Exercise sheet 10.

May 5th

Due May 12th in class.

Exercise 26. In this and the next exercise we do some baby Bruhat-Tits theory. Let

$$\mathrm{Iw} = \begin{bmatrix} \mathcal{O}_F^\times & \mathcal{O}_F \\ \mathfrak{m}_F & \mathcal{O}_F^\times \end{bmatrix} = \{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, d \in \mathcal{O}_F^\times,\, b \in \mathcal{O}_F,\, c \in \mathfrak{m}_F \}$$

be an Iwahori subgroup similar to that in Exercise 24.

(1) Show that $N_{\mathrm{GL}_2(F)}(\mathrm{Iw})/\mathrm{Iw} \cong \mathbb{Z}$, and this map agrees with the valuation of the determinant. [CC: This is the Kottwitz map.]

(2) Show that the normalizer $N_{\mathrm{GL}_2(F)}(\mathrm{Iw})$ is open and compact mod center, i.e. $N_{\mathrm{GL}_2(F)}(\mathrm{Iw})/F^\times$ is compact.

Exercise 27. We work with $\mathrm{Iw} \subset \mathrm{GL}_2(F)$ as in the previous exercise. Suppose $t \in \mathrm{GL}_2(F)$ has two eigenvalues in $E \backslash F$ for a ramified quadratic extension E/F, i.e. such that $\varpi_F \cdot \mathcal{O}_E = \mathfrak{m}_E^2$. Show that there exists $g \in \mathrm{GL}_2(F)$ such that

$$T = Z_G(t) \subset \mathrm{Ad}(g) N_{\mathrm{GL}_2(F)}(\mathrm{Iw}) \text{ and } T \cap \mathrm{Ad}(g)\mathrm{Iw} \cong \mathcal{O}_E^\times.$$

[CC: As in Exercise 24, the Iwahori subgroup, or in general Bruhat-Tits theory, will provide us a way to give provide a sharp bound on the neighborhood on which local character expansions are valid, which is done in [?]. We are not getting anywhere close in this course.]