Learning seminar: Stable envelopes and quantum groups

Spring 2023 Schedule

Meeting place and time: Thursdays 9:30am-11:30am in 509 Lake Hall

Organizers: Elie Casbi, Hunter Dinkins, Iva Halacheva, Valerio Toledano Laredo, Josh Wen, Yan Zhou

The main resource for the seminar will be the book: *D. Maulik and A. Okounkov, Quantum Groups and Quantum Cohomology*. (see AMS bookstore and the arXiv)

Syllabus

Schedule

Jan 19: LECTURE 1. Overview of the seminar theme. *Speaker: Hunter Dinkins*

Jan 26: LECTURE 2. Geometric invariant theory. Exercises *Speaker: Josh Wen*

Main references:

- [BL, Section 2 (p.3-6)] (if time, also [D, Section 6.1, some of 6.2 (p.91-97)])
- [G, Section 2.2 (p.6-8)]
- [Ki, Sections 9.1-9.4 (p.159-171)]

Further references:

- [T, Sections 1-3 (p.1-13)]
- [S, Sections 1.2-1.4]
- [PV, Section 4.6] (See also: https://gauss.math.yale.edu/~il282/Inv.html)
- [D, Ch 6-9, p. 121, examples]
- [H1, H2]

Feb 2: LECTURE 3. Hamiltonian reduction. Exercises Notes *Speaker: Hunter Dinkins*

Main references:

- [Ki, Sections 9.5-9.10 (p.171-191)]
- [BL, Section 4 (p.10-17)]
- [G, Section 4 (p.14-21)]

Further references:

ection 4 (p.13-23)]

Feb 9: LECTURE 4. Quiver varieties I (definition, stability conditions). Notes *Speaker: Sean Carroll*

Main references:

- [Ki, Sections 10.1-10.4]
- [MO, Sections 2.1-2.2 (p. 33-42)]

Further references:

- [G, Sections 4-5]

Feb 16: LECTURE 5. Quiver varieties II (more on stability conditions). (See Lecture 4 Notes)
Speakers: Sean Carroll and Ryan Kannanaikal

Main references:

- [Ki, Sections 10.1-10.4]
- [MO, Sections 2.1-2.2 (p. 33-42)]

Feb 23: Quiver varieties III (examples, tautological bundles). **Notes** *Speaker: Ryan Kannanaikal Mentor: Josh Wen*

Main references:

- [Ki, Sections 10.5-10.7, 10.9, 11.1-11.2]
- [MO, Section 2]

Mar 2: LECTURE 7. Equivariant cohomology I (definition, torus case examples, localization).
Notes *Speaker: Hongqin Zou Mentor: Valerio Toledano Laredo*

Main references:

- [B, Section 1]
- [Ty, Section 1-2]

Further references:

- [AB, Sections 1-3]

Mar 9: (Spring Break)

Mar 16: LECTURE 8. Equivariant cohomology II (further examples, generalizations, Chern classes of tautological bundles). **Notes** *Speaker: Rahul Hirwani Mentor: Josh Wen*

Main references:

- [B, Section 2]
- [Ty, Sections 3-6]

Further references:

- , Sections 1-3]
- [Bo]

Huang Mentor: Yan Zhou

Main reference:

- [MO, Sections 3.1-3.4, 4.1]

Further references:

- [Mi, Feb 9 lecture]
- [O, Section 1]

Mar 30: LECTURE 10. Geometry of stable envelopes II (examples, existence). *Speaker: Hunter Dinkins*

Main reference:

- [MO, Sections 3.5-3.7, 4.1-4.2]

Further references:

- [BMO]

Apr 6: LECTURE 11. Hopf algebras and quantum groups. *Speaker: Aria Masoomi Mentor: Elie Casbi*

Main reference:

- [ES]

Further references

- [M, Section 2]
- [CP, Section 12]
- [Mi, Mar 9 lecture]

Apr 13: LECTURE 12. Yangians and the (algebraic) FRT construction. *Speaker: Anadil Saeed Rao Mentor: Elie Casbi*

Main references:

- [W, Sections 1-3]
- [CP, Section 12.1]
- [M, Sections 1, 2.1-2.5]
- [Mi, Mar 9 lecture]

Further references:

- [MO, Section 5.2]
- [Mc, Sections 2, 3.3-3.4, 4.5]

): LECTURE 13. Geometric R-matrices and the FRT procedure, properties of stable envelopes. *Speaker: Ivan Karpov Mentor: Hunter Dinkins*

- [Mc, Section 5]
- [MO, Section 5]

Apr 27: LECTURE 14. ADE setting: algebraic vs geometric Yangians. *Speaker: Vasily Krylov*
Mentor: Hunter Dinkins

Main reference:

- [Mc, Section 6]

(Possible further topics: Bow varieties and 3D mirror symmetry, Quantum cohomology.)

References

[AB] M. Atiyah and R. Bott, The moment map and equivariant cohomology

[BL] B. Bolognese and I. Losev, A general introduction to the Hilbert scheme of points on the plane

[Bo] R. Bott, An introduction to equivariant cohomology

[B] M. Brion, Equivariant cohomology and equivariant intersection theory

[BMO] A. Braverman, D. Maulik, A. Okounkov, Quantum cohomology of the Springer resolution

[CP] V. Chari and A. Pressley, A guide to quantum groups

[D] I. Dolgachev, Lectures on Invariant theory

[ES] P. Etingof and M. Semenyakin, A brief introduction to quantum groups

[G] V. Ginzburg, Lectures on Nakajima's quiver varieties

[H1] V. Hoskins, Moduli problems and geometric invariant theory

[H2] V. Hoskins, Geometric invariant theory and symplectic quotients

[Ka] J. Kamnitzer, Symplectic resolutions, symplectic duality and Coulomb branches

[Ki] A. Kirillov, Quiver representations and quiver varieties

[Mi] A. Minets, Notes from stable envelopes reading group

[MO] D. Maulik and A. Okounkov, Quantum Groups and Quantum Cohomology

[M] M. McBreen, Quantum cohomology of hypertoric varieties and geometric representations of
ns

[PV] V. Popov and E. Vinberg, Invariant theory in Algebraic geometry IV, Encyclopaedia of Mathematical Sciences, vol. 55, Springer Verlag.

[S] A. Schmitt, Geometric Invariant Theory and Decorated Principal Bundles. Zurich lectures in advanced mathematics, EMS, 2008.

[W] C. Wendlandt, The R-matrix presentation for the Yangian of a simple Lie algebra

[T] R. Thomas, Notes on GIT and symplectic reduction for bundles and varieties

[Ty] J. Tymoczko, An introduction to equivariant cohomology and homology, following Goresky, Kottwitz, and MacPherson