Exercise sheet 11.

May 12th

Due May 19th in class.

Exercise 28. Prove Lemma 9.1, that the restriction map $X^*(\mathbb{M}) \to X^*(\mathbb{S})$ is injective with finite cokernel.

Exercise 29. Let Z be any finite abelian group. Show that there exists a connected reductive $\mathbb{G}/_F$ (for any field F of your choice, possibly depending on Z, and not necessarily a p-adic field) such that $\operatorname{coker}(X^*(\mathbb{M}) \to X^*(\mathbb{S})) \cong Z$.

Exercise 30. Let $\mathbb{G} = \mathrm{SL}_2$ and F be some undetermined finite extension of $\mathbb{Q}_2$. Let $\iota : \mathfrak{sl}_2(F)^* \xrightarrow{\sim} \mathfrak{sl}_2(F)$ be given by $\iota(X)(Y) = \operatorname{Tr}(XY)$. Let $\mathfrak{K} = \mathfrak{sl}_2(\mathcal{O}_F) \subset \mathfrak{g} = \mathfrak{sl}_2(F)$. Show that there is no universal constant $N \in \mathbb{Z}$ such that $\mathfrak{K}_N \subset \iota(\mathfrak{K}^\perp) \subset \mathfrak{K}_{-N}$ for every F.