\left\{ erf\left[\sqrt{\frac{1}{2}}\left(\frac{-Q_2}{P_2}\right)\right] - erf\left[\sqrt{\frac{1}{2}}\left(\frac{-Q_1}{P_1}\right)\right]\right\}.$$

For $I_2(2)$, we observe that

$$\frac{1}{4\sqrt{2\pi}} \int_L^U -2e^{c_1 A + c_2 A^2 + \left(-\nu^2/2\right)} d\nu$$

$$= \frac{-1}{2\sqrt{2\pi}} \int_L^U e^{c_1\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right) + c_2\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right)^2 + \left(-\nu^2/2\right)} d\nu$$

$$= \frac{-1}{2\sqrt{2\pi}} \int_L^U e^{-\left[\frac{-c_2 P_1^2 + 1}{2}\nu^2 + 2\frac{-P_1\left(c_1 + \sqrt{2}c_2 Q_1\right)}{2\sqrt{2}}\nu + \frac{-Q_1\left(\sqrt{2}c_1 + c_2 Q_1\right)}{2}\right]} d\nu\,,$$

and use (B.4), we obtain

$$I_2(2) = \frac{-1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}\, e^{\left(\frac{g_{10}{}^2 - g_9 g_{11}}{g_9}\right)} \left\{ erf\left[\sqrt{g_9}\left(\frac{-Q_2}{P_2}\right) + \frac{g_{10}}{\sqrt{g_9}}\right] - erf\left[\sqrt{g_9}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{10}}{\sqrt{g_9}}\right]\right\}.$$

For $I_2(3)$, we observe that

$$\frac{1}{4\sqrt{2\pi}} \int_L^U e^{c_1 A + c_2 A^2 + c_1 B + c_2 B^2 + \left(-\nu^2/2\right)} d\nu$$

$$= \frac{1}{4\sqrt{2\pi}} \int_L^U e^{c_1\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right) + c_2\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right)^2 + c_1\left(\frac{P_2\nu + Q_2}{\sqrt{2}}\right) + c_2\left(\frac{P_2\nu + Q_2}{\sqrt{2}}\right)^2 + \left(-\nu^2/2\right)} d\nu$$

$$= \frac{1}{4\sqrt{2\pi}} \int_L^U e^{-\left[\frac{-c_2\left(P_1^2 + P_2^2\right) + 1}{2}\nu^2 + 2\frac{-c_1\left(P_1 + P_2\right) - \sqrt{2}c_2\left(P_1 Q_1 + P_2 Q_2\right)}{2\sqrt{2}}\nu + \frac{-\sqrt{2}c_1\left(Q_1 + Q_2\right) - c_2\left(Q_1{}^2 + Q_2{}^2\right)}{2}\right]} d\nu\,,$$

and use (B.4), we obtain

$$I_2(3)=\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{14}{}^2-g_1g_{15}}{g_1}\right)}\left\{erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{14}}{\sqrt{g_1}}\right]-erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{14}}{\sqrt{g_1}}\right]\right\}.$$

For $I_2(4)$, we observe that

$$\frac{1}{4\sqrt{2\pi}}\int_L^U -2e^{c_1B+c_2B^2+\left(-v^2/2\right)}dv$$

$$=\frac{-1}{2\sqrt{2\pi}}\int_L^U e^{c_1\left(\frac{P_2v+Q_2}{\sqrt{2}}\right)+c_2\left(\frac{P_2v+Q_2}{\sqrt{2}}\right)^2+\left(-v^2/2\right)}dv$$

$$=\frac{-1}{2\sqrt{2\pi}}\int_L^U e^{-\left[\frac{-c_2P_2{}^2+1}{2}v^2+2\frac{-P_2\left(c_1+\sqrt{2}c_2Q_2\right)}{2\sqrt{2}}v+\frac{-Q_2\left(\sqrt{2}c_1+c_2Q_2\right)}{2}\right]}dv\,,$$

and use (B.4), we derive

$$I_2(4)=\frac{-1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_{12}{}^2-g_4g_{13}}{g_4}\right)}\left\{erf\left[\sqrt{g_4}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{12}}{\sqrt{g_4}}\right]-erf\left[\sqrt{g_4}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{12}}{\sqrt{g_4}}\right]\right\}.$$

Accordingly,

$$I_2=\frac{1}{2}\left\{erf\left[\sqrt{\frac{1}{2}}\left(\frac{-Q_2}{P_2}\right)\right]-erf\left[\sqrt{\frac{1}{2}}\left(\frac{-Q_1}{P_1}\right)\right]\right\}$$

$$-\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}e^{\left(\frac{g_{10}{}^2-g_9g_{11}}{g_9}\right)}\left\{erf\left[\sqrt{g_9}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{10}}{\sqrt{g_9}}\right]-erf\left[\sqrt{g_9}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{10}}{\sqrt{g_9}}\right]\right\}$$

$$+\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{14}{}^2-g_1g_{15}}{g_1}\right)}\left\{erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{14}}{\sqrt{g_1}}\right]-erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{14}}{\sqrt{g_1}}\right]\right\}$$

$$-\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_{12}{}^2-g_4g_{13}}{g_4}\right)}\left\{erf\left[\sqrt{g_4}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{12}}{\sqrt{g_4}}\right]-erf\left[\sqrt{g_4}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{12}}{\sqrt{g_4}}\right]\right\}.$$

For I_3, we first observe that

$$I_3=\frac{1}{4}\int_U^\infty[1+erf(A)][1+erf(B)]\phi(v)\,dv$$

$$=\frac{1}{4\sqrt{2\pi}}\int_U^\infty\left(1+1-e^{c_1A+c_2A^2}\right)\left(1-1+e^{-c_1B+c_2B^2}\right)e^{\left(-v^2/2\right)}dv$$

$$=\frac{1}{4\sqrt{2\pi}}\int_U^\infty\left(2-e^{c_1A+c_2A^2}\right)\left(e^{-c_1B+c_2B^2}\right)e^{\left(-v^2/2\right)}dv$$

$$=\frac{1}{4\sqrt{2\pi}}\int_U^\infty\left[2e^{-c_1B+c_2B^2-\left(v^2/2\right)}-e^{c_1A+c_2A^2-c_1B+c_2B^2-\left(v^2/2\right)}\right]dv$$

$$= I_3(1) + I_3(2),$$

where

$$I_3(1) = \frac{1}{4\sqrt{2\pi}} \int_U^\infty 2e^{-c_1 B + c_2 B^2 - (v^2/2)} \, dv,$$

$$I_3(2) = \frac{1}{4\sqrt{2\pi}} \int_U^\infty - e^{c_1 A + c_2 A^2 - c_1 B + c_2 B^2 - (v^2/2)} \, dv.$$

For $I_3(1)$, we observe that

$$\frac{1}{4\sqrt{2\pi}} \int_U^\infty 2e^{-c_1 B + c_2 B^2 - (v^2/2)} \, dv$$

$$= \frac{1}{2\sqrt{2\pi}} \int_U^\infty e^{\left[-c_1\left(\frac{P_2 v + Q_2}{\sqrt{2}}\right) + c_2\left(\frac{P_2 v + Q_2}{\sqrt{2}}\right)^2 - (v^2/2)\right]} dv$$

$$= \frac{1}{2\sqrt{2\pi}} \int_U^\infty e^{-\left[\frac{-c_2 P_2^2 + 1}{2} v^2 + 2\frac{P_2\left(c_1 - \sqrt{2} c_2 Q_2\right)}{2\sqrt{2}} v + \frac{Q_2\left(\sqrt{2} c_1 - c_2 Q_2\right)}{2}\right]} dv,$$

and use (B.4), we obtain

$$I_3(1) = \frac{1}{4\sqrt{2}} \sqrt{\frac{1}{g_4}} e^{\left(\frac{g_5^2 - g_4 g_6}{g_4}\right)} \left\{ erf\left[\sqrt{g_4}(\infty) + \frac{g_5}{\sqrt{g_4}}\right] - erf\left[\sqrt{g_4}\left(\frac{-Q_2}{P_2}\right) + \frac{g_5}{\sqrt{g_4}}\right]\right\}$$

$$= \frac{1}{4\sqrt{2}} \sqrt{\frac{1}{g_4}} e^{\left(\frac{g_5^2 - g_4 g_6}{g_4}\right)} \left\{ 1 - erf\left[\sqrt{g_4}\left(\frac{-Q_2}{P_2}\right) + \frac{g_5}{\sqrt{g_4}}\right]\right\}.$$

For $I_3(2)$, we observe that

$$\frac{1}{4\sqrt{2\pi}} \int_U^\infty - e^{c_1 A + c_2 A^2 - c_1 B + c_2 B^2 - (v^2/2)} \, dv$$

$$= \frac{-1}{4\sqrt{2\pi}} \int_U^\infty e^{c_1\left(\frac{P_1 v + Q_1}{\sqrt{2}}\right) + c_2\left(\frac{P_1 v + Q_1}{\sqrt{2}}\right)^2 - c_1\left(\frac{P_2 v + Q_2}{\sqrt{2}}\right) + c_2\left(\frac{P_2 v + Q_2}{\sqrt{2}}\right)^2 - (v^2/2)} dv$$

$$= \frac{-1}{4\sqrt{2\pi}} \int_U^\infty e^{-\left[\frac{-c_2\left(P_1^2 + P_2^2\right) + 1}{2} v^2 + 2\frac{-\sqrt{2} c_2 (P_1 Q_1 + P_2 Q_2) - c_1 (P_1 - P_2)}{2\sqrt{2}} v + \frac{-\sqrt{2} c_1 (Q_1 - Q_2) - c_2\left(Q_1^2 + Q_2^2\right)}{2}\right]} dv,$$

and use (B.4), we derive

$$I_3(2) = \frac{-1}{8\sqrt{2}} \sqrt{\frac{1}{g_1}} e^{\left(\frac{g_7^2 - g_1 g_8}{g_1}\right)} \left\{ erf\left[\sqrt{g_1}(\infty) + \frac{g_7}{\sqrt{g_1}}\right] - erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right) + \frac{g_7}{\sqrt{g_1}}\right]\right\}$$

$$= \frac{-1}{8\sqrt{2}} \sqrt{\frac{1}{g_1}} e^{\left(\frac{g_7^2 - g_1 g_8}{g_1}\right)} \left\{ 1 - erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right) + \frac{g_7}{\sqrt{g_1}}\right]\right\}.$$

Combining the above results, we have

$$I_3 = \frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_5^2 - g_4 g_6}{g_4}\right)}\left\{1 - erf\left[\sqrt{g_4}\left(\frac{-Q_2}{P_2}\right) + \frac{g_5}{\sqrt{g_4}}\right]\right\}$$
$$-\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_7^2 - g_1 g_8}{g_1}\right)}\left\{1 - erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right) + \frac{g_7}{\sqrt{g_1}}\right]\right\}.$$

Therefore, for case 2, the likelihood function becomes

$$L_i = I_1 + I_2 + I_3 =$$
$$\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}e^{\left(\frac{g_{16}^2 - g_9 g_{17}}{g_9}\right)}\left\{1 + erf\left[\sqrt{g_9}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{16}}{\sqrt{g_9}}\right]\right\}$$
$$-\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{18}^2 - g_1 g_{19}}{g_1}\right)}\left\{1 + erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{18}}{\sqrt{g_1}}\right]\right\}$$
$$+\frac{1}{2}\left\{erf\left[\sqrt{\frac{1}{2}}\left(\frac{-Q_2}{P_2}\right)\right] - erf\left[\sqrt{\frac{1}{2}}\left(\frac{-Q_1}{P_1}\right)\right]\right\}$$
$$-\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}e^{\left(\frac{g_{10}^2 - g_9 g_{11}}{g_9}\right)}\left\{erf\left[\sqrt{g_9}\left(\frac{-Q_2}{P_2}\right) + \frac{g_{10}}{\sqrt{g_9}}\right] - erf\left[\sqrt{g_9}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{10}}{\sqrt{g_9}}\right]\right\}$$
$$+\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{14}^2 - g_1 g_{15}}{g_1}\right)}\left\{erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right) + \frac{g_{14}}{\sqrt{g_1}}\right] - erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{14}}{\sqrt{g_1}}\right]\right\}$$
$$-\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_{12}^2 - g_4 g_{13}}{g_4}\right)}\left\{erf\left[\sqrt{g_4}\left(\frac{-Q_2}{P_2}\right) + \frac{g_{12}}{\sqrt{g_4}}\right] - erf\left[\sqrt{g_4}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{12}}{\sqrt{g_4}}\right]\right\}$$
$$+\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_5^2 - g_4 g_6}{g_4}\right)}\left\{1 - erf\left[\sqrt{g_4}\left(\frac{-Q_2}{P_2}\right) + \frac{g_5}{\sqrt{g_4}}\right]\right\}$$
$$-\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_7^2 - g_1 g_8}{g_1}\right)}\left\{1 - erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right) + \frac{g_7}{\sqrt{g_1}}\right]\right\}.$$

Case 3:

For I_1, we first observe that

$$I_1 = \frac{1}{4}\int_{-\infty}^{L}[1 + erf(A)][1 + erf(B)]\phi(v)\,dv$$

$$= \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} \left[1 + 1 - e^{c_1 A + c_2 A^2}\right]\left[1 - 1 + e^{-c_1 B + c_2 B^2}\right] e^{\left(-\nu^2/2\right)}\, d\nu$$

$$= \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} \left(2 - e^{c_1 A + c_2 A^2}\right)\left(e^{-c_1 B + c_2 B^2}\right) e^{\left(-\nu^2/2\right)}\, d\nu$$

$$= \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} \left[2e^{-c_1 B + c_2 B^2 - \left(\nu^2/2\right)} - e^{c_1 A + c_2 A^2 - c_1 B + c_2 B^2 - \left(\nu^2/2\right)}\right] d\nu$$

$$= I_1(1) + I_1(2),$$

where

$$I_1(1) = \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} 2e^{-c_1 B + c_2 B^2 - \left(\nu^2/2\right)}\, d\nu,$$

$$I_1(2) = \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} - e^{c_1 A + c_2 A^2 - c_1 B + c_2 B^2 - \left(\nu^2/2\right)} d\nu.$$

For $I_1(1)$, we observe that

$$\frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} 2e^{-c_1 B + c_2 B^2 - \left(\nu^2/2\right)}\, d\nu$$

$$= \frac{1}{2\sqrt{2\pi}} \int_{-\infty}^{L} e^{\left[-c_1\left(\frac{P_2\nu + Q_2}{\sqrt{2}}\right) + c_2\left(\frac{P_2\nu + Q_2}{\sqrt{2}}\right)^2 - \left(\nu^2/2\right)\right]} d\nu$$

$$= \frac{1}{2\sqrt{2\pi}} \int_{-\infty}^{L} e^{-\left[\frac{-c_2 P_2{}^2 + 1}{2}\nu^2 + 2\frac{P_2\left(c_1 - \sqrt{2}c_2 Q_2\right)}{2\sqrt{2}}\nu + \frac{Q_2\left(\sqrt{2}c_1 - c_2 Q_2\right)}{2}\right]} d\nu,$$

and use (B.4), we obtain

$$I_1(1) = \frac{1}{4\sqrt{2}} \sqrt{\frac{1}{g_4}} e^{\left(\frac{g_5{}^2 - g_4 g_6}{g_4}\right)} \left\{ erf\left[\sqrt{g_4}\left(\frac{-Q_1}{P_1}\right) + \frac{g_5}{\sqrt{g_4}}\right] - erf\left[\sqrt{g_4}(-\infty) + \frac{g_5}{\sqrt{g_4}}\right] \right\}$$

$$= \frac{1}{4\sqrt{2}} \sqrt{\frac{1}{g_4}} e^{\left(\frac{g_5{}^2 - g_4 g_6}{g_4}\right)} \left\{ 1 + erf\left[\sqrt{g_4}\left(\frac{-Q_1}{P_1}\right) + \frac{g_5}{\sqrt{g_4}}\right] \right\}.$$

For $I_1(2)$, we observe that

$$\frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} - e^{c_1 A + c_2 A^2 - c_1 B + c_2 B^2 - \left(\nu^2/2\right)} d\nu$$

$$= \frac{-1}{4\sqrt{2\pi}} \int_{-\infty}^{L} e^{c_1\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right) + c_2\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right)^2 - c_1\left(\frac{P_2\nu + Q_2}{\sqrt{2}}\right) + c_2\left(\frac{P_2\nu + Q_2}{\sqrt{2}}\right)^2 - \left(\frac{\nu^2}{2}\right)} d\nu$$

$$= \frac{-1}{4\sqrt{2\pi}} \int_{-\infty}^{L} e^{-\left[\frac{-c_2\left(P_1{}^2 + P_2{}^2\right) + 1}{2}\nu^2 + 2\frac{-\sqrt{2}c_2(P_1 Q_1 + P_2 Q_2) - c_1(P_1 - P_2)}{2\sqrt{2}}\nu + \frac{-\sqrt{2}c_1(Q_1 - Q_2) - c_2\left(Q_1{}^2 + Q_2{}^2\right)}{2}\right]} d\nu,$$

and use (B.4), $I_1(2)$ becomes

$$I_1(2) = \frac{-1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_7{}^2 - g_1 g_8}{g_1}\right)}\left\{erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_7}{\sqrt{g_1}}\right]-erf\left[\sqrt{g_1}(-\infty)+\frac{g_7}{\sqrt{g_1}}\right]\right\}$$

$$= \frac{-1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_7{}^2 - g_1 g_8}{g_1}\right)}\left\{1+erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_7}{\sqrt{g_1}}\right]\right\}.$$

Accordingly, we have

$$I_1 = \frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_5{}^2 - g_4 g_6}{g_4}\right)}\left\{1+erf\left[\sqrt{g_4}\left(\frac{-Q_1}{P_1}\right)+\frac{g_5}{\sqrt{g_4}}\right]\right\}$$

$$-\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_7{}^2 - g_1 g_8}{g_1}\right)}\left\{1+erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_7}{\sqrt{g_1}}\right]\right\}.$$

For I_2, we first observe that

$$I_2 = \frac{1}{4}\int_L^U [1+erf(A)][1+erf(B)]\phi(\nu)\,d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_L^U \left(1-1+e^{-c_1 A + c_2 A^2}\right)\left(1-1+e^{-c_1 B + c_2 B^2}\right)e^{\left(-\nu^2/2\right)}\,d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_L^U e^{\left[-c_1 A + c_2 A^2 - c_1 B + c_2 B^2 - \left(\nu^2/2\right)\right]}d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_L^U e^{\left[\frac{-c_1(P_1\nu+Q_1)}{\sqrt{2}}+\frac{c_2(P_1\nu+Q_1)^2}{2}+\frac{-c_1(P_2\nu+Q_2)}{\sqrt{2}}+\frac{c_2(P_2\nu+Q_2)^2}{2}+\left(\frac{-\nu^2}{2}\right)\right]}d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_L^U e^{-\left[\frac{-c_2\left(P_1^2+P_2{}^2\right)+1}{2}\nu^2+2\frac{-\sqrt{2}c_2(P_1Q_1+P_2Q_2)+c_1(P_1+P_2)}{2\sqrt{2}}\nu+\frac{\sqrt{2}c_1(Q_1+Q_2)-c_2\left(Q_1{}^2+Q_2{}^2\right)}{2}\right]}d\nu,$$

and use (B.4), we derive

$$I_2 = \frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_2{}^2 - g_1 g_3}{g_1}\right)}\left\{erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_2}{\sqrt{g_1}}\right]-erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_2}{\sqrt{g_1}}\right]\right\}.$$

For I_3, we first observe that

$$I_3 = \frac{1}{4}\int_U^\infty [1+erf(A)][1+erf(B)]\phi(\nu)\,d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_U^\infty \left(1-1+e^{-c_1 A + c_2 A^2}\right)\left(1+1-e^{c_1 B + c_2 B^2}\right)e^{\left(-\nu^2/2\right)}d\nu$$

$$= \frac{1}{4\sqrt{2\pi}} \int_U^\infty \left(e^{-c_1 A + c_2 A^2}\right)\left(2 - e^{c_1 B + c_2 B^2}\right) e^{\left(-\nu^2/2\right)} d\nu$$

$$= \frac{1}{4\sqrt{2\pi}} \int_U^\infty \left[2e^{-c_1 A + c_2 A^2 - \left(\nu^2/2\right)} - e^{-c_1 A + c_2 A^2 + c_1 B + c_2 B^2 - \left(\nu^2/2\right)}\right] d\nu$$

$$= I_3(1) + I_3(2),$$

where

$$I_3(1) = \frac{1}{4\sqrt{2\pi}} \int_U^\infty 2e^{-c_1 A + c_2 A^2 - \left(\nu^2/2\right)}\, d\nu,$$

$$I_3(2) = \frac{1}{4\sqrt{2\pi}} \int_U^\infty - e^{-c_1 A + c_2 A^2 + c_1 B + c_2 B^2 - \left(\nu^2/2\right)} d\nu.$$

For $I_3(1)$, we observe that

$$\frac{1}{4\sqrt{2\pi}} \int_U^\infty 2e^{-c_1 A + c_2 A^2 - \left(\nu^2/2\right)}\, d\nu$$

$$= \frac{1}{2\sqrt{2\pi}} \int_U^\infty e^{\left[-c_1\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right) + c_2\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right)^2 - \left(\nu^2/2\right)\right]} d\nu$$

$$= \frac{1}{2\sqrt{2\pi}} \int_U^\infty e^{-\left[\frac{-c_2 P_1^2 + 1}{2}\nu^2 + 2\frac{-P_1\left(-c_1 + \sqrt{2}c_2 Q_1\right)}{2\sqrt{2}}\nu + \frac{-Q_1\left(-\sqrt{2}c_1 + c_2 Q_1\right)}{2}\right]} d\nu,$$

and use (B.4), we obtain

$$I_3(1) = \frac{1}{4\sqrt{2}} \sqrt{\frac{1}{g_9}} e^{\left(\frac{g_{16}{}^2 - g_9 g_{17}}{g_9}\right)} \left\{ erf\left[\sqrt{g_9}(\infty) + \frac{g_{16}}{\sqrt{g_9}}\right] - erf\left[\sqrt{g_9}\left(\frac{-Q_2}{P_2}\right) + \frac{g_{16}}{\sqrt{g_9}}\right]\right\}$$

$$= \frac{1}{4\sqrt{2}} \sqrt{\frac{1}{g_9}} e^{\left(\frac{g_{16}{}^2 - g_9 g_{17}}{g_9}\right)} \left\{1 - erf\left[\sqrt{g_9}\left(\frac{-Q_2}{P_2}\right) + \frac{g_{16}}{\sqrt{g_9}}\right]\right\}.$$

For $I_3(2)$, we observe that

$$\frac{1}{4\sqrt{2\pi}} \int_U^\infty - e^{-c_1 A + c_2 A^2 + c_1 B + c_2 B^2 - \left(\nu^2/2\right)} d\nu$$

$$= \frac{-1}{4\sqrt{2\pi}} \int_U^\infty e^{-c_1\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right) + c_2\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right)^2 + c_1\left(\frac{P_2\nu + Q_2}{\sqrt{2}}\right) + c_2\left(\frac{P_2\nu + Q_2}{\sqrt{2}}\right)^2 - \left(\nu^2/2\right)} d\nu$$

$$= \frac{-1}{4\sqrt{2\pi}} \int_U^\infty e^{-\left[\frac{-c_2\left(P_1^2 + P_2^2\right) + 1}{2}\nu^2 + 2\frac{-\sqrt{2}c_2\left(P_1 Q_1 + P_2 Q_2\right) + c_1\left(P_1 - P_2\right)}{2\sqrt{2}}\nu + \frac{\sqrt{2}c_1\left(Q_1 - Q_2\right) - c_2\left(Q_1{}^2 + Q_2{}^2\right)}{2}\right]} d\nu,$$

and use (B.4), $I_3(2)$ becomes

$$I_3(2)=\frac{-1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{18}{}^2-g_1g_{19}}{g_1}\right)}\left\{erf\left[\sqrt{g_1}(\infty)+\frac{g_{18}}{\sqrt{g_1}}\right]-erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{18}}{\sqrt{g_1}}\right]\right\}$$

$$=\frac{-1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{18}{}^2-g_1g_{19}}{g_1}\right)}\left\{1-erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{18}}{\sqrt{g_1}}\right]\right\}.$$

Combining the derived, we have

$$I_3=\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}e^{\left(\frac{g_{16}{}^2-g_9g_{17}}{g_9}\right)}\left\{1-erf\left[\sqrt{g_9}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{16}}{\sqrt{g_9}}\right]\right\}$$

$$-\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{18}{}^2-g_1g_{19}}{g_1}\right)}\left\{1-erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{18}}{\sqrt{g_1}}\right]\right\}.$$

Thus, for case 3, the pairwise likelihood function becomes

$$L_i=I_1+I_2+I_3=$$

$$\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_5{}^2-g_4g_6}{g_4}\right)}\left\{1+erf\left[\sqrt{g_4}\left(\frac{-Q_1}{P_1}\right)+\frac{g_5}{\sqrt{g_4}}\right]\right\}$$

$$-\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_7{}^2-g_1g_8}{g_1}\right)}\left\{1+erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_7}{\sqrt{g_1}}\right]\right\}$$

$$+\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_2{}^2-g_1g_3}{g_1}\right)}\left\{erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_2}{\sqrt{g_1}}\right]-erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_2}{\sqrt{g_1}}\right]\right\}$$

$$+\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}e^{\left(\frac{g_{16}{}^2-g_9g_{17}}{g_9}\right)}\left\{1-erf\left[\sqrt{g_9}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{16}}{\sqrt{g_9}}\right]\right\}$$

$$-\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{18}{}^2-g_1g_{19}}{g_1}\right)}\left\{1-erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{18}}{\sqrt{g_1}}\right]\right\}.$$

Case 4:

For I_1, we first observe that

$$I_1=\frac{1}{4}\int_{-\infty}^{L}\left[1+erf(A)\right]\left[1+erf(B)\right]\phi(v)\,dv$$

$$=\frac{1}{4\sqrt{2\pi}}\int_{-\infty}^{L}\left[1+1-e^{c_1A+c_2A^2}\right]\left[1+1-e^{c_1B+c_2B^2}\right]e^{\left(-v^2/2\right)}\,dv$$

$$= \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} \left(2 - e^{c_1 A + c_2 A^2}\right)\left(2 - e^{c_1 B + c_2 B^2}\right) e^{\left(-\nu^2/2\right)} d\nu$$

$$= \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} \left[4e^{\left(-\nu^2/2\right)} - 2e^{c_1 A + c_2 A^2 + \left(-\nu^2/2\right)} - 2e^{c_1 B + c_2 B^2 + \left(-\nu^2/2\right)} + e^{c_1 A + c_2 A^2 + c_1 B + c_2 B^2 + \left(-\nu^2/2\right)}\right] d\nu$$

$$= I_1(1) + I_1(2) + I_1(3) + I_1(4),$$

where

$$I_1(1) = \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} \left[4e^{\left(-\nu^2/2\right)}\right] d\nu,$$

$$I_1(2) = \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} \left[-2e^{c_1 A + c_2 A^2 + \left(-\nu^2/2\right)}\right] d\nu,$$

$$I_1(3) = \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} \left[-2e^{c_1 B + c_2 B^2 + \left(-\nu^2/2\right)}\right] d\nu,$$

$$I_1(4) = \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} \left[e^{c_1 A + c_2 A^2 + c_1 B + c_2 B^2 + \left(-\nu^2/2\right)}\right] d\nu.$$

For $I_1(1)$, by using (B.4), we obtain

$$I_1(1) = \frac{1}{2}\left\{ erf\left[\sqrt{\frac{1}{2}}\left(\frac{-Q_1}{P_1}\right)\right] - erf\left[\sqrt{\frac{1}{2}}(-\infty)\right]\right\} = \frac{1}{2}\left[1 + erf\left(\frac{-Q_1}{\sqrt{2}P_1}\right)\right].$$

For $I_1(2)$, we observe that

$$\frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} -2e^{c_1 A + c_2 A^2 + \left(-\nu^2/2\right)} d\nu$$

$$= \frac{-1}{2\sqrt{2\pi}} \int_{-\infty}^{L} e^{c_1\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right) + c_2\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right)^2 + \left(-\nu^2/2\right)} d\nu$$

$$= \frac{-1}{2\sqrt{2\pi}} \int_{-\infty}^{L} e^{-\left[\frac{-c_2 P_1{}^2 + 1}{2}\nu^2 + 2\frac{-P_1\left(c_1 + \sqrt{2}c_2 Q_1\right)}{2\sqrt{2}}\nu + \frac{-Q_1\left(\sqrt{2}c_1 + c_2 Q_1\right)}{2}\right]} d\nu,$$

and use (B.4), we obtain

$$I_1(2) = \frac{-1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}\, e^{\left(\frac{g_{10}{}^2 - g_9 g_{11}}{g_9}\right)} \left\{ erf\left[\sqrt{g_9}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{10}}{\sqrt{g_9}}\right] - erf\left[\sqrt{g_9}(-\infty) + \frac{g_{10}}{\sqrt{g_9}}\right]\right\}$$

$$= \frac{-1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}\, e^{\left(\frac{g_{10}{}^2 - g_9 g_{11}}{g_9}\right)} \left\{1 + erf\left[\sqrt{g_9}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{10}}{\sqrt{g_9}}\right]\right\}.$$

For $I_1(3)$, we observe that

$$\frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{L} -2e^{c_1 B + c_2 B^2 + \left(-\nu^2/2\right)} d\nu$$

$$=\frac{-1}{2\sqrt{2\pi}}\int_{-\infty}^{L}e^{c_1\left(\frac{P_2\nu+Q_2}{\sqrt{2}}\right)+c_2\left(\frac{P_2\nu+Q_2}{\sqrt{2}}\right)^2+\left(-\nu^2/2\right)}d\nu$$

$$=\frac{-1}{2\sqrt{2\pi}}\int_{-\infty}^{L}e^{-\left[\frac{-c_2P_2^2+1}{2}\nu^2+2\frac{-P_2\left(c_1+\sqrt{2}c_2Q_2\right)}{2\sqrt{2}}\nu+\frac{-Q_2\left(\sqrt{2}c_1+c_2Q_2\right)}{2}\right]}d\nu,$$

and use (B.4), we derive

$$I_1(3)=\frac{-1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_{12}{}^2-g_4g_{13}}{g_4}\right)}\left\{erf\left[\sqrt{g_4}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{12}}{\sqrt{g_4}}\right]-erf\left[\sqrt{g_4}\left(-\infty\right)+\frac{g_{12}}{\sqrt{g_4}}\right]\right\}$$

$$=\frac{-1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_{12}{}^2-g_4g_{13}}{g_4}\right)}\left\{1+erf\left[\sqrt{g_4}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{12}}{\sqrt{g_4}}\right]\right\}.$$

For $I_1(4)$, we observe that

$$\frac{1}{4\sqrt{2\pi}}\int_{-\infty}^{L}e^{c_1A+c_2A^2+c_1B+c_2B^2+\left(-\nu^2/2\right)}d\nu$$

$$=\frac{1}{4\sqrt{2\pi}}\int_{-\infty}^{L}e^{c_1\left(\frac{P_1\nu+Q_1}{\sqrt{2}}\right)+c_2\left(\frac{P_1\nu+Q_1}{\sqrt{2}}\right)^2+c_1\left(\frac{P_2\nu+Q_2}{\sqrt{2}}\right)+c_2\left(\frac{P_2\nu+Q_2}{\sqrt{2}}\right)^2+\left(-\nu^2/2\right)}d\nu$$

$$=\frac{1}{4\sqrt{2\pi}}\int_{-\infty}^{L}e^{-\left[\frac{-c_2\left(P_1^2+P_2^2\right)+1}{2}\nu^2+2\frac{-c_1\left(P_1+P_2\right)-\sqrt{2}c_2\left(P_1Q_1+P_2Q_2\right)}{2\sqrt{2}}\nu+\frac{-\sqrt{2}c_1\left(Q_1+Q_2\right)-c_2\left(Q_1{}^2+Q_2{}^2\right)}{2}\right]}d\nu,$$

and use (B.4), we obtain

$$I_1(4)=\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{14}{}^2-g_1g_{15}}{g_1}\right)}\left\{erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{14}}{\sqrt{g_1}}\right]-erf\left[\sqrt{g_1}\left(-\infty\right)+\frac{g_{14}}{\sqrt{g_1}}\right]\right\}$$

$$=\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{14}{}^2-g_1g_{15}}{g_1}\right)}\left\{1+erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{14}}{\sqrt{g_1}}\right]\right\}.$$

According to the above findings, we have

$$I_1 = \frac{1}{2}\left[1 + erf\left(\frac{-Q_1}{\sqrt{2}P_1}\right)\right]$$

$$-\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}e^{\left(\frac{g_{10}{}^2 - g_9 g_{11}}{g_9}\right)}\left\{1 + erf\left[\sqrt{g_9}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{10}}{\sqrt{g_9}}\right]\right\}$$

$$-\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_{12}{}^2 - g_4 g_{13}}{g_4}\right)}\left\{1 + erf\left[\sqrt{g_4}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{12}}{\sqrt{g_4}}\right]\right\}$$

$$+\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{14}{}^2 - g_1 g_{15}}{g_1}\right)}\left\{1 + erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{14}}{\sqrt{g_1}}\right]\right\}.$$

For I_2, we first observe that

$$I_2 = \frac{1}{4}\int_L^U [1 + erf(A)][1 + erf(B)]\phi(\nu)\,d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_L^U \left(1 - 1 + e^{-c_1 A + c_2 A^2}\right)\left(1 + 1 - e^{c_1 B + c_2 B^2}\right) e^{\left(-\nu^2/2\right)}\,d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_L^U \left(e^{-c_1 A + c_2 A^2}\right)\left(2 - e^{c_1 B + c_2 B^2}\right) e^{\left(-\nu^2/2\right)}\,d\nu$$

$$= \frac{1}{4\sqrt{2\pi}}\int_L^U \left[2e^{-c_1 A + c_2 A^2 - \left(\nu^2/2\right)} - e^{-c_1 A + c_2 A^2 + c_1 B + c_2 B^2 - \left(\nu^2/2\right)}\right]d\nu$$

$$= I_2(1) + I_2(2),$$

where

$$I_2(1) = \frac{1}{4\sqrt{2\pi}}\int_L^U 2e^{-c_1 A + c_2 A^2 - \left(\nu^2/2\right)}\,d\nu,$$

$$I_2(2) = \frac{1}{4\sqrt{2\pi}}\int_L^U -e^{-c_1 A + c_2 A^2 + c_1 B + c_2 B^2 - \left(\nu^2/2\right)}d\nu.$$

For $I_2(1)$, we observe that

$$\frac{1}{4\sqrt{2\pi}}\int_L^U 2e^{-c_1 A + c_2 A^2 - \left(\nu^2/2\right)}\,d\nu$$

$$= \frac{1}{2\sqrt{2\pi}}\int_L^U e^{\left[-c_1\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right) + c_2\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right)^2 - \left(\nu^2/2\right)\right]}d\nu$$

$$= \frac{1}{2\sqrt{2\pi}}\int_L^U e^{-\left[\frac{-c_2 P_1^2 + 1}{2}\nu^2 + 2\frac{-P_1\left(-c_1 + \sqrt{2}c_2 Q_1\right)}{2\sqrt{2}}\nu + \frac{-Q_1\left(-\sqrt{2}c_1 + c_2 Q_1\right)}{2}\right]}d\nu,$$

and use (B.4), we obtain

$$I_2(1)=\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}e^{\left(\frac{g_{16}{}^2-g_9g_{17}}{g_9}\right)}\left\{erf\left[\sqrt{g_9}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{16}}{\sqrt{g_9}}\right]-erf\left[\sqrt{g_9}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{16}}{\sqrt{g_9}}\right]\right\}.$$

For $I_2(2)$, we observe that

$$\frac{1}{4\sqrt{2\pi}}\int_L^U -e^{-c_1A+c_2A^2+c_1B+c_2B^2-\left(v^2/2\right)}dv$$

$$=\frac{-1}{4\sqrt{2\pi}}\int_L^U e^{-c_1\left(\frac{P_1v+Q_1}{\sqrt{2}}\right)+c_2\left(\frac{P_1v+Q_1}{\sqrt{2}}\right)^2+c_1\left(\frac{P_2v+Q_2}{\sqrt{2}}\right)+c_2\left(\frac{P_2v+Q_2}{\sqrt{2}}\right)^2-\left(v^2/2\right)}dv$$

$$=\frac{-1}{4\sqrt{2\pi}}\int_L^U e^{-\left[\frac{-c_2\left(P_1{}^2+P_2{}^2\right)+1}{2}v^2+2\frac{-\sqrt{2}c_2(P_1Q_1+P_2Q_2)+c_1(P_1-P_2)}{2\sqrt{2}}v+\frac{\sqrt{2}c_1(Q_1-Q_2)-c_2\left(Q_1{}^2+Q_2{}^2\right)}{2}\right]}dv,$$

and use (B.4), $I_2(2)$ becomes

$$I_2(2)=\frac{-1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{18}{}^2-g_1g_{19}}{g_1}\right)}\left\{erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{18}}{\sqrt{g_1}}\right]-erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{18}}{\sqrt{g_1}}\right]\right\}.$$

Accordingly,

$$I_2=\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}e^{\left(\frac{g_{16}{}^2-g_9g_{17}}{g_9}\right)}\left\{erf\left[\sqrt{g_9}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{16}}{\sqrt{g_9}}\right]-erf\left[\sqrt{g_9}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{16}}{\sqrt{g_9}}\right]\right\}$$
$$-\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{18}{}^2-g_1g_{19}}{g_1}\right)}\left\{erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right)+\frac{g_{18}}{\sqrt{g_1}}\right]-erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right)+\frac{g_{18}}{\sqrt{g_1}}\right]\right\}.$$

For I_3, we first observe that

$$I_3=\frac{1}{4}\int_U^\infty[1+erf(A)][1+erf(B)]\phi(v)\,dv$$

$$=\frac{1}{4\sqrt{2\pi}}\int_U^\infty\left[1-1+e^{-c_1A+c_2A^2}\right]\left[1-1+e^{-c_1B+c_2B^2}\right]e^{\left(-v^2/2\right)}dv$$

$$=\frac{1}{4\sqrt{2\pi}}\int_U^\infty e^{\left[-c_1A+c_2A^2-c_1B+c_2B^2-\left(v^2/2\right)\right]}dv$$

$$=\frac{1}{4\sqrt{2\pi}}\int_U^\infty e^{\left[\frac{-c_1(P_1v+Q_1)}{\sqrt{2}}+\frac{c_2(P_1v+Q_1)^2}{2}+\frac{-c_1(P_2v+Q_2)}{\sqrt{2}}+\frac{c_2(P_2v+Q_2)^2}{2}+\left(\frac{-v^2}{2}\right)\right]}dv$$

$$=\frac{1}{4\sqrt{2\pi}}\int_U^\infty e^{-\left[\frac{-c_2\left(P_1{}^2+P_2{}^2\right)+1}{2}v^2+2\frac{-\sqrt{2}c_2(P_1Q_1+P_2Q_2)+c_1(P_1+P_2)}{2\sqrt{2}}v+\frac{\sqrt{2}c_1(Q_1+Q_2)-c_2\left(Q_1{}^2+Q_2{}^2\right)}{2}\right]}dv,$$

and use (B.4), I_3 becomes

$$I_3 = \frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_2{}^2 - g_1 g_3}{g_1}\right)}\left\{erf\left[\sqrt{g_1}(\infty) + \frac{g_2}{\sqrt{g_1}}\right] - erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right) + \frac{g_2}{\sqrt{g_1}}\right]\right\}$$

$$= \frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_2{}^2 - g_1 g_3}{g_1}\right)}\left\{1 - erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right) + \frac{g_2}{\sqrt{g_1}}\right]\right\}.$$

Therefore, for case 4, the pairwise likelihood function becomes

$$L_i = I_1 + I_2 + I_3 =$$

$$\frac{1}{2}\left[1 + erf\left(\frac{-Q_1}{\sqrt{2}P_1}\right)\right]$$

$$-\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}e^{\left(\frac{g_{10}{}^2 - g_9 g_{11}}{g_9}\right)}\left\{1 + erf\left[\sqrt{g_9}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{10}}{\sqrt{g_9}}\right]\right\}$$

$$-\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_4}}e^{\left(\frac{g_{12}{}^2 - g_4 g_{13}}{g_4}\right)}\left\{1 + erf\left[\sqrt{g_4}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{12}}{\sqrt{g_4}}\right]\right\}$$

$$+\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{14}{}^2 - g_1 g_{15}}{g_1}\right)}\left\{1 + erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{14}}{\sqrt{g_1}}\right]\right\}$$

$$+\frac{1}{4\sqrt{2}}\sqrt{\frac{1}{g_9}}e^{\left(\frac{g_{16}{}^2 - g_9 g_{17}}{g_9}\right)}\left\{erf\left[\sqrt{g_9}\left(\frac{-Q_2}{P_2}\right) + \frac{g_{16}}{\sqrt{g_9}}\right] - erf\left[\sqrt{g_9}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{16}}{\sqrt{g_9}}\right]\right\}$$

$$-\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_{18}{}^2 - g_1 g_{19}}{g_1}\right)}\left\{erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right) + \frac{g_{18}}{\sqrt{g_1}}\right] - erf\left[\sqrt{g_1}\left(\frac{-Q_1}{P_1}\right) + \frac{g_{18}}{\sqrt{g_1}}\right]\right\}$$

$$+\frac{1}{8\sqrt{2}}\sqrt{\frac{1}{g_1}}e^{\left(\frac{g_2{}^2 - g_1 g_3}{g_1}\right)}\left\{1 - erf\left[\sqrt{g_1}\left(\frac{-Q_2}{P_2}\right) + \frac{g_2}{\sqrt{g_1}}\right]\right\}.$$