

# Characters of $p$ -adic groups

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**Course Goal:** After introducing some basics of representations and characters of a  $p$ -adic reductive group  $G$ , we plan to cover a proper subset of the following topics, depending on the interests of the audience:

- (a) The theorem of Harish-Chandra and Howe that a complex irreducible admissible character is locally the Fourier transform of a distribution on a nilpotent cone.
- (b) Harish-Chandra's theorem that a complex irreducible admissible character can be represented by a locally- $L^1$  function that is locally constant on  $G^{\text{rs}}$ .
- (c) Further enhancement by Waldspurger, DeBacker and Adler-Korman, using Bruhat-Tits theory, when  $p$  is very large.
- (d) The theorem of Rodier and of Mœglin-Waldspurger that relates the character near the identity to degenerate Whittaker models.

Moreover, we plan to work around analytical arguments of Harish-Chandra with algebraic approaches, with the additional goals to:

- (1) Understand the results for representations and characters over  $\overline{\mathbb{Q}_\ell}$ , where analysis is not always applicable, and
- (2) Generalize these results to mod-ell representations.

We do **not** plan to discuss applications to the Langlands program.

**Time:** Every Tuesday 1:30pm to 4pm (Taiwan time), from Feb. 24th to May 19th.

**Venue:** Room 509, [Cosmology Building](#), NTU.

**Prerequisite:** The audience are assumed to be familiar with standard sophomore algebra and analysis, as well as the theory of complex representations and characters of finite groups.

The audience will be told to assume  $G$  is a classical group if they are not familiar with algebraic reductive groups, but it's better that they are.

At some point we will start using Bruhat-Tits theory, for which the audience are referred to the course of Masao Oi in the same semester, though we'll give examples for classical groups in this course.

**Credit and grading:** 2 credits. Homework 100%, potentially replaced by presentation if the student wishes.

Those who take the course for credits are expected to submit minimally 1, but hopefully 2, homework problems weekly, in the way specified in the course, unless otherwise arranged with the instructor. Homeworks are due at the start of class starting March 10th.

There might be a few optional in-class quizzes that can be used to replace homework scores.

Getting an A+ is only possible with serious effort on presentation(s).