

DAHA, Overview

Fix a (finite) Weyl group W of a reductive group G w/ max. torus T .

One can assign:

Cartan $\mathfrak{h} = \text{Lie } T$, and
(co)weight lattice $\mathbb{Z}P, P$

Hecke algebra $\mathcal{H} = \mathcal{H}_q(W)$	(extended) affine Hecke alg $\text{AHA} = \langle \mathcal{H}, \mathbb{Y}^\lambda \mid \lambda \in \mathbb{Z}P \rangle$ $\approx R(\mathbb{Z}) \otimes \mathcal{H}$	Langland dual Double affine Hecke alg = Cherednik alg $\text{DAHA} = \langle \text{AHA}, X^\mu \mid \mu \in P \rangle$ $\approx R(\mathbb{Z}) \otimes \mathcal{H} \otimes R(T)$
	degenerate AHA = graded AHA $\approx S(\mathfrak{h}^*) \otimes \mathbb{C}[W]$	degenerate DAHA/CA (or trigonometric) $\text{dCA} \approx S(\mathfrak{h}^*) \otimes \mathbb{C}[W] \otimes R(T)$
		doubly deg. DAHA/CA (or rational) $\text{rCA} \approx S(\mathfrak{h}^*) \otimes \mathbb{C}[W] \otimes S(\mathfrak{h})$

Remarks

1. dAHA appear in

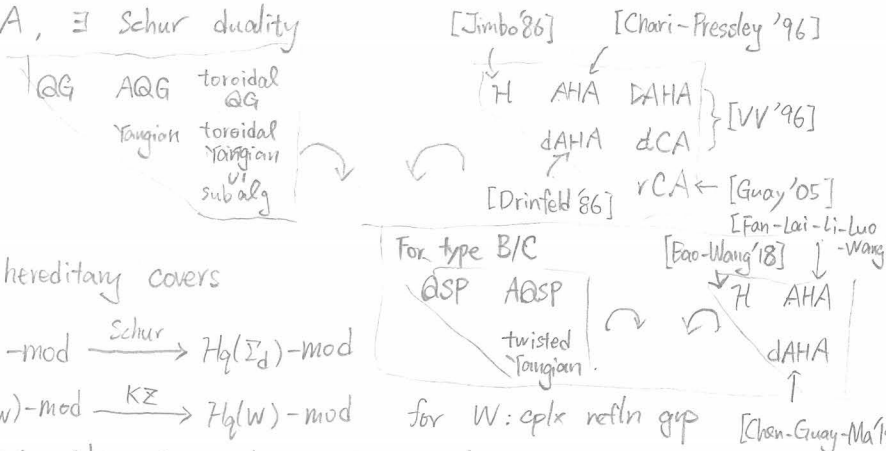
- (a) [Drinfeld '86] for a Schur duality btw (Yangian $\mathcal{Y}(\mathfrak{sl}_n)$, $\text{dAHA}(\Sigma_d)$)
- (b) [Lusztig '88-95] to study rep theory of $\text{AHA}(W)$ via rep theory of $\text{DAHA}(W)$ w/o using K -theory
- (c) [Khovanov '14] categorification of Heisenberg alg

2. DAHA are introduced in [Cherednik '92] to solve Macdonald's orth polyn conj. The deg. versions are obtained via integration of (trig/rat) form of the Knizhnik-Zamolodchikov (KZ) eqns.

3. DAHA is often studied using K -theory & sheaf theory [Vasserot '05]
rCA is --- \mathcal{D} -mod & symp. geom. [Etingof '10]
dCA can be studied both ways

4. Finite dim'l rep theory for all 3 DAHA's are the same;
their (oo-dim'l) categories \mathcal{O} are related [Varagnolo-Vasserot '04]

5. For type A, \exists Schur duality



6. \exists Quasi-hereditary covers

$$S_q(n, d)\text{-mod} \xrightarrow{\text{Schur}} \mathcal{H}_q(\Sigma_d)\text{-mod}$$

$$\mathcal{O}(\text{rCA}_W)\text{-mod} \xrightarrow{\text{KZ}} \mathcal{H}_q(W)\text{-mod}$$

↑ highest weight caty similar to BGG caty $\mathcal{O}(\mathfrak{g})$

7. Presentations:

	0-loop	1-loop	2-loop
Hecke	$\{B \setminus G/B \rightarrow \mathbb{C}\}$ over \mathbb{H}_q		
AHA	$K^A(\text{St}_G)$	$\{B \setminus G/B \rightarrow \mathbb{C}\}$ over 1d local field $\mathbb{H}_q(t)$	
DAHA	Cherednik alg via Dunkl operators	$K^A(\text{St}_G)$ [Garland-Grojnowski '95]	$\{B \setminus G/B \rightarrow \mathbb{C}\}$ over 2d local field [Kapranov '98]

8. Connections:

- Combinatorics on ortho. polyn. [Macdonald '03]
- Integrable systems: Calogero-Moser systems [Etingof '10]
- Geometric Langlands [Bezrukavnikov-Finkelberg-Mirkovic '03]
- Math Phys: Wilson-'t Hooft operators, gauge theory [Kapustin '06]

Plans:

Part I: dAHA

3/04: Schur duality btw dAHA and Yangian [Arakawa '98]

3/11: Rep theory of (deg.) AHA [Solleveld '21 §2-3]

Part II: rDAHA = rCA (following [Etingof '10])

3/18: §2 Dunkl op and Calogero-Moser sys

3/25: §3 Cox \mathcal{O} for rCA

(4/01-29: NCTS minicourse on Whittaker mod and catries for Lie (super)alg)

5/06: §4 fd irreducibles

5/13: §5 Whittaker mod, parabolic ind'n/res'n

5/20: §6 KZ functor

5/27: §7 rCA for orbifolds: incl AHA, DAHA

6/10: §9-10 (quantum) CM space

Part III: DAHA

6/17: Combinatorial picture [Macdonald '03]

6/24: Single loop picture [Varagnolo-Vasserot '09]

7/01: Double loop picture [Kapranov '98, '00]