Exercise sheet 5.

March 31st

Due April 7th in class.

Exercise 15. Suppose G is a p-adic Lie group and H a closed Lie subgroup such that $\Delta_G = 1$ and $\Delta_H = 1$. Show that under Lemma 5.19 we have

(1) For any open compact subgroup $J \subset H$ and any $g \in G$, we have

$$|\omega_{G/H}|(JgH/H) \cdot |\omega_H|(g^{-1}Jg \cap H) = |\omega_G|(J).$$

(2) Denote the measures by dg, dh and $d(gH)$. Show that for any $f \in \mathcal{C}_c^\infty(G)$ we have

$$\int_G f(g)dg = \int_{G/H} \int_H f(gh)dh \cdot d(gH).$$

Exercise 16. (*) Prove indeed that for any smooth H-representation ρ we have

(1) Consider the right H-action on G. There is a G-equivariant isomorphism

$$M_c^\infty(G) \otimes_{M_c^\infty(H)} \rho \xrightarrow{\sim} \mathrm{ind}_H^G \rho$$

where $M_c^\infty(H)$ acts on $M_c^\infty(G)$ by right convolution, and $\mathrm{ind}_H^G \rho$ is the space of locally constant H-equivariant $M_c^\infty(U')$-valued functions on G that is compactly supported mod H.

(2) We have an isomorphism of coinvariants

$$\rho_H \cong (\mathrm{ind}_H^G \rho)_G.$$