Exercise sheet 9.

April 28th

Due May 5th in class.

Exercise 25. Let $n \in \mathbb{Z}_{\geq 1}$, $s \in \mathbb{Z}$ and $d \in \mathbb{Z}_{\geq 0}$ be given. Show that (a) $\implies$ (b) $\implies$ (c) $\implies$ (d) in the following:

(a) $X \in \mathfrak{m}_F^s \cdot \mathfrak{gl}_n(\mathcal{O}_F) \cap \mathfrak{m}_F^{s+nd} \cdot \mathrm{Ad}(\mathrm{GL}_n(F))\mathfrak{gl}_n(\mathcal{O}_F)$.

(b) There exist $k \in \mathrm{GL}_n(\mathcal{O}_F)$, $X_0 \in \mathfrak{m}_F^s \cdot \mathfrak{gl}_n(\mathcal{O}_F)$ and $X_1 \in \mathfrak{m}_F^{s+nd} \cdot \mathfrak{gl}_n(\mathcal{O}_F)$ such that ${}^kX = X_0 + X_1$ and X_0 is strictly upper triangular.

(c) There exist $X_0 \in \mathfrak{m}_F^s \cdot \mathfrak{gl}_n(\mathcal{O}_F)$ and $X_1 \in \mathfrak{m}_F^{s+nd} \cdot \mathfrak{gl}_n(\mathcal{O}_F)$ such that $X = X_0 + X_1$ and X_0 is nilpotent.

(d) $X \in \mathfrak{m}_F^{s+d} \cdot \mathrm{Ad}(\mathrm{GL}_n(F))\mathfrak{gl}_n(\mathcal{O}_F)$.