$W = \langle s_i \mid 1 \le i \le d \rangle /$ braid rel'ns, $s_i^2 = 1$

$W^{aff} = W \ltimes {}^L Q$

$= \langle s_i \mid 0 \le i \le d \rangle / \sim$

$W^{ext} = W \ltimes {}^L P$

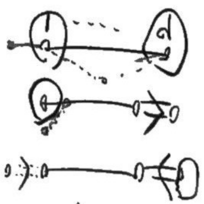
$= W^{aff} \ltimes \Omega$

for $\Omega = \{ w \in W^{ext} \mid \ell(w) = 0 \}$

$= \{ u_j \}_{j \in J}$, $J = \{ 1 \le j \le d \mid \langle \varpi_j^{L}, \theta \rangle = 1 \} \cup \{0\}$

$u_0 = 1$

u_j = shortest elt in $t_{\varpi_j^L} W$



$J = \{0, \dots, d\}$ (type A)

$J = \{0, 1\}$ (type B)

$J = \{0, d\}$ (type C)

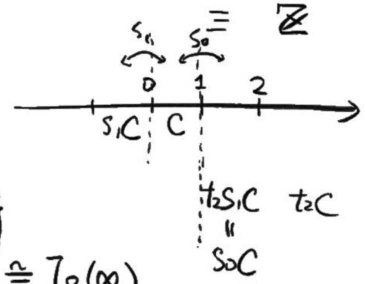
eg type A_1

${}^L Q = Q = \mathbb{Z}\alpha_1 \subsetneq {}^L P = P = \frac{1}{2}\mathbb{Z}\alpha_1$

$\cong 2\mathbb{Z} \qquad \cong \mathbb{Z}$

$W = \langle s_1 \rangle / \sim$

$\cong \Sigma_2$



$W^{aff} = \langle s_1 \rangle \ltimes \{ t_{2n} \}$

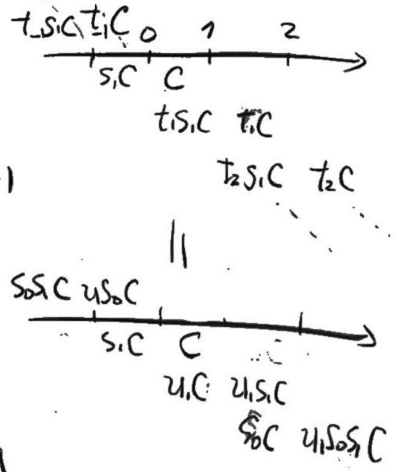
$= \langle s_0, s_1 \rangle / \sim \cong I_2(\infty)$

$W^{ext} = \langle s_1 \rangle \ltimes \{ t_n \}$

${}_2\Omega = \{ 1, u_1 \}$

$u_1 = t_1 s_1 : \lambda \mapsto -\lambda + 1$

= refl'n wrt $1/2$



Thm

$Z(H_q^{ext}(W)) = R(T)^W [q, q^{\pm 1}]$

$\rightsquigarrow H_q(W) = \langle T_i \mid 1 \le i \le d \rangle /$ braid rel'ns

$(T_i + 1)(T_i - q) = 0$

$T_i^2 + (1-q) T_i = q$

$\Downarrow$

$T_i^{-1} = q^{-1} T_i + (q^{-1} - 1)$

$H_q^{aff}(W) = \langle T_i \mid 0 \le i \le d \rangle / \sim$

Chriss-Ginzburg

T = max torus of ${}^L G$

recall $\mathbb{Z}[{}^L P] \cong R({}^L T)$

$\lambda \mapsto e^\lambda \qquad e^\lambda \cdot e^\mu = e^{\lambda + \mu}$

$H_q^{ext}(W) = \langle H_q(W), R({}^L T)[q^{\pm 1}] \rangle /$

(1) $T_i e^\lambda = e^\lambda T_i$ if $\langle \lambda, \alpha_i^\vee \rangle = 0$

(2) $T_i e^{s_i(\lambda)} T_i = q e^\lambda$ —— $= 1$

Lem

(1) & (2) $\iff$ Lusztig rel'n

$T_i e^{s_i(\lambda)} - e^\lambda T_i = (1-q) \dfrac{e^\lambda - e^{s_i(\lambda)}}{1 - e^{\alpha_i}}$

Only need $\langle \lambda, \alpha_i^\vee \rangle = 0, 1$ since ${}^L P$ is gen by ${}^L \varpi_i$

(1): $s_i(\lambda) = \lambda$ ✓

(2): $s_i(\lambda) = \lambda - \alpha_i$

(2) $\iff T_i e^{s_i(\lambda)} = q e^\lambda T_i^{-1} = e^\lambda T_i + (1-q) e^\lambda$

while $\dfrac{e^\lambda - e^{\lambda - \alpha_i}}{1 - e^{-\alpha_i}} = e^\lambda$ ✗