Exercise sheet 8.

April 21st

Due April 28th in class.

Exercise 23. Given $n \in \mathbb{Z}_{\geq 2}$, consider $\mathbb{G} = \mathrm{GL}_n$. Show that for $\Lambda_0 = \mathfrak{gl}_n(\mathcal{O}_F)$, the subset $\mathrm{Ad}(G)\Lambda_0 \subset \mathfrak{g}$ is closed. Give an example (and justify it) of a lattice $\Lambda \subset \mathfrak{g}$ such that $\mathrm{Ad}(G)\Lambda \subset \mathfrak{g}$ is not closed.

Exercise 24. (*) Let

$$\mathrm{iw} = \begin{bmatrix} \mathcal{O}_F & \mathcal{O}_F \\ \mathfrak{m}_F & \mathcal{O}_F \end{bmatrix} = \{ \begin{bmatrix} a & b \\ c & d \end{bmatrix},\ a, b, d \in \mathcal{O}_F,\ c \in \mathfrak{m}_F \}.$$

(This is an Iwahori subalgebra.) Show that for any $f \in \mathcal{C}_c(\mathfrak{gl}_2(F)/\mathrm{iw})$, there exists $f' \in \mathcal{C}(\mathfrak{gl}_2(\mathcal{O}_F)/\mathrm{iw})$ such that $d(f') = d(f)$ for any $d \in D(\mathrm{Ad}(\mathrm{GL}_2(F))\mathfrak{gl}_2(\mathcal{O}_F))^{\mathrm{GL}_2(F)}$. Conclude that the pairing

$$D(\mathrm{Ad}(\mathrm{GL}_2(F)\mathfrak{gl}_2(\mathcal{O}_F))^{\mathrm{GL}_2(F)} \times \mathcal{C}_c(\mathfrak{gl}_2(F)/\mathrm{iw}) \to \mathcal{C}$$

has rank at most 2.

(It has rank exactly 2, and that implies that Proposition 8.1 is incorrect for $n = 0$ with $\Lambda_0 = \mathfrak{gl}_2(\mathcal{O}_F)$.)