Exercise sheet 7.

April 14th

Due April 21st in class.

Exercise 19. Let $\mathcal{C} = \mathbb{C}$, prove directly without using Theorem C' that the function t_π in Exercise 4 is locally-L^1.

Exercise 20. (*) Again $\mathcal{C} = \mathbb{C}$ and we look at the representation $C_c^\infty(\mathbb{P}^1(\mathbb{Q}_p))$ of $\mathrm{GL}_2(\mathbb{Q}_p)$. Prove (as in Theorem 4.13(2), but without using it or Theorem C') locally near the identity we have $\Theta_\pi \overset{\star}{=} \widehat{I_{O^*}}$ where O^* is the unique non-zero nilpotent orbit in $\mathfrak{gl}_2(\mathbb{Q}_p)$.

Exercise 21. Suppose $\mathcal{C} = \mathbb{F}_2$ and p is odd, and we look at the representation $C_c^\infty(\mathbb{P}^1(\mathbb{Q}_p))$ of $\mathrm{GL}_2(\mathbb{Q}_p)$. Show that its character as in Definition 1.10 is a non-zero generalized function.
(On the other hand, the function t_π as in Exercise 4 is identically 0 mod 2, i.e. the character is identically zero on G^{rs}.)

Exercise 22. (**) Suppose $\mathcal{C} = \mathbb{C}$. The **wave-front set** of an irreducible admissible $\mathbb{C}$-representation is the support $\mathrm{supp}(\delta_n) \subset (\mathfrak{g}^*)^{\mathrm{nil}}$ where δ_n is as in Theorem C'(2) applied to the case $s = e$ (i.e. Theorem 4.13(2)). Likewise, the **wave-front set**[1] of $\widehat{\delta}$ in Theorem C'(1) applied to the case $X_s = 0$ (i.e. Theorem 4.13(1)) is the support $\mathrm{supp}(\delta_n) \subset (\mathfrak{g}^*)^{\mathrm{nil}}$.

Suppose $\mathbb{G} = \mathrm{GL}_n$. Consider any $X \in \mathfrak{gl}_n(F)$. Let $I_{\mathrm{Ad}(G)X}$ be as in Theorem 3.11 or equivalently Corollary 5.32. Show that the wave-front set of $\widehat{I_{\mathrm{Ad}(G)X}}$ is equal to the wave-front set of $\mathrm{Ad}(G)X$ defined in Exercise 10.

(That this holds for arbitrary $\mathbb{G}$ when p is sufficiently large is an unpublished result of DeBacker and myself. I do not know if this is true for arbitrary $\mathbb{G}$ when p is small.)

[1] These are the p-adic analogue of that in https://en.wikipedia.org/wiki/Wave_front_set.