

0. Prerequisites

- Graduate alg: module theory, linear alg, \otimes ... etc
- Basic caty theory: equiv. of caties, isom. of functors, adjunctions... etc
- Basic homological alg: projectives, Ext, derived caties... etc

Helpful to know but not required:

- Basic Lie theory: Lie alg, root systems, ... etc
- Cohomology: of $\mathbb{C}P^1$... etc
- Algebraic geometry: proper maps, fibers of a morphism of varieties... etc

1. Motivation Study $\text{Rep}(\mathfrak{g})$, \mathfrak{g} : simple Lie alg

• 1970's: BGG caty \mathcal{O} is the "right" caty to study Chp 13-14

\Rightarrow suffices to study its principal block \mathcal{O}_0

\hookrightarrow Verma's 1966 thesis

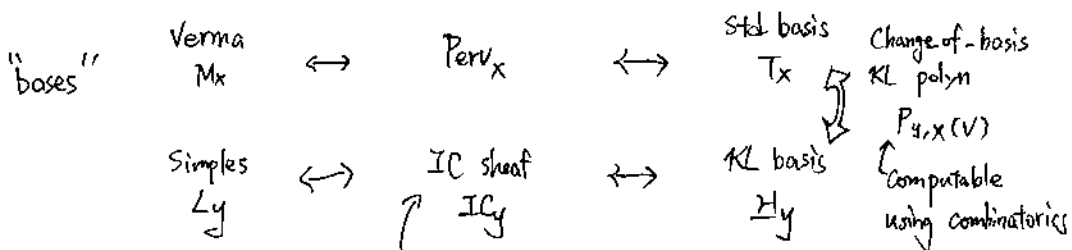
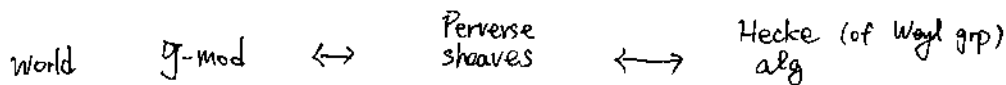
\rightarrow characterize simple mod $L_\lambda \in \mathcal{O}_0$

\rightarrow suffices to compute

Jordan Hölder multy $[M_\lambda : L_\mu]$

\mathcal{O}_0
 \downarrow (oo-dim'l)
 M_λ Verma module
 \uparrow
 $x \in W$: Weyl group Chp 1-2
 (special case of Coxeter grp)

• 1979-1981: Kazhdan-Lusztig theory Chp 15-16



Intersection cohomology

• 1990: Soergel: reformulation of KL theory using
 Soergel modules + decomposition thm (deep/difficult thm in geometry)

• 1992-2006: Soergel establish theory of (now called) Soergel bimodules so its cat

SBim categorifies Hecke alg

Chp 3-5

• Argument goes through for Coxeter grp (not need Weyl grps)
 provided an analog of decompn thm

• 2016: Elias-Williamson: diagrammatic theory

Chp 7-12

SBim \Rightarrow diag. Hecke caty

• Reduces complicated polyn computations in SBim
 to doable manipulations of diagrams

• Hodge theory \Rightarrow alg pt of decomp thm (for Coxeter grps)
Chp 17-20 \Rightarrow alg pt of KL theory (for Coxeter grps)

• over \mathbb{Z} $\xrightarrow[\text{mod } p]{\text{reduction}}$ modular repr

\Rightarrow counterexample to Lusztig's conjecture Chp 27

• Applications in

• Knot theory: Jones polyn, HOMFLY polyn Chp 21

• Koszul duality Chp 25-26

• Positivity of $P_{\lambda, \mu}$