The order of the problems are randomized. 題目的排序經過隨機調換

True/False questions; Write down **only T/F or O/X** (or whatever you are used to) for each problem. For each correct answer 2 points are awarded, and for each incorrect answer **1 point is taken**. 這一頁只有是非題

(1) Let $\mathbb{G}$ be a connected reductive group over $\mathbb{Q}_p$ with $\dim \mathbb{G} > 0$. Then $\mathbb{G}(\mathbb{Q}_p)$ is non-compact.

(2) Let $\mathbb{G} = \mathrm{GL}_2$, $G = \mathbb{G}(\mathbb{Q}_p)$ and $X \in \mathfrak{gl}_2(\mathbb{Q}_p) = \mathrm{Lie}(G)$. Let $O = \mathrm{Ad}(G)X$. Suppose X is not a scalar multiple of the identity matrix. Then O is a 2-dimensional manifold over $\mathbb{Q}_p$.

(3) If a p-adic Lie group H is abelian (as an abstract group), then for $X, Y \in \mathrm{Lie}(H)$ we have $[X, Y] = 0$.

(4) If a p-adic Lie group H is connected (meaning that the underlying analytic manifold is connected), then $H = \{e\}$.

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Short answers; write down **only an integer** for each problem. For each correct answer 3 points are awarded. 這一頁每題的答案都是整數

(i) Let $p = 3$. Let

$$X := (\mathbb{Z}_3 \backslash 3^2 \mathbb{Z}_3)^2 = \{(x, y) \in (\mathbb{Z}_3)^2 \mid |x|_3, |y|_3 \in \{1, 3^{-1}\}\}$$
$$= \{(x, y) \in (\mathbb{Z}_3)^2 \mid \mathrm{val}_{\mathbb{Q}_3}(x), \mathrm{val}_{\mathbb{Q}_3}(y) \in \{0, 1\}\} \subset (\mathbb{Z}_3)^2$$

which is a 3-adic analytic manifold. Suppose

$$\int_X |\frac{1}{x} dx dy| = \frac{a}{b}$$

where $a \in \mathbb{Z}$ is coprime to 3 and b is an integral power of 3. What is a?

(ii) Let V be a 4-dimensional non-degenerate quadratic space over $F = \mathbb{Q}_p(\sqrt{p})$. Consider $G = \mathrm{O}_V(F)$, which is the group of orthogonal operators on V. It has dimension N when viewed as a Lie group over $\mathbb{Q}_p$ (**not** viewed as a Lie group over $\mathbb{Q}_p(\sqrt{p})$). What is N?

(iii) Let $p = 5$. Consider the group $G = \mathrm{GL}_3(\mathbb{Q}_5)$. It has an irreducible admissible complex representation given by the trivial representation, i.e. the trivial map $\pi : \mathrm{GL}_3(\mathbb{Q}_5) \to \{1\} \subset \mathrm{GL}_1(\mathbb{C})$. The representation π has a character Θ_π. Let $g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5^2 \end{bmatrix}$. It happens that Θ_π is represented by a unique locally constant function $t_\pi : G \to \mathbb{C}$, and that $t_\pi(g)$ is an integer. What is this integer?

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