

The Microwave SQUID Multiplexer

by

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The Microwave SQUID Multiplexer

Thesis directed by Dr. Kent Irwin

This thesis describes a multiplexer of Superconducting Quantum Interference Devices (SQUIDs) with low-noise, ultra-low power dissipation, and great scalability. The multiplexer circuit measures the magnetic flux in a large number of unshunted rf SQUIDs by coupling each SQUID to a superconducting microwave resonator tuned to a unique resonance frequency and driving the resonators from a common feedline. A superposition of microwave tones measures each SQUID simultaneously using only two coaxial cables between the cryogenic device and room temperature. This multiplexer will enable the instrumentation of arrays with hundreds of thousands of low-temperature detectors for new applications in cosmology, materials analysis, and nuclear non-proliferation.

The driving application of the Microwave SQUID Multiplexer is the readout of large arrays of superconducting transition-edge sensors, by some figures of merit the most sensitive detectors of electromagnetic signals over a span of more than nine orders of magnitude in energy, from 40 GHz microwaves to 200 keV gamma rays. Modern transition-edge sensors have noise-equivalent power as low as $10^{-20} \text{ W}/\sqrt{\text{Hz}}$ and energy resolution as good as 2 eV at 6 keV. These per-pixel sensitivities approach theoretical limits set by the underlying signals, motivating a rapid increase in pixel count to access new science. Compelling applications, like the non-destructive assay of nuclear material for treaty verification or the search for primordial gravity waves from inflation use arrays of these detectors to increase collection area or tile a focal plane.

We developed three generations of SQUID multiplexers, optimizing the first for flux noise ($0.17 \mu\Phi_0/\sqrt{\text{Hz}}$), the second for input current noise ($19 \text{ pA}/\sqrt{\text{Hz}}$), and the last for practical multiplexing of large arrays of cosmic microwave background polarimeters based on transition-edge sensors. Using the last design we demonstrated multiplexed readout of prototype polarimeters with the performance required for the future development of a large-scale astronomical instrument.

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I have also been fortunate to associate with Prof. Konrad Lehnert. He introduced me to the theory of superconducting microwave resonators and taught me experimental techniques at microwave frequencies. Three weeks in his lab resulted in my first academic paper.

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Chapter 1

Introduction

The Microwave SQUID Multiplexer is a device for the readout of large arrays of low-temperature detectors with a small number of wires. It was motivated by the dramatic growth in array sizes and will provide the necessary multiplexing factors for megapixel arrays of the future.

1.1 Low-Temperature Detectors

Detectors operating at very low temperature[1] have been studied since 1908 when Bottomley[2] cooled a platinum-platinoid thermojunction to the temperature of liquid nitrogen (77 K) and used it to measure thermal radiation from other bodies. At low temperatures, thermal fluctuations are smaller and detector responsivity is greater[3]. The sensitivity of low-temperature detectors has enabled measurements of the cosmic microwave background, THz imaging for security, optical photon counting for telecommunications, x-ray spectroscopy for materials analysis, γ -ray spectroscopy for nuclear non-proliferation, and more.

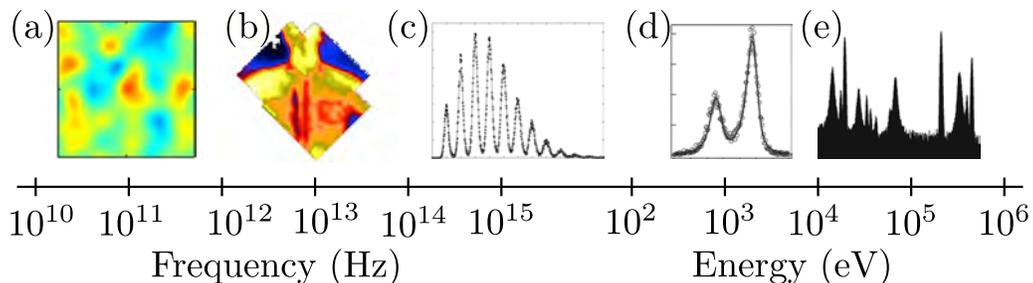


Figure 1.1: : a) Cosmic Microwave Background b) THz imaging c) Optical photon counting d) x-ray spectroscopy e) γ -ray spectroscopy

The first low-temperature detectors were cooled metal strips and thermocouples[2]. In 1941, Andrews used a superconducting film as a “radiometric receiver” [4][5]. In 1957, Boyle cooled a carbon thermocouple and used it to make sensitive measurements of radiation in the far infrared[6]. In 1961, cooled semiconductor detectors were developed and the doped-germanium bolometer[7] became a workhorse of astronomy and particle detection. The past twenty years has seen broad adoption of the superconducting transition-edge sensor[8], and the past ten years has seen work begin on magnetic calorimeters[9][10] and microwave kinetic inductance detectors[11][12]. All of these technologies use low temperature to increase sensitivity.

The Microwave SQUID Multiplexer has been developed in the context of Transition-Edge Sensors and Microwave Kinetic Inductance Detectors.

1.2 Transition-Edge Sensors

A Transition-Edge Sensor (TES) uses the steep change in resistance of a superconducting film at the transition between the superconducting and normal states.

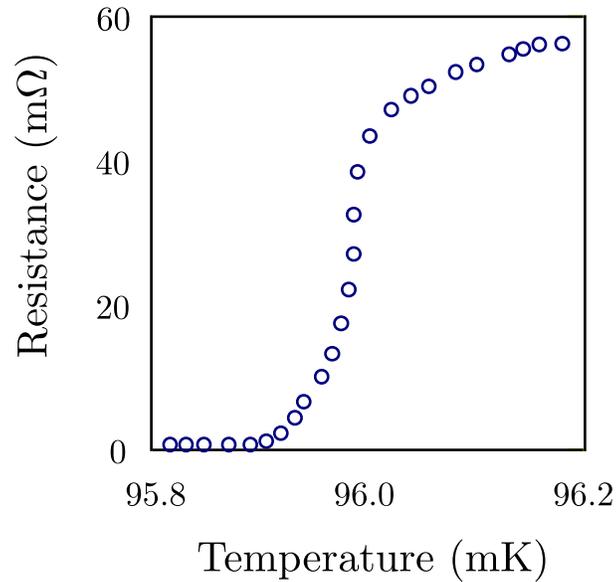


Figure 1.2: (TES).

Biased in temperature in its transition, the film acts as an exquisitely sensitive thermometry and can be used to form a TES bolometer or TES calorimeter.

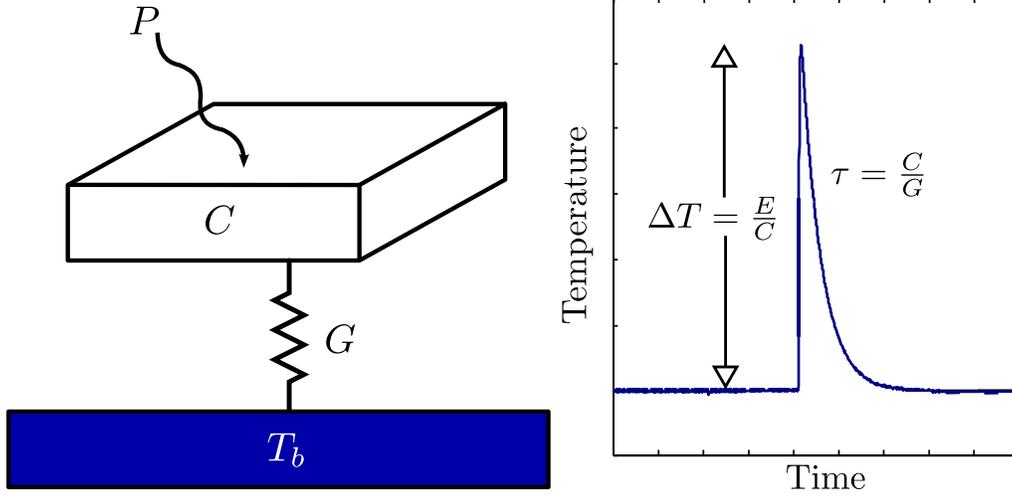


Figure 1.3: Incident power heats the floating heat capacity above the temperature of the bath. A pulse of energy causes a pulse in temperature that decays back to equilibrium with the bath.

A *bolometer* consists of an absorber of heat capacity C connected by a weak thermal conductance G to a bath at temperature T_b (Figure 1.3)[13]. Measurement of the temperature of the absorber constitutes a measurement of incident power P because the power heats the absorber to a temperature $T = T_b + \frac{P}{G}$ [14]. The same device is a *calorimeter* when measuring discrete incident energy E rather than continuous power[15][8][16]; the absorber warms to $T = T_b + \frac{E}{C}$ and returns to the bath temperature with a $\tau = \frac{C}{G}$ time constant.

In a TES bolometer/calorimeter, the superconducting film provides the sensitive thermometry of the floating absorber. These devices provide world-record power and energy sensitivity across more than nine orders of magnitude in wavelength and energy: CMB[17][18], THz[19], sub-mm[20][21], FIR[22], optical[23], x-ray[24][25], γ -ray[26], and α -particles[27].

We describe the sensitivity of a bolometer by a Noise-Equivalent Power (NEP), which is the signal power in a 1 Hz bandwidth at which the signal-to-noise is unity. We describe the sensitivity of a calorimeter by an energy sensitivity ΔE , the full width at half maximum of a spectral peak. The noise-equivalent

power[28] and energy resolution[15] of TESs are limited by heat fluctuating across the thermal conductance:

$$NEP \approx \sqrt{4k_B T^2 G} \quad (1.1)$$

$$\Delta E \approx 2.35 \sqrt{k_B T^2 C} \quad (1.2)$$

This fluctuation is the thermal analog of Johnson noise in a resistor. TESs therefore benefit greatly from operation at low temperatures (Equations 1.1 and 1.2). For example, a typical TES bolometer used for CMB measurement operates at 300 mK with a thermal conductivity of $G \approx 70$ pW/K and a noise-equivalent power of roughly 2×10^{-17} W/ $\sqrt{\text{Hz}}$. TESs have operated at temperatures as low as 20 mK and many now operate at 100 mK.

Practical Transition-Edge Sensors are voltage-biased[15][29] which keeps them in the transition using the V^2/R self-heating. The self-heating provides negative electro-thermal feedback; as the TES temperature and resistance increase, current through the device and joule-heating decrease. The primary advantage of negative electro-thermal feedback is that it allows TESs with different transition temperatures T_c to operate simultaneously as long as the bath temperature is colder than every T_c . In this mode the TES is a low-impedance device, producing a current signal proportional to incident power. The noise power spectral density of this current signal is[15]:

$$S_I = \frac{4k_B T}{R} \left(\frac{(n/\alpha)^2 + (\omega/\omega_{ETF})^2 + n/2}{1 + (\omega/\omega_{ETF})^2} \right) \quad (1.3)$$

where $\alpha \equiv \frac{T}{R} \frac{dR}{dT}$ is a unitless measure of the sharpness of the superconducting transition, $\omega_{ETF} \equiv \frac{G(1+\alpha/n)}{C}$ is the rolloff of the detector response, and n describes the heat loss to the bath $P \propto (T_c^n - T_b^n)$ and can be 4, 5, or 6 depending on the temperature range and physical mechanism of heat exchange. Superconducting films can be made with α as high as 1,000.

The output noise temperature of a TES is therefore between two and three times the transition temperature. For typical TES operating resistance $R_{TES} \approx 1$ m Ω , the current signal is on the order of microamps with fluctuations on the order of 100 pA/ $\sqrt{\text{Hz}}$. Non-degrading detection of such small and

quiet currents requires a low-impedance amplifier with low input current noise that can operate at cryogenic temperatures. The Superconducting Quantum Interference Device is the amplifier of choice.

1.3 Superconducting Quantum Interference Devices

In 1962, Brian Josephson observed[30][31] that the supercurrent tunneling through a superconductor-insulator-superconductor junction should be a periodic function of the phase difference between the superconducting wave-functions on either side of the junction. The Superconducting Quantum Interference Device (SQUID)[32], which consists of a superconducting loop interrupted by one or more Josephson junctions, was invented at Ford labs soon after. The two-junction, or dc-SQUID was invented in 1964[33] and the one-junction, or rf SQUID was invented in 1967[34].

These devices are sensitive to magnetic flux in the loop because the electromagnetic vector potential advances the phase of the superconducting wave function through the canonical momentum of charged particles[35]:

$$H = \frac{(\mathbf{p} - q\mathbf{A})^2}{2m} - q\varphi \quad (1.4)$$

The superconducting wave function therefore accumulates a 2π phase twist around a loop containing $\Phi_0 = \frac{h}{2e} = 2.068 \times 10^{-15}$ Webers, called the *magnetic flux quantum*[36][37]. A SQUID is a circuit that uses Josephson junctions to detect this phase.

The dc SQUID consists of a superconducting loop interrupted by two resistively-shunted Josephson junctions. Current taps are placed on the loop so that a bias current must flow through one junction or the other. Magnetic flux in the loop changes the relationship between the phase differences at the two junctions, effectively modulating the total tunneling supercurrent that can flow between the taps. When the bias current exceeds the maximum tunneling supercurrent through the two Josephson junctions, the excess current flows through the resistive shunts and generates a fluctuating voltage with a dc component between the current taps. Magnetic flux in the SQUID loop modulates this dc voltage. The low-noise and readout simplicity of dc-SQUIDs has made them the most popular SQUID technology today.

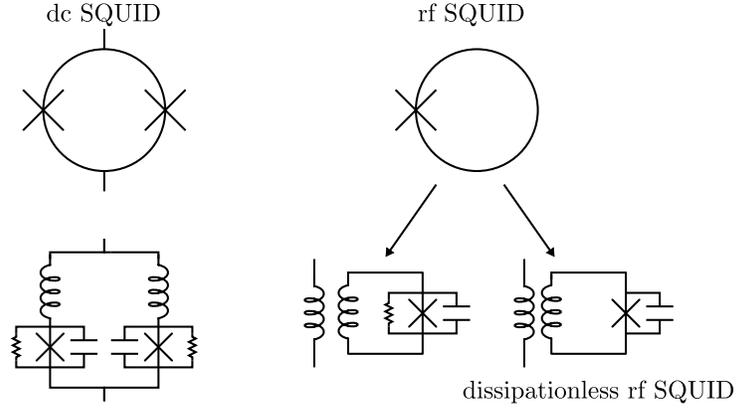


Figure 1.4: Artistic representations and accurate lumped-element models of different SQUIDs.

The standard rf SQUID consists of a superconducting loop interrupted by a single resistively-shunted Josephson junction. Oscillating magnetic flux in the loop dissipates power in the shunt, with the energy dissipation per cycle dependent on the mean value of flux. A tank circuit that inductively couples to the SQUID drives ac flux in the SQUID loop to measure this dissipation. rf SQUIDs generally have higher noise than dc-SQUIDs and require ac readout, but were popular before the development of methods to reliably fabricate multiple Josephson junctions.

The dissipationless rf SQUID in the Microwave SQUID Multiplexer consists of a superconducting loop interrupted by a single unshunted Josephson junction. For small flux oscillations, the SQUID behaves like a loop whose self-inductance depends on the mean value of flux. We inductively couple this SQUID to a microwave resonator so that low-frequency flux in the SQUID shifts the resonance frequency. Although in practice these SQUIDs are not perfectly dissipationless due to sub-gap resistance and the loss tangent of the junction dielectric, they dissipate very little power. We work out the theory of these SQUIDs in section 2.1.

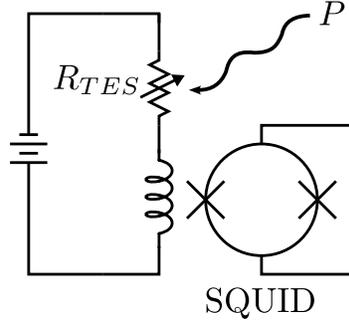


Figure 1.5: Schematic of dc-SQUID readout of a voltage-biased TES.

To measure a TES, we can direct the signal current through an inductor that couples magnetic flux into the SQUID (Figure 1.5). SQUIDS typically have flux noise of order $1 \mu\Phi_0/\sqrt{\text{Hz}}$ so a coupling as low as $M \approx 50 \text{ pH}$ is sufficient to give a current noise ($40 \text{ pA}/\sqrt{\text{Hz}}$) well below the output current noise of many TES designs. The inductive coupler presents a low impedance at typical signal frequencies.

SQUIDS are therefore ideal amplifiers for TESs, and naturally operate at cryogenic temperatures. They have been used for TES readout for the past twenty years[38].

1.4 Arrays and Multiplexing

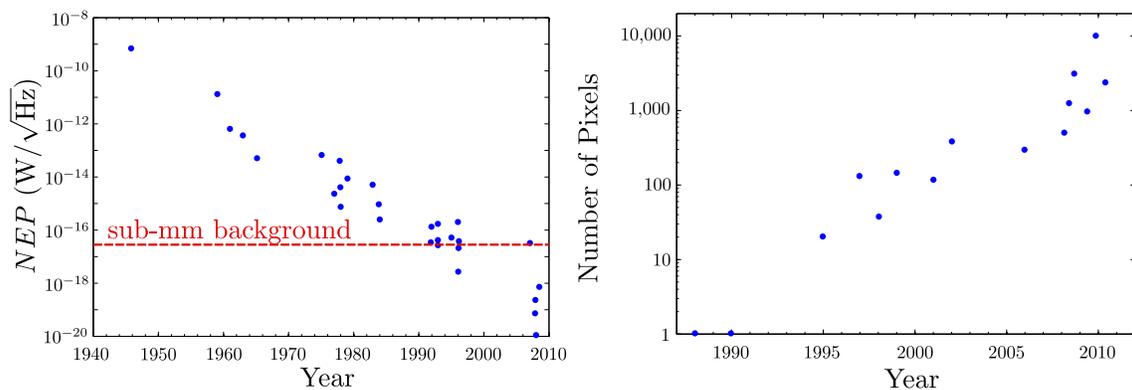


Figure 1.6: [39] The red line shows a typical background noise for ground-based observations in the sub-mm.

The sensitivity of low-temperature detectors has improved dramatically over the past few decades to the point where many applications are limited by other factors. For example, ground-based observations in the sub-mm are limited by shot noise from atmospheric absorption (Figure 1.6 left) and some x-ray spectra are limited by the natural width of the underlying emission lines. To continue accessing new science, increasingly large arrays of detectors have been developed (Figure 1.6 right).

One difficulty posed by low-temperature detector arrays is that of large wire counts between room temperature and the cryogenic stage, which add heat load and cryogenic complexity. We therefore multiplex the detector signals onto a smaller number of wires. Most existing TES arrays use either time-division SQUID multiplexing or frequency-division TES multiplexing schemes.

Time-division multiplexing (TDM) consists of multiple input signals taking turns on an output channel. To satisfy the Nyquist-Shannon sampling theorem [40], the multiplexer must return to each channel with a frequency at least twice the bandwidth of the input channel. Time-division SQUID multiplexing [41][42] switches between dc-SQUIDs by applying bias current to one SQUID at a time. The outputs of all the “first-stage” SQUIDs are summed into a “second-stage” SQUID that amplifies the combined signals onto a single output channel. With n current bias lines and m output channels this technique allows measurement of $n \times m$ detectors with $O(n + m)$ wires. The majority of existing TES arrays [43][44][21][45][26] use time-division SQUID multiplexing.

Frequency-division multiplexing (FDM) consists of modulating multiple input signals at different frequencies on the same output channel [46][47][48]. Frequency-division TES multiplexing uses cold filter circuits to apply a different oscillating voltage bias to each TES. The TES currents are summed into a common SQUID amplifier. The detector signals appear in sidebands of the bias frequencies which must therefore be spaced by more than the expected bandwidth of input signals. Many existing instruments [49][50][51] use frequency-division TES multiplexing.

Code-division multiplexing uses an orthogonal basis set intermediate between time-division and frequency-division. Like time-division multiplexing, readout is broken into multiple timeslots, but unlike time-division multiplexing the signals from all pixels are summed in each timeslot. To allow separation of the input signals, the weight of different input signals changes between timeslots. For example, in a two pixel device

the first timeslot could sum the signals and the second timeslot could take the difference. Development of code-division SQUID multiplexing is just beginning[52][53].

Information theory limits the maximum possible multiplexing factor N with any of these multiplexers to the ratio of the input channel capacity to the output channel capacity, where channel capacity is defined by Shannon[54][40][55] as:

$$C = \int_0^{BW} \log_2 (1 + (SNR)^2) df \quad (1.5)$$

and has units of bits per second (bps).

A single pixel for the CMB application I will discuss has a photon power of 5 pW, a photon shot noise of $4 \times 10^{-17} \text{ W}/\sqrt{\text{Hz}}$, and a bandwidth of 100 Hz. It therefore requires 2.7 kbps of channel capacity for lossless readout. An open-loop SQUID has a linear range of approximately Φ_0/π , a flux noise of $1\mu\Phi_0/\sqrt{\text{Hz}}$, and a bandwidth of a few MHz, providing roughly 100 Mbps of channel capacity. A perfectly efficient multiplexer could therefore read out 40,000 CMB TESs on a single SQUID-amplified output channel.

It is practically difficult to approach the theoretical limits of a multiplexer in an analog system. The maximum multiplexing factor achieved so far with time-division SQUID multiplexing is 40 and the maximum with frequency-division TES multiplexing is 7. Neither multiplexing solution seems likely to provide the multiplexing factors that will be necessary in the next decade.

1.5 Microwave Kinetic Inductance Detectors

The Microwave Kinetic Inductance Detector (MKID) is a low-temperature detector that approaches the photon noise limit in the sub-mm. Although it does not have the sensitivity of TESs at all wavelengths, it provides elegant large-scale multiplexability[12][56][11][57][58]. An MKID consists of a superconducting strip integrated in a microwave resonator. Incident radiation breaks Cooper pairs in the superconductor, changing the surface impedance of the strip, which in turn changes the resonance frequency and quality factor of the resonator (Figure 1.7).

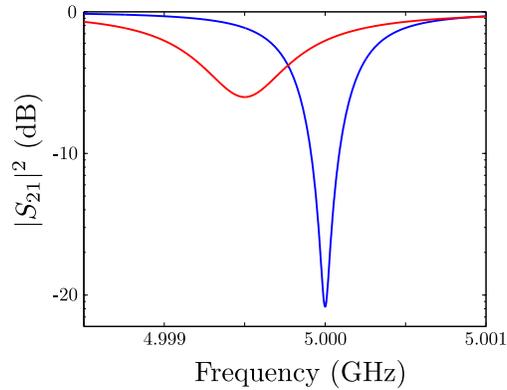


Figure 1.7: Pair-breaking radiation transforms the blue curve into the red curve.

A large number of superconducting microwave resonators can be fabricated on a single chip or wafer (Figure 1.8). Non-overlapping resonances can be read out simultaneously by measuring the complex transmission of a superposition of microwave tones. Only two coaxial cables are therefore necessary between the cryogenic device and room temperature. This technology was enabled by the development of a cryogenic microwave amplifier called a high electron-mobility transistor[59] (HEMT) with a 10 GHz bandwidth, a saturation power of -40 dBm, and a noise temperature of roughly 5 K, implying a Shannon channel capacity of 300 Gbps. Even inefficient use of this channel capacity should allow practical multiplexing factors in the thousands.

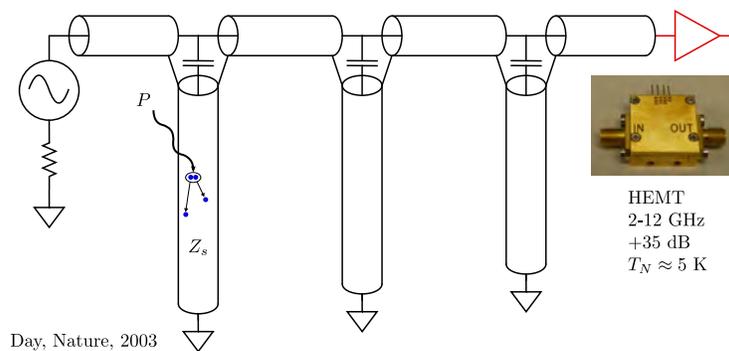


Figure 1.8: , representing the detectors as quarter-wave resonators.

Recent work on MKIDs[60][61] and the remarkable channel capacity of the HEMT inspired the devel-

opment of the Microwave SQUID Multiplexer.

1.6 Microwave SQUID Multiplexer

The Microwave SQUID Multiplexer is an attempt to combine the sensitivity of TESs across a wide range of applications with the multiplexability of MKIDs, and may allow even larger multiplexing factors in combination with other multiplexing technologies (Section 9.5.2).

TESs do not retain their sensitivity in microwave resonant circuits with a HEMT amplifier (Figure 1.9) because of the mismatch between the input noise temperature of the HEMT ($T_N \approx 5$ K) and the output noise temperature of the TESs ($T_N \approx 2T_c \approx 200$ mK). We therefore use SQUIDs to provide gain between the TESs and the resonators.

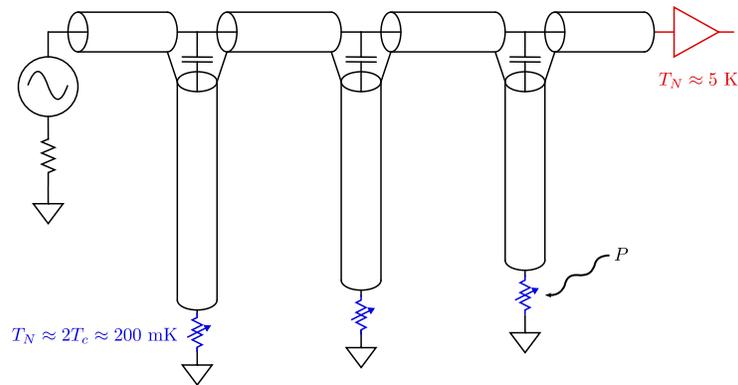


Figure 1.9: . The HEMT noise dominates the TES noise.

The Microwave SQUID Multiplexer couples SQUIDs to superconducting microwave resonators (Figure 1.10). An early device[62] used dc-SQUIDs to modulate the Q of the resonators, but current devices use the change in inductance of dissipationless rf SQUIDs to modulate the resonance frequencies[63][64]. The magnetic flux in thousands of SQUIDs, each modulating a distinct microwave resonance, can then be measured with a pair of coaxial cables.

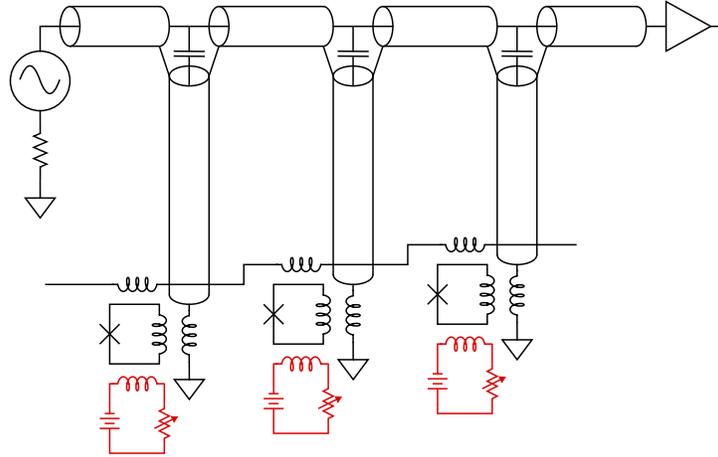


Figure 1.10: . A common flux-bias line is used to linearize all the SQUIDs (Sections 2.6, 6.3.4 and 8.2.4)

By contrast with MKIDs, the Microwave SQUID Multiplexer allows independent optimization of the multiplexer and detectors and can adapt to read out many detector technologies. The SQUID amplifier enables modulation of the detector signal to avoid low-frequency noise in the resonators and HEMT. Finally, the Microwave SQUID Multiplexer does not degrade the sensitivity of the TES detectors, making it useful for a wide range of scientific applications.

This thesis explores the theory, design, and experimental results of the Microwave SQUID Multiplexer. We discuss the predicted and observed flux noise in Sections 2.5, 6.3.3, 7.3.3, and 8.2.3. We discuss the linearization of SQUID readout with *flux-ramp modulation* in Sections 2.6, 6.3.4, and 8.2.4. We conclude by considering the compatibility of the Microwave SQUID Multiplexer with hybrid multiplexing schemes in Section 9.

Hybrid multiplexing could potentially achieve Shannon efficiencies that allow read out of a megapixel array with a handful of coaxial cables and twisted pairs. Let us consider a couple of scientific applications that will require large multiplexed arrays.

1.7 Bolometric Applications

Instruments with TES bolometers currently perform astronomy in the microwave, sub-mm, terahertz, and far infrared, as well as terrestrial terahertz imaging, providing unsurpassed sensitivity in each band. There are too many applications of TES bolometers to accurately describe each, so we will focus on one of the most compelling: the measurement of the polarization of the cosmic microwave background (CMB).

After the inflationary epoch the universe consisted of a hot, dense plasma in thermal equilibrium with a black body population of photons[65]. When the universe cooled enough to form neutral hydrogen (3000 K) the photons began to propagate without scattering. They have since redshifted into the microwave region of the spectrum, with a peak frequency of 160.2 GHz corresponding to a temperature of 2.725 K.

A variety of mechanisms of scientific interest have slightly polarized the CMB. The polarization varies across the sky and can be broken into two parts, a tensor curl-less or *E-mode* component, and a tensor divergence-less or *B-mode* component. Primordial gravity waves from inflation impart a B-mode polarization signature on the CMB[66]. The B-mode polarization component due to primordial gravity waves is expected to be no more than 100 nK (Figure 1.11)[67]. Its detection would confirm theories of inflation in the early universe and thus has great scientific importance.

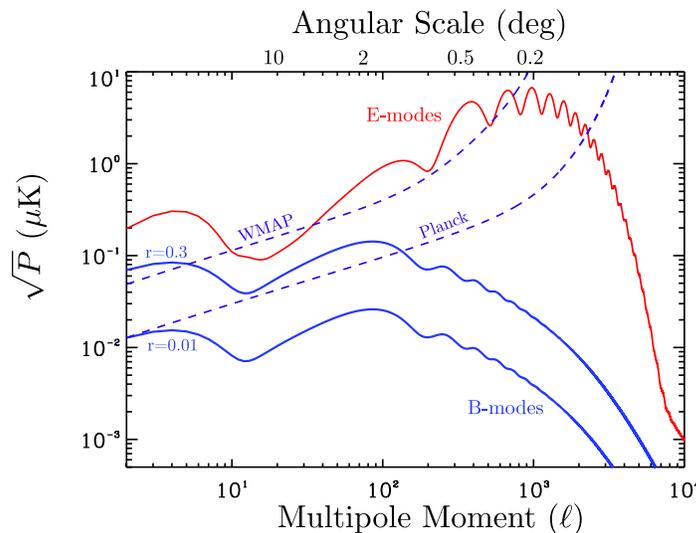


Figure 1.11: [67].

The two primary terrestrial observing sites for CMB astronomy are the Atacama desert in Chile[68][18] and the south pole[69][50][70]. At the south pole the noise-equivalent temperature (NET) is $200 \mu\text{K}\sqrt{\text{s}}$ in the 150 GHz band and higher for higher frequency observing windows on the CMB[70].

The search for inflationary B-modes requires observation of a large area of sky from low multipole moment $\ell \approx 2$ to $\ell \approx 100$. Surveying a hemisphere to that angular resolution with a single pixel would take many hundreds of years. Therefore new instruments[71][72] for this work are being designed with tens of thousands of pixels.

1.8 Spectroscopic Applications

Instruments with TES calorimeters currently perform optical photon counting for telecommunication applications, x-ray spectroscopy for materials analysis, imaging x-ray spectroscopy for astronomy, γ -ray spectroscopy for nuclear materials analysis, and α -particle spectroscopy for nuclear forensics, providing record non-dispersive resolution in all of these applications.

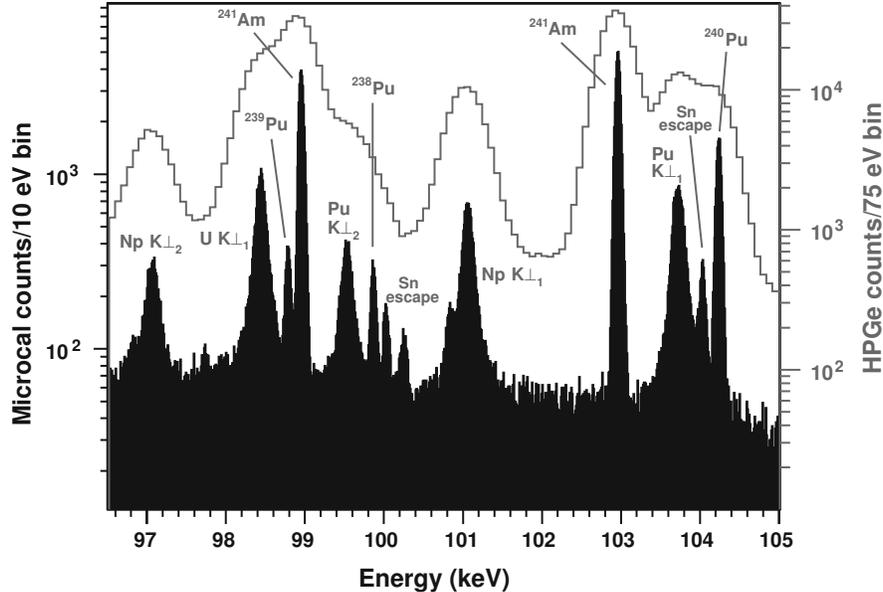


Figure 1.12: using a state-of-the-art high-purity germanium detector (grey line) and a TES microcalorimeter (solid black). (Andrew Hoover, LANL)

One compelling application is the non-destructive assay of nuclear materials for nuclear non-proliferation and treaty verification. TES microcalorimeters can distinguish between the nearby peaks of ^{238}Pu , ^{239}Pu , ^{240}Pu , and ^{241}Pu (Figure 1.12) which a doped-germanium detector cannot. The ratio of ^{240}Pu to ^{239}Pu in fuel from a nuclear reactor provides important evidence of whether it is being used to generate power or make weapons.

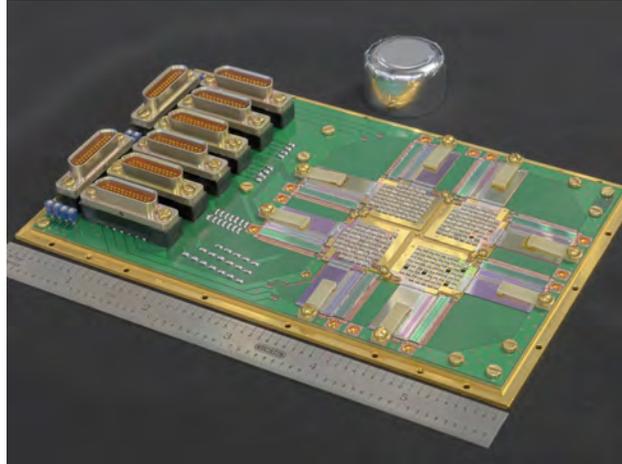


Figure 1.13: with 256 pixels and a planar germanium detector of similar collection area.

Although these TES calorimeters have much better resolution than other detector technologies they have less collection area per pixel ($\Delta E \propto \sqrt{C} \propto \sqrt{V}$). To increase the count rate we therefore assemble arrays of detectors (Figure 1.13) which require multiplexed readout. In the future we desire arrays of many thousands of TES microcalorimeters.

Chapter 2

Theory

This section will explore the physical and electrical theory of the Microwave SQUID Multiplexer circuit shown in Figure 2.1. Each input channel consists of a dissipationless rf SQUID coupled to a superconducting quarter-wave resonator. The input channels are combined into a common output channel by capacitive coupling to a microwave feedline.

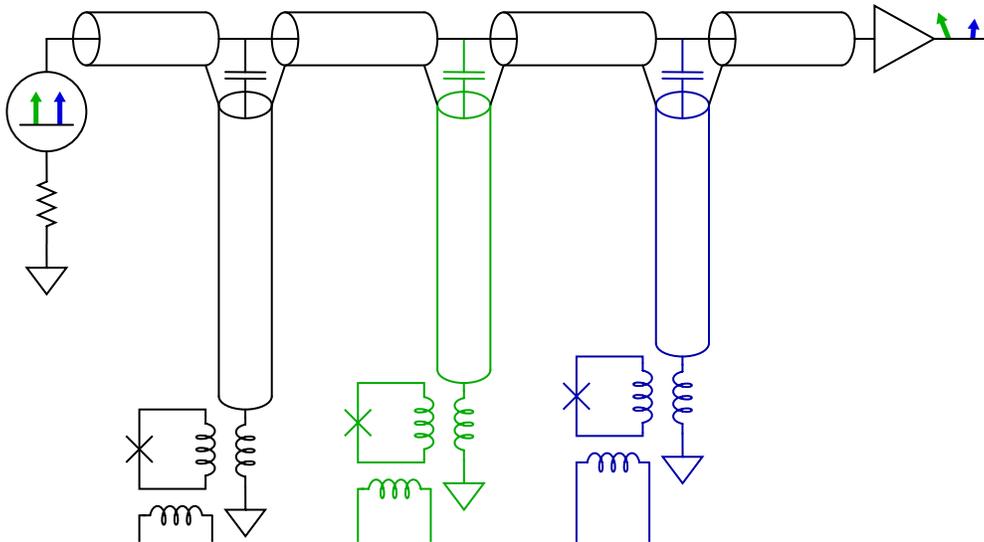


Figure 2.1: . Current on the input coil of a SQUID-coupled resonator modulates the transmitted amplitude and phase of an on-resonance microwave tone.

We will derive the flux-variable resonance frequency of a SQUID-coupled resonator and the corresponding modulation of the transmission of an on-resonance probe tone. We will match the resonance frequency shift to the resonance bandwidth and derive the optimal readout power. We will conclude with a noise theory that predicts the noise referred to flux in the SQUID.

2.1 Dissipationless rf SQUID

We use a dissipationless rf SQUID[73][74] to transduce a change in current in an input coil into a change in inductive load of a microwave resonator. The SQUID consists of a superconducting loop interrupted by an insulating tunnel barrier, called a *Josephson junction*.

2.1.1 Josephson Junction Inductance

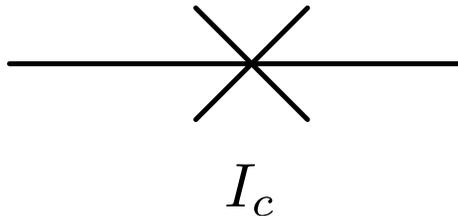


Figure 2.2: Schematic representation of a Josephson Junction.

The tunneling supercurrent across a Josephson junction (Figure 2.2) depends on the difference in phase of the superconducting wave function between the two sides of the junction[30][74]:

$$\boxed{I = I_c \sin \phi} \quad (2.1)$$

where I_c is the so-called *critical current* of the junction and ϕ is the phase difference across the junction.

A voltage drop across the junction makes the phase evolve faster on the high-voltage side than on the low-voltage side. Therefore the phase difference across the junction evolves in time:

$$\boxed{\frac{d\phi}{dt} = \frac{2eV}{\hbar}} \quad (2.2)$$

These two equations are called the *Josephson relations* and have been verified by many experiments[31]. In combination, they imply an effective self-inductance of the junction. Near any particular value of ϕ the rate of change of current through the junction is:

$$\frac{dI}{dt} = I_c \cos(\phi) \frac{d\phi}{dt} \quad (2.3)$$

$$= I_c \cos(\phi) \frac{2eV}{\hbar} \quad (2.4)$$

which implies

$$V = \frac{\hbar}{2eI_c \cos(\phi)} \frac{dI}{dt} \quad (2.5)$$

This voltage-current relation describes an effective inductance called the *Josephson inductance*:

$$\boxed{L(\phi) = L_J \sec(\phi) \quad \text{where} \quad L_J \equiv \frac{\hbar}{2eI_c} = \frac{\Phi_0}{2\pi I_c}} \quad (2.6)$$

where $\Phi_0 = \frac{h}{2e} \approx 2 \times 10^{-15}$ Webers is the quantum of magnetic flux. We can adjust I_c to achieve different values of L_J . Note that this relation holds only for small oscillations in ϕ .

2.1.2 Non-hysteretic rf SQUIDS

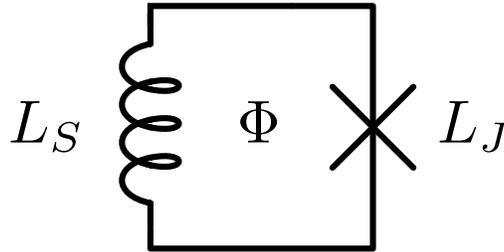


Figure 2.3: Schematic representation of an rf SQUID.

An rf SQUID consists of a superconducting loop interrupted by a single Josephson Junction as shown in Figure 2.3. The phase difference across the junction is initially $\phi = 0$ and evolves with magnetic flux as:

$$\phi = \frac{2e}{\hbar} \int \frac{d\Phi}{dt} dt \quad (2.7)$$

$$= \frac{2e\Phi}{\hbar} \quad (2.8)$$

$$= 2\pi \frac{\Phi}{\Phi_0} \quad (2.9)$$

Because the loop has a self inductance L_S the same current that tunnels across the junction also drives magnetic flux the loop and therefore magnetic flux in the loop Φ is not in general a single-valued function of externally applied magnetic flux Φ_e .

$$\Phi_e = \Phi - I_c L_S \sin\left(2\pi \frac{\Phi}{\Phi_0}\right) \quad (2.10)$$

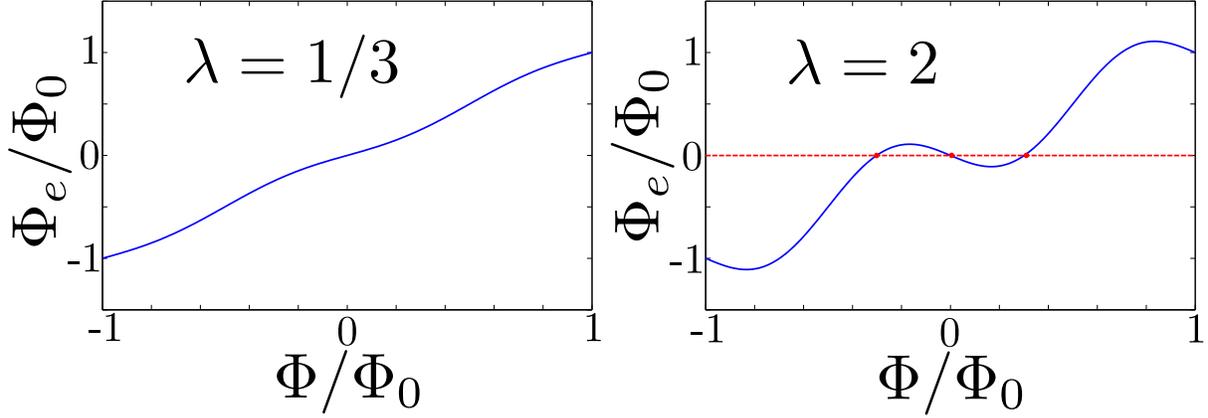


Figure 2.4: for two values of $\lambda \equiv L_S/L_J$. When $\lambda > 1$ the total flux may have multiple acceptable values for a given value of applied flux.

To avoid hysteresis the total flux must be a single-valued function of the applied flux. Therefore $\Phi_e(\Phi)$ must be monotonic.

$$0 < \frac{d\Phi_e}{d\Phi} \quad (2.11)$$

$$< 1 - I_c L_S \cos\left(2\pi \frac{\Phi}{\Phi_0}\right) \frac{2\pi}{\Phi_0} \quad (2.12)$$

$$< 1 - \frac{2\pi I_c L_S}{\Phi_0} \quad (2.13)$$

$$< 1 - \frac{L_S}{L_J} \quad (2.14)$$

We define $\lambda \equiv L_S/L_J$. When $\lambda < 1$, the rf SQUID is non-hysteretic and when $\lambda > 1$, the rf SQUID is hysteretic and can perform flux jumps between metastable states (Figure 2.4). A resistive shunt of the junction is necessary to make these transitions predictable. Most rf SQUIDs use resistive shunts, but we

desire a dissipationless SQUID to minimize the heat load in a large array and therefore operate without shunts, targeting $\lambda \approx 1/3$.

2.1.3 Measuring the SQUID Inductance

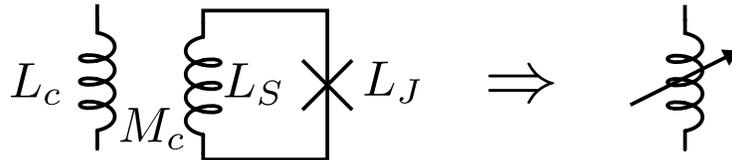


Figure 2.5: Circuit diagram of an rf SQUID screening another inductor.

To measure the inductance of the non-hysteretic rf SQUID we use it to screen an inductor in another circuit (Figure 2.5). That other inductor therefore has an effective flux-variable inductance (Appendix B):

$$L(\Phi) = L_c - \frac{M_c^2}{L_S + L_J \sec(2\pi\Phi/\Phi_0)} \quad (2.15)$$

$$= L_c - \frac{M_c^2}{L_S} \frac{\lambda \cos(2\pi\Phi/\Phi_0)}{1 + \lambda \cos(2\pi\Phi/\Phi_0)} \quad (2.16)$$

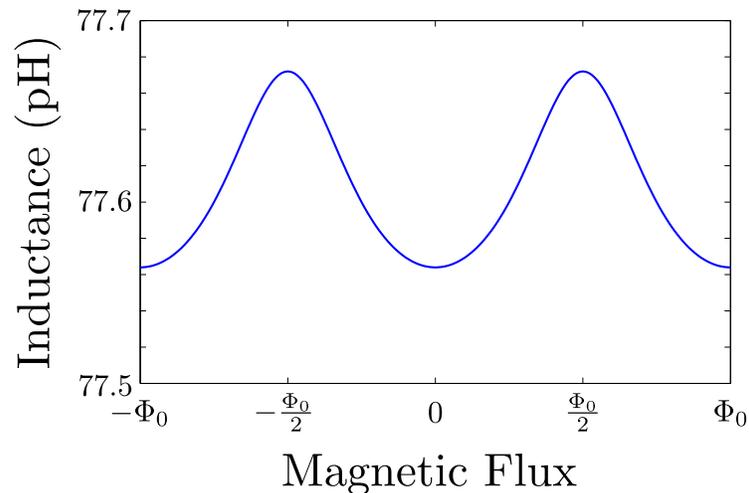


Figure 2.6: for $L_c = 77.6$ pH, $M_c = 1.65$ pH, $L_S = 18.9$ pH, and $\lambda = 1/3$. These values come from the design in Section 7.1.5.2.

For small values of λ this looks like a nearly cosinusoidal function of flux (Figure 2.6). The peak-to-peak change in inductance is:

$$L_{pp} = \frac{M_c^2}{L_S} \left(\frac{1}{1 + \lambda^{-1}} - \frac{1}{1 - \lambda^{-1}} \right) \quad (2.17)$$

$$= \frac{M_c^2}{L_S} \frac{2\lambda^{-1}}{\lambda^{-2} - 1} \quad (2.18)$$

$$= \frac{M_c^2}{L_S} \frac{2\lambda}{1 - \lambda^2} \quad (2.19)$$

We are also interested in the rate of change of inductance with respect to flux at different values of flux:

$$\frac{dL}{d\phi} = \frac{M_c^2 - \lambda \sin(\phi) (1 + \lambda \cos(\phi)) + \lambda \sin(\phi) (\lambda \cos(\phi))}{L_S (1 + \lambda \cos(\phi))^2} \quad (2.20)$$

$$= -\frac{M_c^2}{L_S} \frac{\lambda \sin(\phi)}{(1 + \lambda \cos(\phi))^2} \quad (2.21)$$

In particular, the maximum rate of change of inductance at small λ occurs at $\phi = \pi/2$ and is:

$$\boxed{\frac{dL}{d\phi_{max}} = -\frac{\lambda M_c^2}{L_S}} \quad (2.22)$$

2.1.4 Junction Resistance and Capacitance

Practical Josephson junctions have some capacitance C_J and leakage resistance R_{sg} which shunt the junction inductance (Figure 2.7).

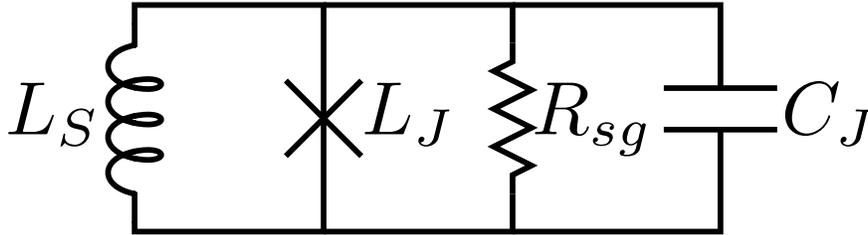


Figure 2.7: SQUID circuit including leakage resistance and junction capacitance.

Typical values of SQUID and junction inductance are $L_S \approx 20$ pH and $L_J \approx 60$ pH. From the junction thickness and area we predict a parallel-plate capacitance of $C_J \approx 100$ fF. This circuit resonates at between

90 and 130 GHz depending on the average phase across the junction. We operate between 4 and 8 GHz, and therefore this resonance does not affect our measurements.

For analysis of the screening currents that flow in the SQUID we consider L_S in a series loop with the parallel combination of the other three components (Appendix B.1). The effective load impedance then becomes:

$$Z_{eff} \approx i\omega \left(L_c - \frac{M_c^2}{L_S + L_J \sec \phi} \right) + \frac{(\omega M_c)^2 (i\omega C_J + 1/R)}{(1 + \lambda \cos \phi)^2} \quad (2.23)$$

For typical SQUIDs, the variation in effective load impedance due to the junction capacitance is therefore less than 5% of the primary variation in effective load impedance due to junction inductance.

In similar junctions we measure leakage resistance of $R_{sg} \approx 100 \Omega$. For inductive coupling of $M_c \approx 1$ pH, this adds a real component of less than $50 \mu\Omega$ to the effective load impedance, setting a limit on internal Q (Section 2.4.3) of roughly one million.

These shunts do not substantially change the flux screening behavior of the SQUID. They do not dramatically increase the loss. We therefore neglect C_J and R_{sg} in the rest of the analysis.

2.2 Resonance Frequency

To multiplex the SQUIDs in frequency space, we coupled each SQUID to a different resonator with a unique resonance frequency. We therefore had to design resonators to resonate at microwave frequencies and adjust resonator parameters so that the resonances do not overlap. Let us begin with the quarter-wave resonator and then extend our analysis, first by capacitive coupling to the readout circuit, and then by inductive coupling to the input circuit.

2.2.1 Ideal Quarter-Wave Resonator

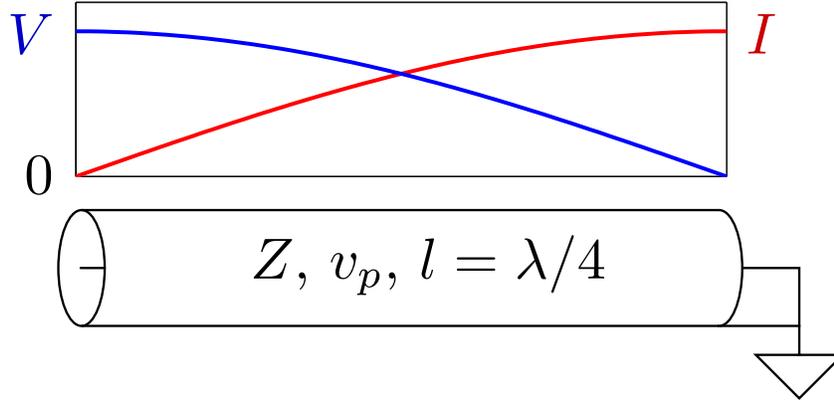


Figure 2.8: , which consists of a transmission line that is open at one end and shorted at the other.

The ideal quarter-wave resonator is a dissipationless transmission line that is open at one end and shorted at the other (Figure 2.8). No current flows at the open end. No voltage exists at the shorted end. The only standing waves that can match these boundary conditions are those for which $(2n+1)\frac{\lambda}{4} = l$, where l is the length of the transmission line.

For a length l of transmission line of phase velocity v_p the frequency of the first mode is:

$$\boxed{f_1 = \frac{v_p}{4l}} \quad (2.24)$$

More precisely, voltage and current waves of frequency ω on a transmission line with phase velocity v_p can be written as:

$$V(z) = V_0^+ e^{-i\beta z} + V_0^- e^{i\beta z} \quad (2.25)$$

$$I(z) = \frac{V_0^+}{Z_1} e^{-i\beta z} - \frac{V_0^-}{Z_1} e^{i\beta z} \quad (2.26)$$

where $\beta \equiv \frac{\omega}{v_p}$. Since the transmission line shorts to ground at $z = 0$ we have

$$V(0) = V_0^+ + V_0^- = 0 \quad (2.27)$$

$$I(0) = \frac{V_0^+}{Z_1} - \frac{V_0^-}{Z_1} \equiv I, \quad (2.28)$$

where I is the magnitude of the current oscillation at the shorted end of the quarter-wave transmission line.

This implies a standing wave configuration of fields within the transmission line:

$$V(z) = -iIZ_1 \sin(\beta z) \quad (2.29)$$

$$I(z) = I \cos(\beta z) \quad (2.30)$$

Note that the voltage and current are $\pi/2$ out of phase.

2.2.2 Capacitive Coupling

To couple the quarter-wave resonator to the external world we replace the open with a small capacitance C_c to the center conductor of another transmission line (Figure 2.9).

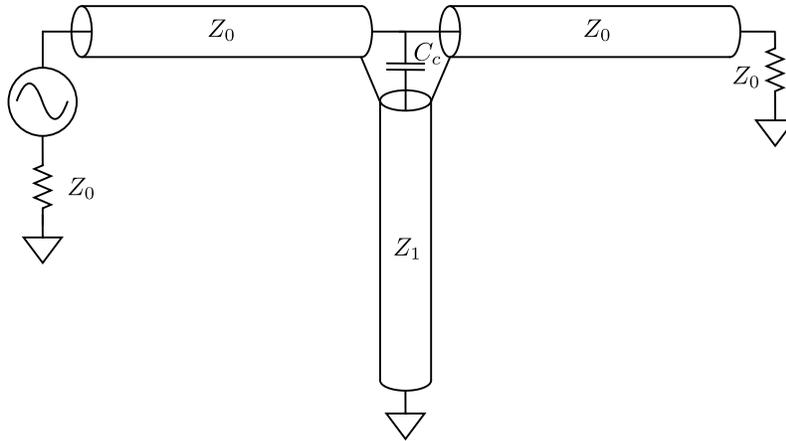


Figure 2.9: Circuit diagram of a quarter-wave resonator capacitively coupled to a microwave feedline.

The resonance frequency of this structure is the frequency at which the capacitively coupled resonator presents an effective short to the feedline. This occurs at the frequency where the reactance of the coupling capacitor exactly cancels the reactance of the quarter-wave transmission line. A length of lossless transmission line transforms a load impedance (Figure 2.10) as follows[75]:

$$Z = Z_1 \frac{Z_L + iZ_1 \tan\left(\omega \frac{l}{v_p}\right)}{Z_1 + iZ_L \tan\left(\omega \frac{l}{v_p}\right)}. \quad (2.31)$$



Figure 2.10: A length of transmission line, terminated with a load impedance.

For a shorted quarter-wave transmission line the transformed impedance is simply

$$Z = iZ_1 \tan\left(\omega \frac{l}{v_p}\right). \quad (2.32)$$

This impedance is in series with the impedance of the coupling capacitor. The resonance condition is thus:

$$0 = \frac{1}{i\omega_0 C_c} + iZ_1 \tan\left(\omega_0 \frac{l}{v_p}\right), \quad (2.33)$$

which means that the resonance frequency must satisfy

$$\omega_0 C_c Z_1 = \cot\left(\omega_0 \frac{\pi}{2\omega_1}\right) \quad (2.34)$$

where $\omega_1 = \frac{\pi v_p}{2l}$ is the resonance frequency of the uncoupled resonator. This equation is transcendental but can be solved for small $\omega_0 C_c Z_1$ by expanding the cotangent around $\pi/2$.

$$\omega_0 C_c Z_1 = \cot\left(\frac{\pi}{2} + \frac{\pi}{2\omega_1}(\omega_0 - \omega_1)\right) \quad (2.35)$$

$$= 0 - \frac{\pi}{2\omega_1}(\omega_0 - \omega_1) + O((\omega_0 - \omega_1)^2) \quad (2.36)$$

$$2\omega_0\omega_1 C_c Z_1 / \pi \approx \omega_1 - \omega_0 \quad (2.37)$$

$$\omega_0 (1 + 2\omega_1 C_c Z_1 / \pi) \approx \omega_1 \quad (2.38)$$

$$\omega_0 \approx \frac{\omega_1}{1 + 2\omega_1 C_c Z_1 / \pi}. \quad (2.39)$$

Therefore the loaded resonance frequency is close to the quarter-wave resonance frequency but reduced by the coupling capacitor:

$$f_0 = \frac{f_1}{1 + 4f_1 C_c Z_1} \quad (2.40)$$

When only weak capacitance $C_c \ll 1/4f_0 Z_1$ couples the resonators to the feedline, $f_0 \approx f_1$ and the resonances can be spaced by their physical length.

2.2.3 Inductive Load

Instead of shorting the other end of the quarter-wave resonator, we terminate it with an inductor that is screened by the SQUID (Figure 2.11). A change in the flux coupled to the SQUID changes the SQUID inductance, and therefore the parameters of the resonator, including particularly the resonance frequency. A resonator may thus be used to measure the flux in the SQUID.

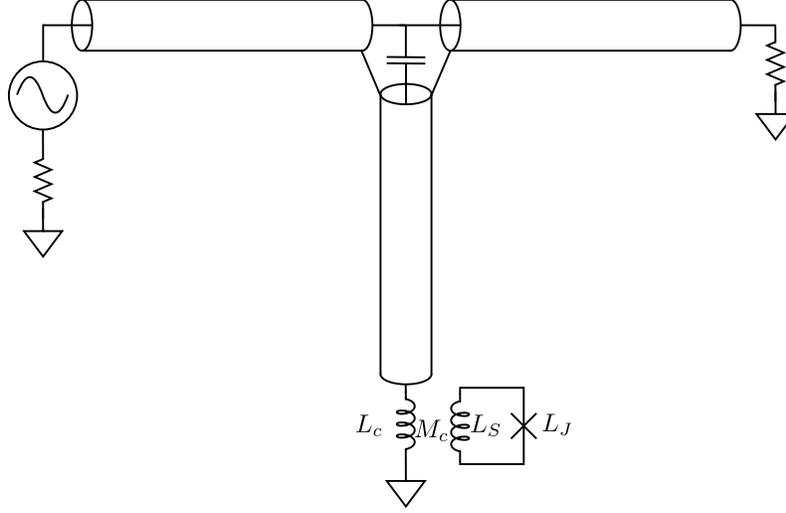


Figure 2.11: A quarter-wave resonator coupled to a microwave feedline and loaded by an inductor screened by an rf SQUID.

$$0 = \frac{1}{i\omega_0 C_c} + Z_1 \frac{i\omega_0 L \cot\left(\omega_0 \frac{l}{v_p}\right) + iZ_1}{Z_1 \cot\left(\omega_0 \frac{l}{v_p}\right) - \omega_0 L} \quad (2.41)$$

$$= (\omega_0 C_c Z_1) \left(\omega_0 L \cot\left(\omega_0 \frac{l}{v_p}\right) + Z_1 \right) - Z_1 \cot\left(\omega_0 \frac{l}{v_p}\right) - \omega_0 L \quad (2.42)$$

Expanding around the quarter-wave resonance frequency ω_1 :

$$0 = \omega_1^2 LC_c Z_1 \left(\frac{\pi}{2} - \frac{\pi\omega_0}{2\omega_1} \right) + \omega_0 C_c Z_1^2 - Z_1 \left(\frac{\pi}{2} - \frac{\pi\omega_0}{2\omega_1} \right) - \omega_0 L \quad (2.43)$$

$$= \omega_1^2 LC_c \left(1 - \frac{\omega_0}{\omega_1} \right) + 2\omega_0 C_c Z_1 / \pi - \left(1 - \frac{\omega_0}{\omega_1} \right) - 2\omega_0 L / \pi Z_1 \quad (2.44)$$

$$= \frac{\omega_0}{\omega_1} \left(1 + 2\omega_1 C_c Z_1 / \pi + 2\omega_1 L / \pi Z_1 - \omega_1^2 LC_c \right) - 1 + \omega_1^2 LC_c \quad (2.45)$$

Therefore the adjusted resonance frequency is close to the quarter-wave resonance frequency but reduced by both the coupling capacitor and the load inductor.

$$\frac{\omega_0}{\omega_1} = \frac{1 - \omega_1^2 LC_c}{1 + 2\omega_1 C_c Z_1 / \pi + 2\omega_1 L / \pi Z_1 - \omega_1^2 LC_c} \quad (2.46)$$

Since we will design the coupling capacitor so that $\frac{1}{\omega_1 C_c} \gg Z_1$ and the load inductor so that $\omega_1 L \ll Z_1$, we can discard the quadratic terms:

$$f_0 = \frac{f_1}{1 + 4f_1 C_c Z_1 + 4f_1 L / Z_1} \quad (2.47)$$

2.2.4 Variation in Load Inductance

The resonance frequency ω_0 therefore shifts with small changes in L :

$$\frac{\partial \omega_0}{\partial L} = \frac{-\omega_1}{(1 + 2\omega_1 C_c Z_1 / \pi + 2\omega_1 L / \pi Z_1)^2} (2\omega_1 / \pi Z_1) \quad (2.48)$$

$$= -\frac{2\omega_0^2}{\pi Z_1} \quad (2.49)$$

or

$$\frac{\partial f_0}{\partial L} = -\frac{4f_0^2}{Z_1} \quad (2.50)$$

Combining this with the small changes of load inductance with flux in the SQUID we see

$$f_0(\phi) \approx f_1 - 4f_1^2 C_c Z_1 - \frac{4f_1^2 L_c}{Z_1} + \frac{4f_1^2 \lambda M_c^2}{Z_1 L_S} \cos \phi \quad (2.51)$$

Figure 2.12 shows this variation in resonance frequency for the actual design parameters from Section 7.1.5.2.

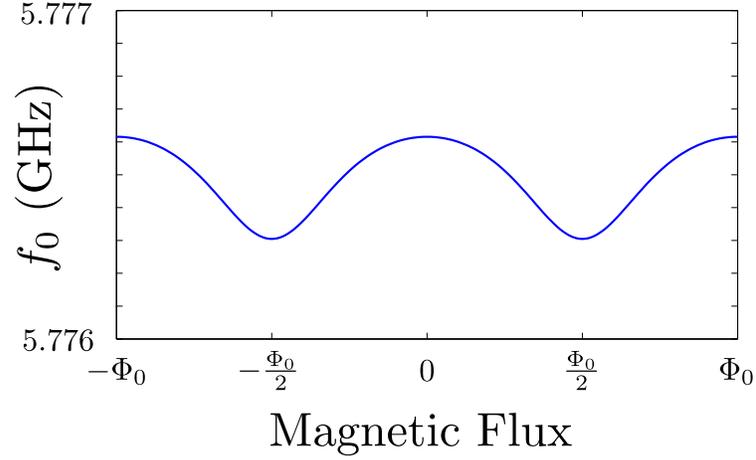


Figure 2.12: for $L_c = 77.6$ pH, $M_c = 1.65$ pH, $L_S = 18.9$ pH, $\lambda = 1/3$, and an unperturbed resonance frequency of 6 GHz. These values come from the design in Section 7.1.5.2.

2.3 Resonator Bandwidth

On resonance, the resonator looks like a perfect short and reflects all microwave power. Far from resonance, the resonator looks like an open and therefore all microwave power passes it by unperturbed. We now consider how quickly the resonator transitions from reflection to transmission.

2.3.1 Resonance Shape

The reflection coefficient for a microwave signal encountering an impedance mismatch is:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (2.52)$$

In our setup the resonator is an impedance in parallel with a Z_0 termination. This means that:

$$\Gamma^{-1} = \frac{Z_L + Z_0}{Z_L - Z_0} \quad (2.53)$$

$$= \frac{1 + Z_0/Z_L}{1 - Z_0/Z_L} \quad (2.54)$$

$$= \frac{1 + Z_0 \left(\frac{1}{Z_0} + \frac{1}{Z_R} \right)}{1 - Z_0 \left(\frac{1}{Z_0} + \frac{1}{Z_R} \right)} \quad (2.55)$$

$$= \frac{2 + Z_0/Z_R}{-Z_0/Z_R} \quad (2.56)$$

$$= -(1 + 2Z_R/Z_0) \quad (2.57)$$

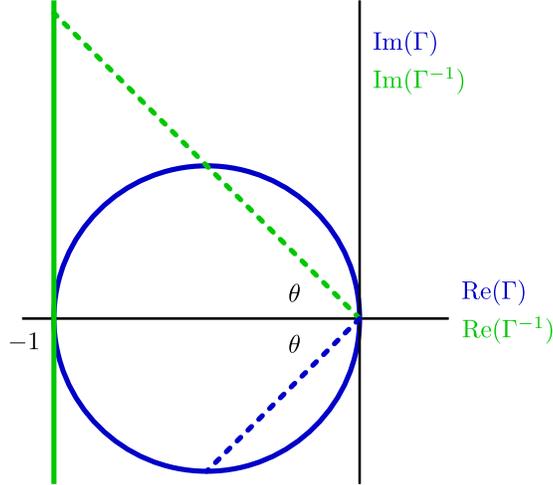


Figure 2.13: describing a vertical line in the complex plane and Γ describing a circle.

If the resonator has negligible losses then Z_R is purely imaginary and Γ^{-1} is a vertical line in the complex plane that passes through $(-1, 0)$ as in Figure 2.13. If a set of complex numbers forms a straight line in the complex plane then the set of their multiplicative inverses forms a circle. Specifically, if $\Gamma^{-1} = -1 - i \tan \theta$ then

$$\Gamma = -\cos^2 \theta + i \sin \theta \cos \theta \quad (2.58)$$

$$= -\frac{1}{2} \cos 2\theta - \frac{i}{2} \sin 2\theta \quad (2.59)$$

which describes a circle of radius $1/2$ around $(-1/2, 0)$, also shown in Figure 2.13. We usually measure transmission $S_{21} = 1 + \Gamma$ rather than reflection, but this clearly describes a circle in the complex plane too.

The most familiar way to describe the shape of a resonance is by the peak in reflected power or dip in transmitted power.

$$|\Gamma|^2 = \frac{-1}{1 + 2Z_R/Z_0} \frac{-1}{1 + 2Z_R^*/Z_0} \quad (2.60)$$

$$= \frac{1}{1 + 4\text{Re}(Z_R)/Z_0 + 4|Z_R|^2/Z_0^2} \quad (2.61)$$

Assuming negligible losses in the resonator and considering frequencies only slightly detuned from resonance:

$$|\Gamma|^2 = \frac{1}{1 + \left(\frac{2\partial|Z_R|/\partial\omega}{Z_0}\right)^2 (\omega - \omega_0)^2} \quad (2.62)$$

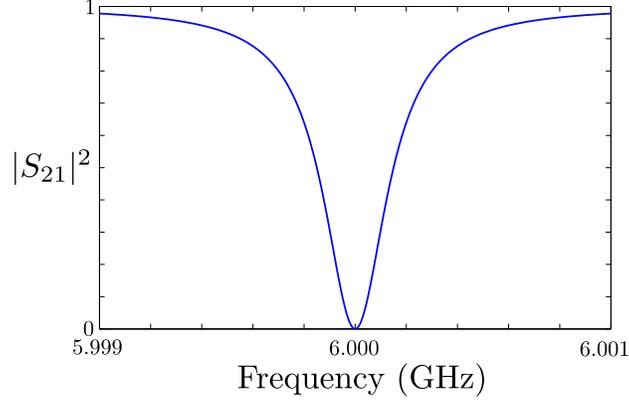


Figure 2.14: for $f_0 = 6$ GHz and $Q = 20,000$.

which is recognizable as a Lorentzian lineshape (Figure 2.14) with a full-width half-maximum bandwidth of

$$BW = \frac{Z_0}{\partial |Z_R| / \partial \omega} \quad (2.63)$$

We often describe resonance widths by a dimensionless number $Q \equiv \omega_0 / BW$ called the quality factor:

$$Q = \frac{\partial(|Z_R|/Z_0)}{\partial(\omega/\omega_0)} \quad (2.64)$$

Similarly,

$$|S_{21}|^2 = 1 - |\Gamma|^2 \quad (2.65)$$

$$= \frac{1}{1 + \left(\frac{BW/2}{f - f_0}\right)^2} \quad (2.66)$$

2.3.2 Coupled Q

We previously derived an expression for the resonator impedance:

$$Z_R(\omega) = \frac{1}{i\omega C_c} + Z_1 \frac{i\omega L \cot\left(\omega \frac{l}{v_p}\right) + iZ_1}{Z_1 \cot\left(\omega \frac{l}{v_p}\right) - \omega L} \quad (2.67)$$

letting $x \equiv \omega/\omega_0$,

$$|Z_R(\omega)|/Z_0 \approx \frac{Z_1 \omega_0 x L \cot(x\pi/2) + Z_1}{Z_0 Z_1 \cot(x\pi/2) - \omega_0 x L} - \frac{x^{-1}}{\omega_0 C_c Z_0} \quad (2.68)$$

so that

$$\left. \frac{\partial |Z_R(\omega)|/Z_0}{\partial x} \right|_{x=1} = \frac{Z_1 - (\omega_0 L \pi/2) (\omega C_c Z_1 (\omega L \cot(\pi/2) + Z_1)) + (Z_1 \pi/2 + \omega_0 L) ((\omega L \cot(\pi/2) + Z_1))}{Z_0 (\omega C_c Z_1 (\omega L \cot(\pi/2) + Z_1))^2} + \frac{1}{\omega_0 C_c Z_0} \quad (2.69)$$

$$= \frac{Z_1 - (\omega_0 L \pi/2) (\omega C_c Z_1) + (Z_1 \pi/2 + \omega_0 L)}{Z_0 (\omega C_c Z_1)^2 (\omega L \cot(\pi/2) + Z_1)} + \frac{1}{\omega_0 C_c Z_0} \quad (2.70)$$

$$\approx \frac{Z_1}{Z_0} \frac{\pi}{2 (\omega_0 C_c Z_1)^2} \quad (2.71)$$

By this calculation the coupled Q is:

$$Q_c = \frac{\pi}{2\omega_0^2 C_c^2 Z_0 Z_1} \quad (2.72)$$

2.3.3 Response to Frequency Shift

On resonance, the response of Γ to a small detuning is the same as the response of Γ^{-1} , but negative (see Figure 2.13). We are most concerned with the voltage signal in the imaginary direction, since the response to flux is mostly in this direction:

$$\frac{d\Gamma}{d\omega} = \frac{2}{Z_0} \frac{dZ_R}{d\omega} \quad (2.73)$$

$$= 2iQ_c/\omega_0 \quad (2.74)$$

We measure the SQUID by interrogating the resonance with a fixed tone as its resonance frequency shifts. For small frequency shifts, the result of shifting the resonator away from the tone is just the inverse of the result of shifting the tone.

$$\frac{dS_{21}}{d\omega_0} = -\frac{2iQ_c}{\omega_0} \quad (2.75)$$

2.3.4 Energy in the Resonator

Let us now explicitly consider the energy in the resonator. This will allow us to confirm the calculations we have already performed from an impedance perspective and yield some new insights. Stored energy in the resonator sloshes back and forth between the electric field and the magnetic field, with minimum dissipation occurring at the resonance frequency.

Consider first only the quarter-wave resonator and coupling capacitor. The energy in the capacitor is:

$$E = \frac{1}{2} C_c V^2 \quad (2.76)$$

The rest of the energy is stored in the electric and magnetic fields between the inner and outer conductors of the transmission line. The line has a capacitance per unit length C and inductance per unit length \mathcal{L} , which are related to the characteristic impedance and phase velocity:

$$Z_1 = \sqrt{\frac{\mathcal{L}}{C}} \quad C = \frac{1}{Z_1 v_p}$$

$$\Leftrightarrow$$

$$v_p = \frac{1}{\sqrt{\mathcal{L}C}} \quad \mathcal{L} = \frac{Z_1}{v_p}$$

The energy stored in the electric field can be integrated over the length of the transmission line:

$$E = \int_0^l \frac{1}{2} C (V(z))^2 dz \quad (2.77)$$

$$= \frac{I^2 Z_1^2}{2 Z_1 v_p} \int_0^l \sin^2(\beta z) dz \quad (2.78)$$

$$= \frac{I^2 Z_1}{2 v_p} \left[\frac{z}{2} - \frac{\sin(2\beta z)}{4\beta} \right]_0^l \quad (2.79)$$

$$= \frac{I^2 Z_1 l}{4 v_p} \left(1 - \frac{\sin(2\beta l)}{2\beta l} \right) \quad (2.80)$$

$$= I^2 Z_1 \left(\frac{\pi}{8\omega_1} - \frac{\sin(2\beta l)}{8\beta v_p} \right) \quad (2.81)$$

Similarly, the energy stored in the magnetic field is:

$$E = \int_0^l \frac{1}{2} \mathcal{L} (I(z))^2 dz \quad (2.82)$$

$$= I^2 Z_1 \left(\frac{\pi}{8\omega_1} + \frac{\sin(2\beta l)}{8\beta v_p} \right) \quad (2.83)$$

Resonance occurs when the energy in the capacitor accounts for the difference between the energy in the

electric and magnetic fields in the transmission line.

$$\frac{1}{2}C_c V^2 = \frac{I^2 Z_1 \sin(2\beta l)}{4\beta v_p} \quad (2.84)$$

$$C_c \frac{I^2 \cos^2(\beta l)}{\omega_0^2 C_c^2} = \frac{I^2 Z_1 \sin(\beta l) \cos(\beta l)}{\beta v_p} \quad (2.85)$$

$$C_c^{-1} = \frac{Z_1 \omega_0^2 \tan(\beta l)}{\beta v_p} \quad (2.86)$$

$$\omega_0 C_c Z_1 = \cot\left(\omega_0 \frac{l}{v_p}\right) \quad (2.87)$$

This is a good check of our result from matching impedances. The consideration of energy stored in the load inductance follows a similar argument. Finally, note that although the sinusoidally varying term in the energy is critical to determining the resonance frequency it is a small fraction of the total energy stored in the resonator.

$$E \approx \frac{I^2 Z_1}{16 f_0} \quad (2.88)$$

2.3.5 Driven Steady-State

On resonance, the resonator looks like a short between the conductors of the feedline. This means that in steady-state the resonator enforces a voltage node on the feedline. If a voltage wave arrives from the left it must be reflected back to the left, inverted.

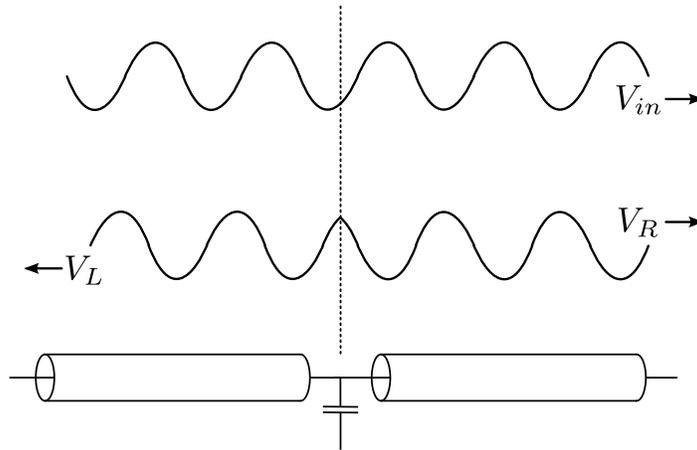


Figure 2.15: Illustration of the voltage waves for a resonator driven on resonance.

We can view this as a superposition of voltage waves as in Figure 2.15. By symmetry the two voltage waves, of amplitude V_L and V_R , generated at the resonator must propagate identically to the left and right. The third voltage wave, of amplitude V_{in} propagates from the source to the right. The sum of these waves must have a voltage node at the resonator:

$$V_L = V_R = -V_{in} \quad (2.89)$$

Although the resonator enforces a voltage node, current still flows in and out of the resonator from the feedline.

$$-i\omega_0 \mathcal{L}I = \frac{dV}{dz} \quad (2.90)$$

The voltage wave from the source is continuously differentiable, and therefore supplies no current to the resonator. All the current into the resonator comes from the discontinuity in the derivative of the resonator voltage wave.

$$I \cos(\beta l) = \frac{iv_p}{\omega_0 Z_0} ((i\beta V_R) - (-i\beta V_L)) \quad (2.91)$$

$$= \frac{2v_p \beta V_{in}}{\omega_0 Z_0} \quad (2.92)$$

$$= \frac{2V_{in}}{Z_0} \quad (2.93)$$

We can therefore describe the energy in the resonator in terms of the voltage wave on the feedline

$$E = \frac{Z_1 \pi}{8\omega_0} I^2 \quad (2.94)$$

$$= \frac{Z_1 \pi}{8\omega_0} \frac{4V_{in}^2}{Z_0^2} \sec^2(\beta l) \quad (2.95)$$

$$= \frac{\pi P_{in} Z_1}{\omega_0 Z_0} \sec^2(\beta l) \quad (2.96)$$

$$\approx \frac{\pi P_{in}}{\omega_0} \frac{1}{\omega_0^2 C_c^2 Z_0 Z_1} \quad (2.97)$$

$$= \frac{2Q_c P_{in}}{\omega_0} \quad (2.98)$$

2.3.6 Power Dissipation in the Terminations

The steady-state calculation does not determine how quickly the resonator adjusts to a change in drive. No power enters or leaves the resonator. All input power reflects back to the source. We must calculate

power loss from an excited resonator in the absence of a input drive to know how quickly the resonator rings up or down.

Each time a traveling wave inside the resonator reflects from the coupling capacitor, some power leaks onto the feedline and dissipates at the terminations. The energy in the transmission line propagates down and back in $\tau = \frac{2l}{v_p}$. Thus, the internal power incident on the capacitor is

$$P_i = \frac{E}{\tau} \quad (2.99)$$

$$\approx \frac{E\omega_0}{\pi} \quad (2.100)$$

This power is constantly reflecting from a load that looks like $Z_L = \frac{1}{i\omega C_c} + \frac{Z_0}{2}$, which has a reflection coefficient of

$$\Gamma = \frac{Z_L - Z_1}{Z_L + Z_1} \quad (2.101)$$

$$= \frac{\frac{1}{i\omega_0 C_c} + \frac{Z_0}{2} - Z_1}{\frac{1}{i\omega_0 C_c} + \frac{Z_0}{2} + Z_1} \quad (2.102)$$

$$= \frac{1 + i\omega_0 C_c \left(\frac{Z_0}{2} - Z_1\right)}{1 + i\omega_0 C_c \left(\frac{Z_0}{2} + Z_1\right)} \quad (2.103)$$

For small capacitance, almost all the power reflects, but some disappears into the terminations:

$$1 - |\Gamma|^2 = 1 - \frac{1 + i\omega_0 C_c \left(\frac{Z_0}{2} - Z_1\right)}{1 + i\omega_0 C_c \left(\frac{Z_0}{2} + Z_1\right)} \frac{1 - i\omega_0 C_c \left(\frac{Z_0}{2} - Z_1\right)}{1 - i\omega_0 C_c \left(\frac{Z_0}{2} + Z_1\right)} \quad (2.104)$$

$$= 1 - \frac{1 + \omega_0^2 C_c^2 \left(\frac{Z_0}{2} - Z_1\right)^2}{1 + \omega_0^2 C_c^2 \left(\frac{Z_0}{2} + Z_1\right)^2} \quad (2.105)$$

$$= \frac{2\omega_0^2 C_c^2 Z_0 Z_1}{1 + \omega_0^2 C_c^2 \left(\frac{Z_0}{2} + Z_1\right)^2} \quad (2.106)$$

$$\approx 2\omega_0^2 C_c^2 Z_0 Z_1 \quad (2.107)$$

which makes the dissipated power

$$P_{diss} = \frac{2E\omega_0}{\pi} \omega_0^2 C_c^2 Z_0 Z_1 \quad (2.108)$$

This power loss is twice the drive necessary to maintain the resonator at an internal energy E , which makes sense if we consider that each voltage wave emanating from the resonator would carry P_{in} if there were not

other voltage waves on the line. This power loss also gives the coupled Q :

$$Q_c = \frac{\omega_0 E}{P_{diss}} \quad (2.109)$$

$$= \frac{\pi}{2\omega_0^2 C_c^2 Z_0 Z_1} \quad (2.110)$$

in full agreement with the impedance-based bandwidth calculation.

2.3.7 Antinode Current

The coupling quality factor Q_c determines energy in the resonator for a given input power and therefore the current oscillation at the resonator short:

$$E = \frac{Z_1 \pi}{8\omega_0} I^2 = \frac{2P_{in} Q_c}{\omega_0} \quad (2.111)$$

$$I^2 = \frac{16P_{in} Q_c}{\pi Z_1} \quad (2.112)$$

$$I = 4\sqrt{Q_c P_{in} / \pi Z_1} \quad (2.113)$$

In terms of voltage oscillation on the feedline this means

$$I = \frac{2V_{in}}{Z_0} \sqrt{\frac{2Q_c Z_0}{\pi Z_1}} \quad (2.114)$$

This current oscillation will modulate the flux in the SQUID and therefore limit the microwave power we can apply for SQUID readout. Let p be the peak-to-peak measure of flux oscillating in the SQUID in units of magnetic flux quanta.

$$p \equiv \frac{2IM_c}{\Phi_0} \quad (2.115)$$

$$= V_{in} \frac{4M_c}{\Phi_0 Z_0} \sqrt{\frac{2Q_c Z_0}{\pi Z_1}} \quad (2.116)$$

Conversely, we can describe the voltage wave on the feedline in terms of flux in the SQUID:

$$V_{in} = \frac{p\Phi_0 Z_0}{4M_c} \sqrt{\frac{\pi Z_1}{2Q_c Z_0}} \quad (2.117)$$

We can also write the internal power in terms of flux in the SQUID:

$$P_i = 2f_0 E = \frac{p^2 \Phi_0^2 Z_1}{32M_c^2} \quad (2.118)$$

2.3.8 Matching Frequency Shift to Bandwidth

In general, we choose the bandwidth for a resonator and then try to match the SQUID response to that bandwidth. If the peak-to-peak SQUID response is less than the bandwidth, we sacrifice possible SQUID gain. If the peak-to-peak SQUID response is greater than the bandwidth, there are two problems. First, SQUID response becomes more of a square wave than sinusoidal. Second, measurement at high microwave power risks resonator bistability[76]. Let $\eta = \Delta\omega/BW$ be the coupling strength:

$$\eta = \frac{2\omega_0^2}{\pi Z_1} \left(\frac{M_c^2}{L_S} \frac{2\lambda}{1-\lambda^2} \right) \frac{Q_c}{\omega_0} \quad (2.119)$$

$$= \frac{4\omega_0 Q_c}{\pi Z_1} \frac{M_c^2}{L_S} \frac{\lambda}{1-\lambda^2} \quad (2.120)$$

This allows us to write

$$Q_c = \frac{\eta \pi Z_1 L_S}{4\omega_0 M_c^2} \frac{1-\lambda^2}{\lambda} \quad (2.121)$$

When the bandwidth matches the frequency shift the transmission looks like Figure 2.16.

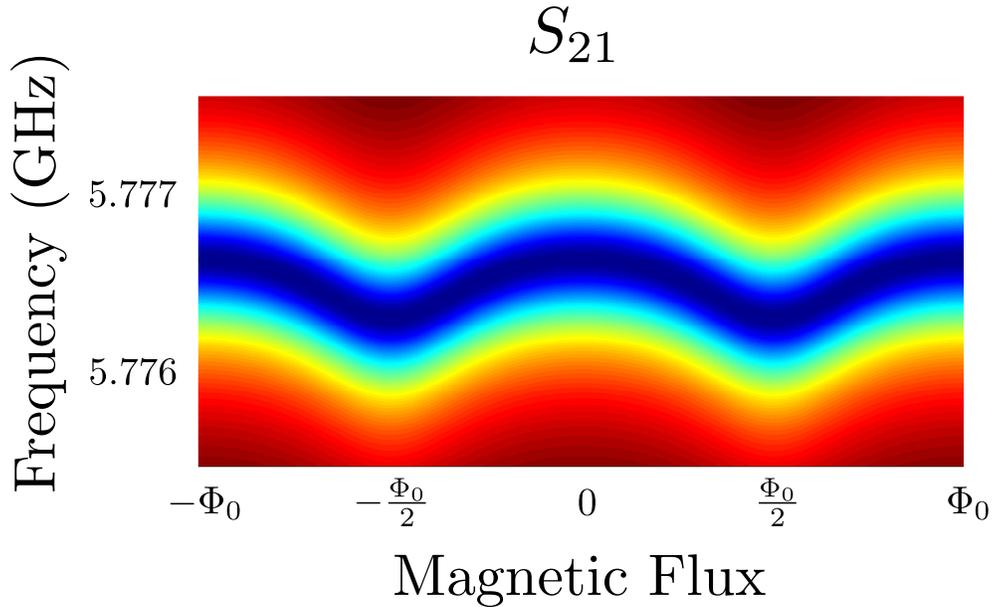


Figure 2.16: through the SQUID for a matched resonator. Red is high transmission and blue is low transmission.

Tracking the resonance in frequency space with a phase-locked loop could allow overcoupled operation,

although it does not relax the constraint on readout power due to resonator bistability. This technique may be useful in the future, for example to read out magnetic calorimeters[9][10], which may require lower flux noise.

2.3.9 Crosstalk and Resonance Spacing

Consider a voltage wave travelling along the feedline and passing two resonators of variable resonance frequency. The transmission coefficient for the system is then:

$$S_{21} = (1 + \Gamma_1)(1 + \Gamma_2)(1 + \Gamma_1\Gamma_2 + \Gamma_1^2\Gamma_2^2 + \dots) \quad (2.122)$$

$$= \frac{(1 + \Gamma_1)(1 + \Gamma_2)}{1 - \Gamma_1\Gamma_2} \quad (2.123)$$

$$= \frac{(\Gamma_1^{-1} + 1)(\Gamma_2^{-1} + 1)}{\Gamma_1^{-1}\Gamma_2^{-1} - 1} \quad (2.124)$$

$$= \frac{4 \frac{Z_{R1}Z_{R2}}{Z_0^2}}{4 \frac{Z_{R1}Z_{R2}}{Z_0^2} + 2 \frac{Z_{R1}}{Z_0} + 2 \frac{Z_{R2}}{Z_0}} \quad (2.125)$$

$$= \frac{2}{2 + \frac{Z_0}{Z_{R1}} + \frac{Z_0}{Z_{R2}}} \quad (2.126)$$

If the resonators are lossless and we are in the regime where $Z_R \propto i\omega$ we can rewrite S_{21} in terms of BW :

$$S_{21} = \frac{2}{2 - i \left(\frac{BW}{\omega - \omega_1} + \frac{BW}{\omega - \omega_2} \right)} \quad (2.127)$$

Let us consider the response to small changes in resonance frequency for ω on resonance with an unperturbed ω_1 , with an unperturbed ω_2 spaced n bandwidths away:

$$\Delta S_{21} = \frac{-2}{\left(2 - i \left(-\frac{BW}{\Delta\omega_1} - \frac{BW}{nBW} \right) \right)^2} \left(-iBW \frac{-1}{(nBW)^2} \right) (-\Delta\omega_2) + O(\Delta\omega_2^2) \quad (2.128)$$

$$\approx \frac{i}{2n^2 \left(1 + i \left(\frac{BW}{2\Delta\omega_1} + \frac{1}{2n} \right) \right)^2} \frac{\Delta\omega_2}{BW} \quad (2.129)$$

The crosstalk vanishes for $\Delta\omega_1 = 0$ as we expect because the voltage wave is fully reflected and never reaches the second resonator. One can show that the crosstalk into the imaginary component of S_{21} is maximized for $\frac{BW}{2\Delta\omega_1} + \frac{1}{2n} = \sqrt{3}$ so that the maximum crosstalk is:

$$\text{Im}[\Delta S_{21}] \approx \frac{1}{2n^2} \frac{1 - 3}{(1 + 3)^2} \frac{\Delta\omega_2}{BW} \quad (2.130)$$

$$\approx \frac{-1}{16n^2} \frac{\Delta\omega_2}{BW} \quad (2.131)$$

To keep crosstalk between neighboring resonators at less than a part per 1,000 we therefore space resonators by at least ten times their bandwidth.

2.4 Losses in the Resonator Circuit

We have already considered loss in the Josephson junctions in Section 2.1.4. We will consider three additional loss mechanisms: loss to free-space radiation, loss in the transmission line, and loss in the flux input circuit. If we design the multiplexer correctly, none of these losses should compare to the power dissipated in the terminations of the input and output ports.

2.4.1 CPW Radiation

The losses due to radiation from a quarter-wave resonator with dielectric below and free-space above can be given as[77]:

$$Q_{rad} = \frac{\pi(1 + \epsilon)^2 \eta_0}{2\epsilon^{5/2}} \frac{1}{Z_0} \frac{1}{I'(\epsilon, n)} \frac{1}{n - 1/2} \left(\frac{L}{s}\right)^2 \quad (2.132)$$

where $\eta_0 = 377 \Omega$ is the impedance of free space, n is the mode number, ϵ is the dielectric constant of the substrate, and s is the spacing between the centerlines of the two slots. $I'(\epsilon, n)$ is an integral that can be calculated numerically[77], e.g. for $\epsilon = 10$ and $n = 1$:

$$I'(10, 1) = 1.62 \quad (2.133)$$

and thus we find that for our fundamental oscillations with $\epsilon \approx 10$

$$Q_{rad} \approx 5.6 \left(\frac{L}{s}\right)^2 \quad (2.134)$$

for a typical design we might have $L = 5300 \mu\text{m}$ and $s = 16 \mu\text{m}$ so

$$Q_{rad} \approx 600,000 \quad (2.135)$$

This analytic calculation is a poor substitute for computational modeling, because it does not consider such things as the conducting enclosure which is closer than a free-space wavelength to the CPW. We have measured much higher internal quality factors in real devices.

2.4.2 Dielectric Loss

Let us add lossy dielectric to the resonator. The attenuation constant in a coaxial transmission line with a lossy dielectric between the ground and the center conductor is

$$\alpha = \frac{\omega_0 \tan \delta}{2v_p}. \quad (2.136)$$

In a coaxial geometry the electric field is distributed evenly throughout the dielectric, but in a coplanar waveguide geometry the electric field is non-uniform and mostly exists outside of the dielectric. We can describe this with a loss tangent δ for the dielectric and a filling factor F that accounts for the distribution of the electric field in the dielectric.

$$\alpha \approx \frac{\omega_0 F \delta}{2v_p} \quad (2.137)$$

This attenuation constant means that a voltage wave starting at the capacitor, travelling to the inductive load, reflecting, and returning to the capacitor in time $\tau = \frac{2l}{v_p}$ gets attenuated by $e^{-2\alpha l}$. For weak attenuation this means that

$$Q_d = \frac{E\omega_0}{P} \quad (2.138)$$

$$= \frac{E\omega_0}{\frac{E}{\tau} \left(1 - (e^{-2\alpha l})^2\right)} \quad (2.139)$$

$$\approx \frac{\omega_0 \tau}{4\alpha l} \quad (2.140)$$

$$= \frac{\omega_0}{2\alpha v_p} \quad (2.141)$$

$$= \frac{1}{F\delta} \quad (2.142)$$

$F\delta \sim 10^{-5}$ has been reported[78] at low power for niobium CPW ($s = 10 \mu\text{m}$) resonators on sapphire. We operate at higher power, which can saturate the two-level systems, and have measured $Q_i > 2 \times 10^6$ for bare resonators of niobium on high resistivity silicon.

2.4.3 Loss in the Flux Input Circuit

The circuit that feeds a magnetic flux signal to the SQUID (Figure 2.17) may present an impedance with a real component. Microwave power in the resonator may therefore dissipate in the input circuit.

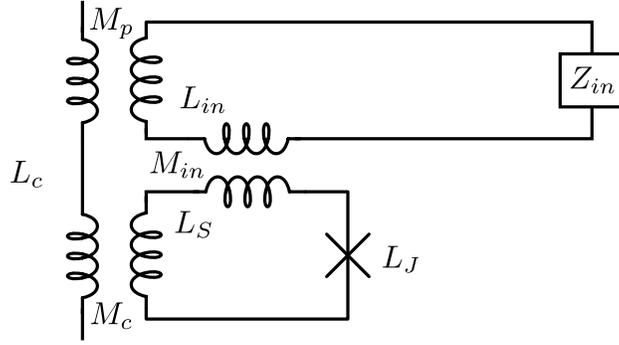


Figure 2.17: coupling to both the resonator and each other.

There are two ways for the microwave power to couple to the input circuit. First, there is an unavoidable parasitic coupling (M_p) from the resonator termination into the input coil. Second, the resonator drives (M_c) microwave currents in the SQUID loop which then couple to the input coil (M_{in}). Maximum coupling occurs when $\phi = \pi$ and the SQUID effectively "anti-screens" flux from the resonator. Solving the coupled set of linear equations gives (Appendix B.2):

$$\text{Re}[Z_L] \approx R_F \frac{M_c^2 M_{in}^2 + 4L_S (L_S M_p^2 + M_{in} M_c M_p)}{(2L_S(L_{in} + L_F) + M_{in}^2)^2} \quad (2.143)$$

where we have modeled the input impedance as an L/R -filter because in practice we place inductive chokes L_F shunted by resistors R_F on the input coils to prevent microwave power from reaching the input devices. A resistive termination on the quarter-wave resonator results in a power loss of

$$P = \frac{I^2 \text{Re}[Z_L]}{2} \quad (2.144)$$

$$= \frac{4\omega_0 E \text{Re}[Z_L]}{\pi Z_1}. \quad (2.145)$$

from which we can calculate a quality factor

$$Q_f = \frac{\omega_0 E}{P} \quad (2.146)$$

$$= \frac{\pi Z_1}{4 \text{Re}[Z_L]} \quad (2.147)$$

Plugging in the transformed filter resistance:

$$Q_f = \frac{\pi Z_1}{4R_F} \frac{(2L_S(L_{in} + L_F) + M_{in}^2)^2}{M_c^2 M_{in}^2 + 4L_S(L_S M_p^2 + M_{in} M_c M_p)} \quad (2.148)$$

which we can increase by increasing L_F and decreasing R_F .

2.4.4 S_{21}^{min}

We measure the transmitted and reflected power before it dissipates in the matched terminations of the external ports, and so Q_c differs from the other Q s in its effect on the resonance circle. Q_c sets the bandwidth of the resonance but has no effect on S_{21}^{min} . If the resonator is lossless, $S_{21}^{min} = 0$ because on resonance the resonator becomes a perfect short and reflects all power. If power dissipates in the resonator:

$$P_{diss} = (1 - |S_{21} - 1|^2 - |S_{21}|^2) P_{in} \quad (2.149)$$

$$= 2P_{in} (\text{Re}[S_{21}] - |S_{21}|^2) \quad (2.150)$$

On resonance, the impedance of a lossy resonator becomes purely real, so

$$P_{diss} = 2P_{in} (S_{21}^{min} - (S_{21}^{min})^2) \quad (2.151)$$

$$= 2P_{in} S_{21}^{min} (1 - S_{21}^{min}) \quad (2.152)$$

$$= \frac{\omega_0 E}{Q} S_{21}^{min} (1 - S_{21}^{min}) \quad (2.153)$$

$$\approx \frac{\omega_0 E}{Q} S_{21}^{min} \quad (2.154)$$

and therefore

$$S_{21}^{min} = \frac{Q}{Q_i} \quad (2.155)$$

Note that

$$P_{diss} \approx 2P_{in} \frac{Q}{Q_i} \quad (2.156)$$

will be useful for calculating cold-stage heating, but that for a particular SQUID design, biased for maximum signal-to-noise, $P_{in} Q$ is constant.

$$2P_{in} Q = \frac{Z_1 \pi p^2 \Phi_0^2}{32M^2} \quad (2.157)$$

Therefore improvements in Q_i reduce the per-pixel heat load.

2.5 Flux Noise

We will consider four sources of noise: Johnson noise in the flux input circuit, intrinsic flux noise in the SQUID, HEMT noise, and two-level system (TLS) noise in the resonator.

2.5.1 Johnson Noise

The lossy input filter can drive noise currents through the input coil of $S_I = 4k_B T/R_F$ that couples to the SQUID through M_{in} . For example, at a base temperature of 300 mK, $R_F \approx 0.2 \Omega$, and $M_{in} \approx 88 \text{ pH}$, this produces roughly $0.2 \mu\Phi_0/\sqrt{\text{Hz}}$. This noise is small compared to the other sources of noise in the system.

2.5.2 SQUID Noise

The dissipationless rf SQUID lacks an analog to Johnson noise to the extent that it is truly dissipationless, but there are other common mechanisms of noise in SQUIDs that can apply. One is fluctuations of the junction critical current[79][80] and another is the flipping of magnetic dipoles on the SQUID loop[81]. Both mechanisms produce noise where the spectral density scales inversely with frequency, so-called $1/f$ noise.

Experiments have shown that in the NIST dc SQUIDs Johnson noise in the shunt resistors dominates to well below $1 \mu\Phi_0/\sqrt{\text{Hz}}$ at 1 Hz. Since we use a similar layout and similar fabrication process to the NIST dc SQUID, we expect these noise contributions to be small.

2.5.3 HEMT noise

Let us calculate the noise due to the HEMT amplifier, referred to flux in the SQUID. This is entirely determined by the SQUID gain at maximum power:

$$\left. \frac{dV}{d\Phi} \right|_{\omega=\omega_0, \Phi=\Phi_0/2} = V_{in} \frac{d\text{Im}(S_{21})}{d\omega} \frac{d\omega_0}{dL} \frac{dL}{d\phi} \frac{2\pi}{\Phi_0} \quad (2.158)$$

$$= \frac{p\Phi_0 Z_0}{4M_c} \sqrt{\frac{\pi Z_1}{2Q_c Z_0} \frac{2Q_c}{\omega_0} \frac{-2\omega_0^2}{\pi Z_1} \frac{-\lambda M_c^2}{L_S} \frac{2\pi}{\Phi_0}} \quad (2.159)$$

$$= \omega_0 p \lambda \frac{M_c}{L_S} \sqrt{2\pi Q_c Z_0 / Z_1} \quad (2.160)$$

$$\approx \omega_0 p \sqrt{\frac{\pi^2 \eta \lambda Z_0}{2\omega_0 L_S}} \quad (2.161)$$

we will generally desire $\eta \approx 1$ and $p \approx 1/\pi$, i.e. matched coupling with a strongly driven but not over-driven SQUID:

$$\boxed{\left. \frac{dV}{d\Phi} \right|_{\omega=\omega_0, \Phi=\Phi_0/2} \approx \omega_0 \sqrt{\frac{Z_0}{2\omega_0 L_J}}} \quad (2.162)$$

At this gain, the voltage noise of the HEMT, $S_V = 4k_B T_N Z_0$ refers to a flux noise in the SQUID of

$$\boxed{S_\Phi|_{f=f_0, \Phi=\Phi_0/2} \approx \frac{4k_B T_N L_J}{\pi f_0}} \quad (2.163)$$

For $T_N = 6$ K, $f_0 = 6$ GHz, and $L_J = 60$ pH this is a noise of $0.58 \mu\Phi_0/\sqrt{\text{Hz}}$.

Note that it is possible to improve the gain at the steepest slope of the SQUID response curve by overcoupling $\eta > 1$. However, this reduces the gain at other values of flux.

2.5.4 TLS noise

Another major source of noise is fluctuations in two-level systems (TLS) in the resonator[58][82][78]. Where these have an electric dipole moment and feel the electric field of the resonator, they affect the distributed capacitance when they switch state. They therefore produce fluctuations in the resonance frequency.

Considering only the gain from flux to frequency:

$$\frac{1}{f_0} \left. \frac{df_0}{d\Phi} \right|_{f=f_0, \Phi=\Phi_0/2} = \frac{1}{\omega_0} \frac{d\omega_0}{dL} \frac{dL}{d\phi} \frac{2\pi}{\Phi_0} \quad (2.164)$$

$$= \frac{4\lambda\omega_0 M_c^2}{Z_1 L_S \Phi_0} \quad (2.165)$$

$$= \frac{\eta\pi}{Q_c \Phi_0} (1 - \lambda^2) \quad (2.166)$$

we again consider the matched state, $\eta \approx 1$, and small λ :

$$\boxed{\frac{1}{f_0} \left. \frac{df_0}{d\Phi} \right|_{f=f_0, \Phi=\Phi_0/2} \approx \frac{\pi}{Q_c \Phi_0}} \quad (2.167)$$

for $Q_c \approx 20,000$ this gain becomes $\frac{1}{f_0} \frac{df_0}{d\Phi} = 1.4 \times 10^{-4} / \Phi_0$. Referring frequency noise through this gain:

$$\boxed{S_\Phi|_{f=f_0, \Phi=\Phi_0/2} \approx \left(\frac{Q_c \Phi_0}{\pi f_0} \right)^2 S_{f_0}} \quad (2.168)$$

2.6 Flux-ramp Modulation

A necessary component of any SQUID multiplexer is a method for linearizing the SQUID response. Conventionally one linearizes with an active feedback loop that maintains the SQUID at a particular flux bias point. The feedback current is then directly proportional to the input current. The Microwave SQUID Multiplexer cannot use this technique because it would have to apply feedback to every SQUID in the array, re-introducing the multiplexing problem.

The alternative method we propose is *flux-ramp modulation*. We apply a periodic ramp that sweeps through multiple flux quanta in the SQUIDs, and require the slew rate of this ramp to greatly exceed that of any expected input signal. An input signal therefore looks like a flux offset during the duration of the ramp and produces a phase-shift in the SQUID response.

$$\phi = 2\pi \frac{\Phi}{\Phi_0} \quad (2.169)$$

To measure flux in the SQUID, we track the phase of the SQUID response. This method linearizes the SQUID response up to a slew-rate set by the frequency and amplitude of the flux-ramp. Most importantly, it can linearize all the SQUIDs in a large array using a single low-frequency flux bias line to apply the

flux-ramp to every SQUID in the array. As a bonus, the flux-ramp modulates the input signal of the SQUID up to higher frequencies, and can therefore avoid low-frequency noise in the amplifier chain after the SQUID.

Flux-ramp modulation increases readout noise relative to open-loop readout on the steepest slope of the SQUID response or readout in a flux-locked loop. The noise increases because some measurements during the ramp occur near extrema of the SQUID response curve, where it is insensitive to magnetic flux.

Assuming stationary noise and a sinusoidal SQUID response we can calculate the effective degradation in signal-to-noise. The signal-to-noise of independent measurements adds in quadrature. With stationary noise the signal-to-noise is proportional to the slope of the SQUID response and the degradation factor α is

$$\alpha = \frac{1}{2\pi} \int_0^{2\pi} \left(\frac{d}{d\phi} \sin \phi \right)^2 d\phi \quad (2.170)$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \cos^2 \phi d\phi \quad (2.171)$$

$$= \frac{1}{2} \quad (2.172)$$

We should therefore expect at least a $\sqrt{2}$ increase in flux noise when using flux-ramp modulation. This noise increase is balanced by the ability to linearize the response of all the SQUIDs in an array and modulate their input signals using a single pair of wires.

Chapter 3

Fabrication

These devices were fabricated by Leila Vale and Gene Hilton in the Quantum Fabrication Facility at NIST in Boulder, CO.

3.1 Substrate

To minimize two-level systems in the substrate, these devices are fabricated on 3 inch diameter wafers of high-resistivity silicon[83] ($\rho > 10^4 \Omega \text{ cm}^2$, $380 \mu\text{m}$). The wafers are covered with a 20 nm layer of SiO_2 .

3.2 Junction Fabrication

SQUID fabrication requires Josephson junctions with reliable critical current density J_c and minimal leakage current. The NIST process for Josephson junctions begins with a trilayer deposition in vacuum: deposit niobium (200 nm), deposit a thin layer of aluminum (~ 7 nm), flow oxygen to oxidize it, and deposit more niobium (120 nm). The entire wafer thus begins as a Josephson junction. The trilayer process has been tuned over the last 10 years and produces repeatable critical current density and low leakage across the wafer. The current rf SQUIDs use a $J_c = 0.5 \mu\text{A}/\mu\text{m}^2$ adaptation of the standard $J_c = 5 \mu\text{A}/\mu\text{m}^2$ trilayer recipe used the NIST time-division SQUID multiplexers.

The top two layers are etched away over most of the wafer, leaving isolated junction pillars. An additional niobium wiring layer subsequently connects these junctions to SQUID loops.

3.3 SQUID and Resonator Fabrication

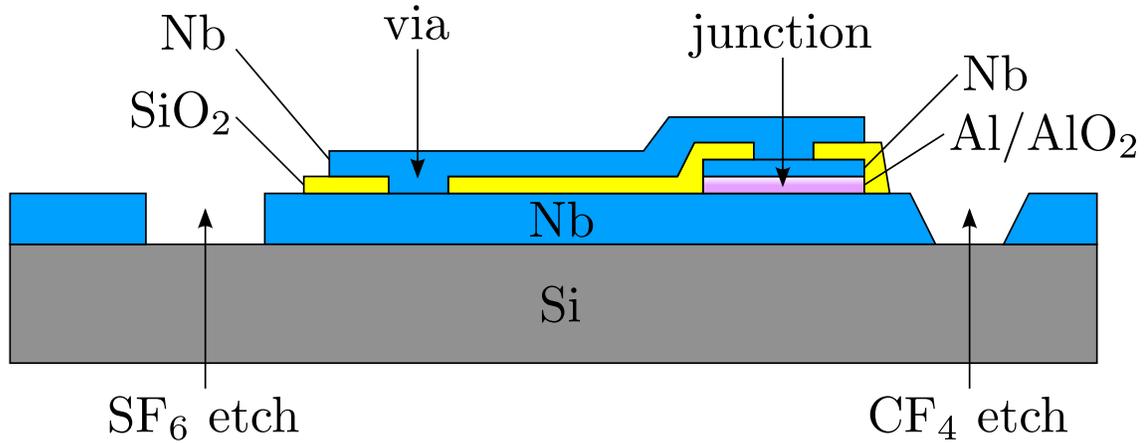


Figure 3.1: Diagram of the main layers of the microwave SQUID process.

Standard photolithography techniques pattern etch masks for the junctions, resonators, and SQUID wiring. The SQUID loop is defined in the base layer of niobium with a CF_4/O_2 reactive ion etch. This etch creates sloped sidewalls and therefore makes wiring crossovers more reliable. A silicon dioxide insulating layer is deposited (350 nm) and vias are etched in it to expose the junction islands and allow reconnection to the base niobium layer. A new layer of niobium is deposited (300 nm) that connects to these junctions and the base niobium. This layer constitutes the second wiring layer, while the base layer of the trilayer deposition constitutes the first wiring layer. These layers are shown in Figure 3.1.

The wiring pattern in the new layer is defined with an SF_6 reactive ion etch. This etch creates vertical sidewalls and appears to give much higher internal Q in niobium resonators. It is also used to define the coplanar waveguide resonators in the first layer of niobium.

Since the resonator is defined in the first layer of niobium, it will have had silicon dioxide deposited over it, then Niobium, and then the second layer of niobium etched away. At the end of the process, the silicon dioxide is etched away wherever possible to reduce the two-level system noise.

There is also a deposition and etch of a $2\Omega/\square$ layer of PdAu (135 nm) for the filter resistors.

Chapter 4

Measurement Setup

Early work (μmux07a , μmux09a) took place in a dilution refrigerator in Konrad Lehnert's lab at JILA, with the HEMT in liquid helium at 4 K and the resonators at 17 mK. Later work (μmux09a , μmux10b) took place in an adiabatic demagnetization refrigerator (ADR) at NIST (Figure 4.1), with the HEMT at 3 K and the resonators at temperatures ranging from 70 mK to 350 mK depending on the measurement.

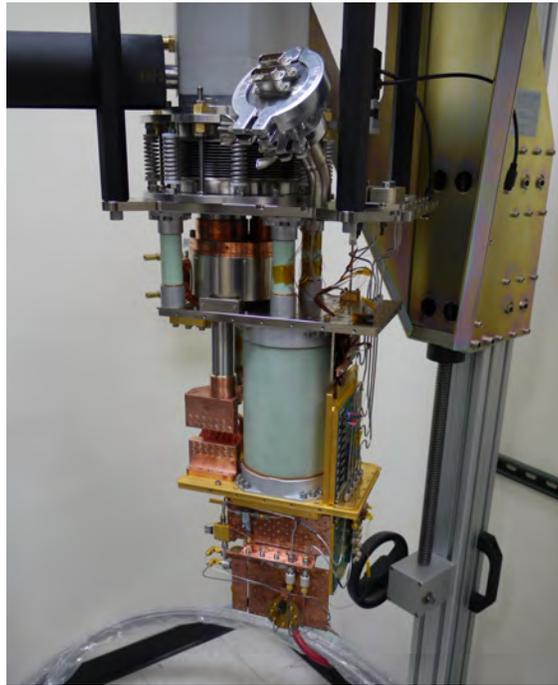


Figure 4.1: (ADR) at NIST, configured for microwave measurements.

Most experiments begin with sweeps on a network analyzer to identify the resonance frequency, Q 's,

and flux-dependence of the resonances. We then interrogate specific resonances using a homodyne setup (Figure 4.2) to measure of the amplitude and phase of the microwave signal transmitted through the multiplexer.

This setup generates a microwave signal and splits it into two arms: a reference arm and a measurement arm. The measurement arm enters the cryostat, passes through the multiplexer, and returns to room temperature through the HEMT amplifier. The two arms mix at room temperature in an I-Q mixer, which extracts the amplitude of the in-phase and quadrature-phase components of the measurement signal with respect to the reference signal. The phase shifter on the reference arm rotates the signal in the I-Q plane to place first-order phase shifts ($\text{Im}[S_{21}]$) in one quadrature.

The expected input power of the local-oscillator (LO) port of the mixer determines the microwave power at the outputs of the splitter and a variable attenuator at room temperature to determines the microwave power entering the cryostat. The measurement signal passes through another attenuator at 3 K and a directional coupler at base temperature. It then enters the multiplexer chip and passes by the resonance structures which modulate the transmitted amplitude and phase. It then passes through a circulator to the HEMT amplifier. A room-temperature amplifier boosts the signal to the expected input power of the RF port of the mixer. The two arms mix and produce two low-frequency signals that we digitize and analyze.

We want the multiplexer chip to see cold 50Ω terminations on both ports to minimize the noise temperature of the feedline. The measurement signal begins with a noise temperature of 300 K. We reduce this noise temperature with a -20 dB attenuator at 3 K and a -20 dB directional coupler at base temperature. The directional coupler dissipates the incident power a different termination than it presents to the multiplexer, so that that one doesn't heat. A circulator provides the cold termination on the output side of the multiplexer and isolates it from the HEMT, which has an input noise temperature of roughly 5 K. Figure 4.3 shows the actual microwave components at 3 K and base temperature.

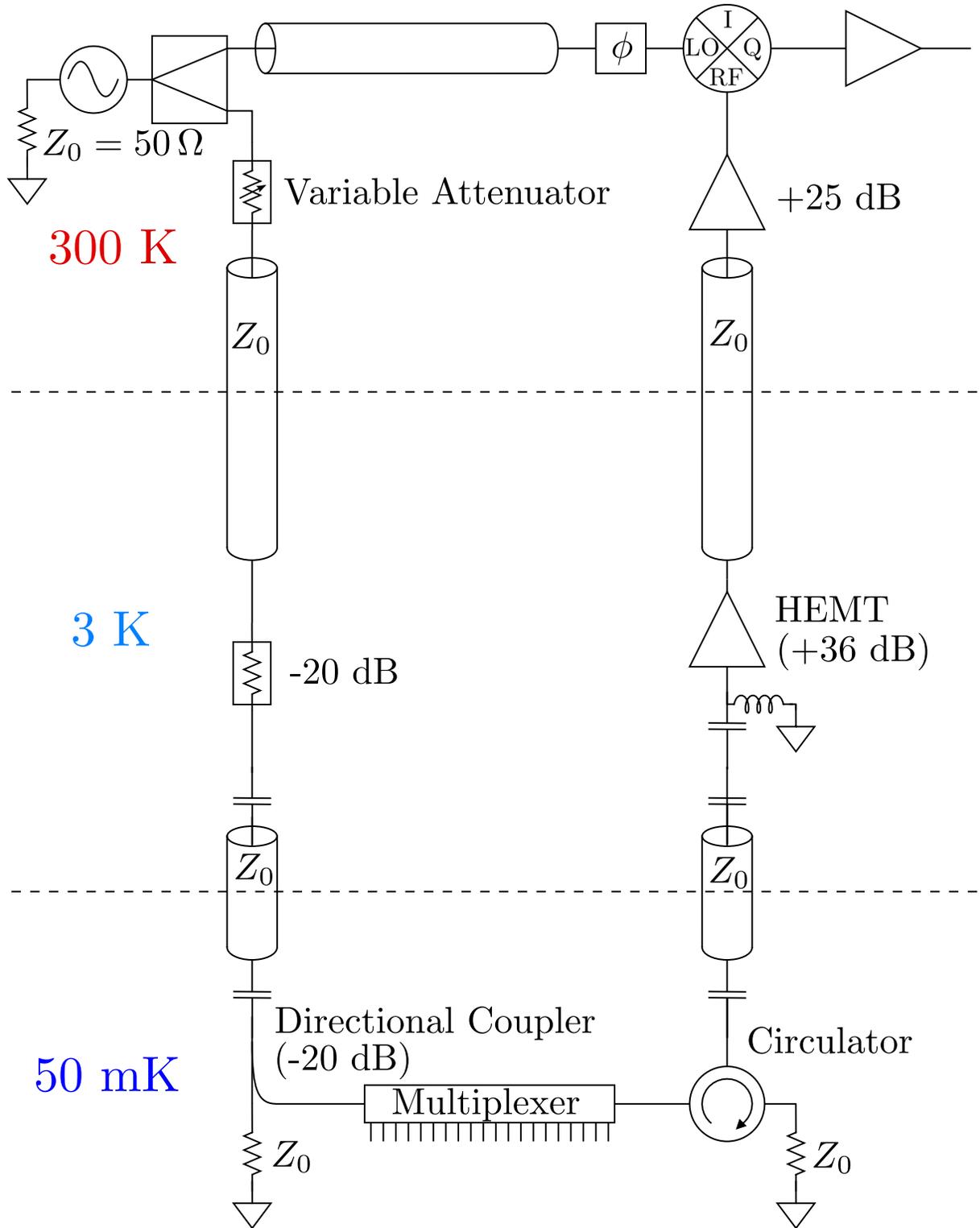


Figure 4.2: Schematic of the measurement apparatus for a single pixel.

To minimize the heat load on the ADR we use copper-nickel coaxial cables between 300 K and 3 K, thin copper-nickel coaxial cables between 3 K and the base temperature on the multiplexer input, and thin niobium-titanium coaxial cables between 3 K and the base temperature on the multiplexer output. We reduce the Wiedemann-Franz thermal conductivity by breaking the dc electrical connection of the coaxial cables with inside/outside dc blocks. Without good heat sinking the center pin of the HEMT can get much hotter than 3 K, so we heat sink it with a bias tee.

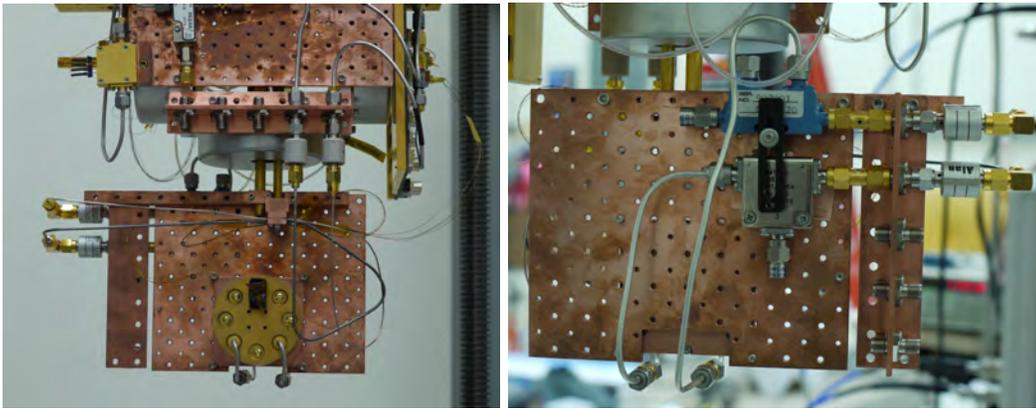


Figure 4.3: Photos of the microwave components in an ADR.

To simultaneously measure the flux in two SQUIDs, we used two synthesizers, two I-Q mixers, and splitters to combine the tones on the input and send a copy of the transmitted signals to each mixer.

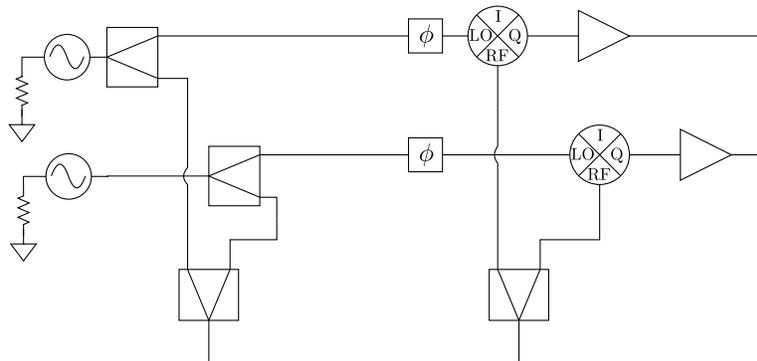


Figure 4.4: Schematic of the setup for measurement with two tones.

Appendix A lists the instruments and components used in this work.

4.1 Open-Source Electronics

A collaboration including our research group, several MKID groups, and several groups of digital electronics engineers, has been developing open-source electronics to read out microwave frequency division multiplexers[84][85]. These electronics perform Software-Defined Radio (SDR), digitally generating a superposition of measurement tones and digitally recording their transmitted amplitude and phase.



Figure 4.5: Photo of the open-source electronics.

The existing open-source electronics (Figure 9.2) uses two 1 GHz, 16-bit DACs to generate the measurement tones. We can mix these tones with a 4-8 GHz carrier to upconvert them to the microwave frequencies of interest, pass them through the multiplexer circuit, and mix them back down to baseband. Two 550 MHz, 12-bit ADCs then extract the amplitude and phase.

Chapter 5

Design Summary

Subject to the constraints of the fabrication process and the physical theory, we designed, fabricated, and tested multiple generations of Microwave SQUID Multiplexer:

- μmux07a was a proof-of-principle. It coupled simple, low-inductance dissipationless rf SQUIDs to microwave resonators and the resonators to a common feedline. It explored overcoupled and undercoupled regimes as well as rudimentary gradiometry and input filters. The primary goal was to perform microwave readout of a dissipationless rf SQUID. The secondary goal was to achieve low flux noise.
- μmux09a built on the success of μmux07a . The SQUID became a second-order gradiometer using slit and slotted washers. Input coupling increased to achieve low current noise for coupling to TES devices.
- μmux10b integrated the lessons of the previous designs to form a practical multiplexer for an array of CMB TESs. This entailed tight packing of the resonances in frequency space, optimizations to improve yield, and a common flux bias coil for flux-ramp modulation. The primary goal was a two-pixel TES multiplexing demonstration.

The next three sections describe the design process and experimental results (resonance spacing, Q , flux-dependence, flux-ramp modulation, crosstalk, noise) for these devices. They describe the major results of μmux07a and μmux09a and give a comprehensive treatment of μmux10b , culminating in a multiplexing demonstration with transition-edge sensors designed for polarimetry of the CMB.

Chapter 6

μ mux07a: Low Flux Noise

6.1 Design

μ mux07a was our first design to use dissipationless rf SQUIDs. It coupled 33 of these SQUIDs to superconducting resonators with resonance frequencies around 5 GHz and bandwidths on the order of 1 MHz. We designed the SQUID-resonator coupling so that some SQUIDs were overcoupled and some were undercoupled, included both gradiometric and non-gradiometric SQUID designs, and used microwave blocking filters on the input coils.

6.1.1 Resonator Design

The transmission lines for the feedline and resonators are coplanar waveguides (CPW) of niobium on high purity silicon. A coplanar waveguide consists of a center conductor with ground plane on both sides separated by a gap (Figure 6.1). The simplicity of this geometry is particularly appealing because it does not require dielectric between the conductors, avoiding a significant mechanism of loss and source of two-level-system noise. The high critical temperature of niobium ($T_c = 9.2$ K) means that at operating temperatures of roughly 300 mK there will be low thermal excitation of quasiparticles and therefore low microwave loss. We have measured $Q_i > 2 \times 10^6$ in niobium resonators on high-purity silicon. (Leduc[83] has measured $Q_i > 3 \times 10^7$ in titanium nitride resonators on high-purity silicon, but we do not need such low loss.)

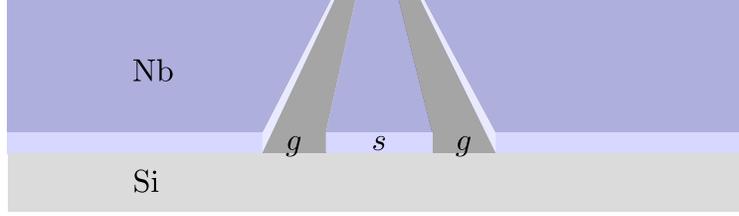


Figure 6.1: showing center conductor with a ground plane on both sides.

The frequency noise $S_{\delta f_0}$ of such CPW resonators exhibits a $f^{-1/2}$ spectral dependence and scales with geometry as $s^{-1.6}$ and power as $P_i^{-1/2}$ [58][78]. Gao reported the frequency noise at 1 kHz for resonators made of various materials[82], including niobium with a 3 μm center strip and 2 μm gaps at various internal powers. The internal power predicted by Equation 2.118 for these SQUIDs was roughly 0.1 μW or -40 dBm, and at this internal power Gao measured $S_{\delta f_0}/f_0^2 \approx 10^{-19}/\text{Hz}$.

This frequency noise implies (Equation 2.168) a flux noise of $0.5 \mu\Phi_0/\sqrt{\text{Hz}}$, exceeding the flux noise due to the HEMT. We designed resonators with a 10 μm center strip and 6 μm gaps to reduce this noise.

We chose to fabricate 33 resonators on a chip, spacing them 30 MHz apart to ensure negligible crosscoupling (Section 2.3.9). We used TX-line[86] to calculate the characteristic impedance $Z_1 = 50.3 \Omega$ and wavelength $\lambda = 20.4 \text{ mm}$ at 6 GHz, using $\epsilon = 11.45$ for the silicon substrate at cryogenic temperatures[87]. The coupling capacitor and coupling inductor combine to lower the resonance frequency by 10%. We therefore designed waveguides between 4 mm and 5 mm long, stepping the length by 25 μm .

6.1.2 Resonator-Feedline Coupling

We designed coupling capacitors for a spread around $Q_c \approx 3,000$ by simulating S_{13} of various designs in Microwave Office, where ports 1 and 2 are the ports of the feedline and port 3 is the resonator side of the coupler. From Section 2.3.6, this transmission coefficient relates to Q_c by:

$$Q_c = \frac{\pi}{1 - |\Gamma|^2} \quad (6.1)$$

$$= \frac{\pi}{2|S_{13}|^2} \quad (6.2)$$

The simulations led to various interdigitated capacitor couplers (Figure 6.2) for resonances of various bandwidths.

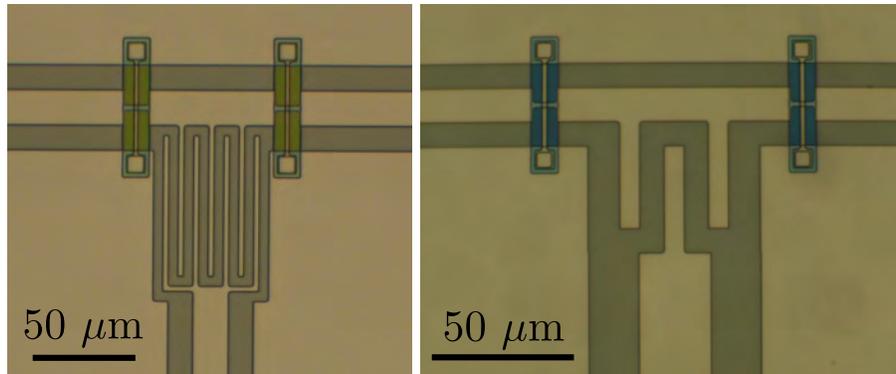


Figure 6.2: Photos of interdigitated capacitors coupling resonators to the feedline.

Straps connected the ground planes on either side of the feedline. These straps are critical for consistent Q and resonance spacing. Without them, oscillating voltage on the capacitor drives a coupled slotline mode on the feedline. This mode cannot propagate off the chip and therefore develops standing waves. These standing waves mean that the feedline presents a different effective impedance to resonators at different positions on the chip.

As the ground strap crosses over the feedline it changes the capacitance to ground and can create an impedance discontinuity. The feedline must constrict under the ground strap to maintain continuous impedance. Simulations in Microwave Office led to the design in Figure 6.3.

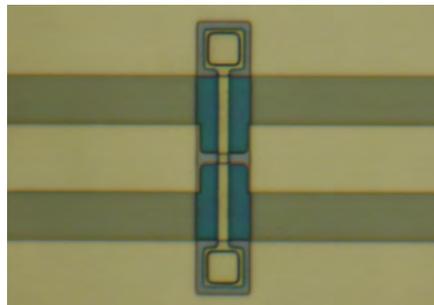


Figure 6.3: on either side of the feedline.

6.1.3 Critical Current

The standard trilayer process for the NIST dc-SQUID has a critical current density of $J_c = 50 \mu\text{A}/\mu\text{m}^2$. A $10 \mu\text{m}^2$ junction requires ($\lambda \approx 1/3$) a SQUID loop inductance of $L_S \approx 2 \text{pH}$ to ensure non-hysteretic operation. As this inductance is prohibitively small, we developed a $J_c = 1.5 \mu\text{A}/\mu\text{m}^2$ trilayer, with which a $10 \mu\text{m}^2$ junction requires a loop inductance of $L_S \approx 8 \text{pH}$, still a small inductance, but reasonable to fabricate in our cleanroom.

From this critical current Equation 2.163 predicts that at the optimal flux bias, the flux noise due to the HEMT amplifier should be roughly $0.35 \mu\Phi_0/\sqrt{\text{Hz}}$ for the SQUIDS with matched coupling and even less for the overcoupled SQUIDS.

6.1.4 Coil Geometry

A SQUID is unfortunately sensitive to flux from magnetic fields in the environment, such as Earth's field, or the fields from magnetic materials in a telescope, cryostat, or office chair. Earth's field drives a magnetic flux quantum through a loop as small as $10 \mu\text{m} \times 10 \mu\text{m}$. This pickup degrades the SQUID as an amplifier of the current through a specific low-temperature detector. Coil gradiometry (Figure 6.4) can greatly reduce this pickup.

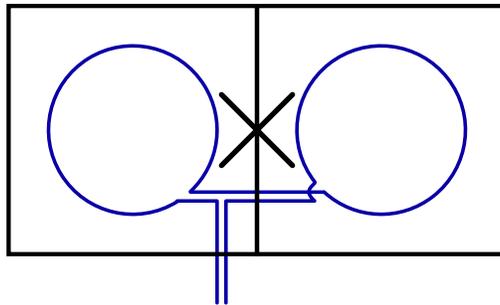


Figure 6.4: Schematic of a parallel, two-lobe (first-order) gradiometer with input coil.

Series gradiometry consists of twisting a loop so that it only couples to field gradients. Parallel gradiometry consists of orienting parallel lobes so that they couple to a uniform field with opposite polarity;

screening current flows around the circumference of the gradiometer, bypassing the Josephson junction, cancelling the flux from a uniform field and leaving equal and opposite flux in the two lobes. Note that in a parallel gradiometer the mutual inductance between input coil and SQUID is that of a single lobe, but the SQUID self-inductance is divided by the number of lobes.

Gradiometric coils couple to higher-order terms in the multipole expansion of the magnetic field. Distant sources produce dipole magnetic fields, and nearer sources can produce quadrupole fields, but the winding of the input coil can generate octopole or even higher-order fields.

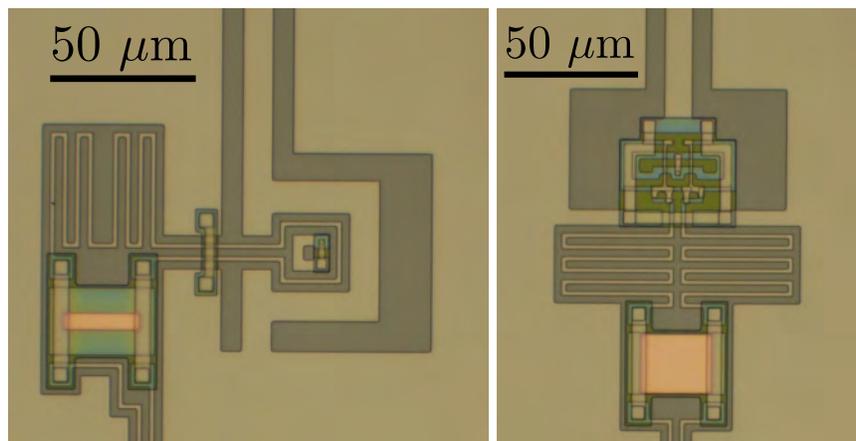


Figure 6.5: each inductively coupled to the current antinode of a resonator in μmux07a . Both photos show the input filter consisting of a meander inductor shunted with a copper resistor.

μmux07a incorporated both non-gradiometric and first-order parallel gradiometric SQUID designs (Figure 6.5). We used simulations of the coupling between the resonators and the SQUIDs to match the peak-to-peak shift in resonance frequency to the resonator bandwidths, intentionally overcoupling some SQUIDs and undercoupling others.

6.2 Filter Design

Microwave power from a resonator can potentially leak out on flux bias lines, degrading Q_i or even affecting the low-temperature detector on the input. To prevent this we placed inductive chokes on the input lines (Figure 6.5).

The inductive chokes may be unnecessary, as any connection with a low-temperature detector is likely to be made with wire bonds (~ 1 nH/mm). We chose to avoid microwave analysis of the low-frequency wiring and of the low-temperature detectors. To further eliminate the low-frequency input circuit from the microwave analysis, we shunted the inductive chokes with resistors, forming high-bandwidth L/R filters on the input lines.

6.3 Results

Measurements of μmux07a confirmed that we could fabricate multiple superconducting microwave resonators on a chip, and that the resonance frequencies of the resonators shifted periodically with magnetic flux in the SQUID. The dissipationless rf SQUIDs showed low open-loop flux noise at 100 kHz.

6.3.1 Resonance Spacing

We surveyed S_{21} of a μmux07a chip across wide frequency range. Not all resonances yielded, and some clearly shifted from their design frequencies (Figure 6.6). The remaining resonances were spaced by roughly 40 MHz with Q_i varying between 20,000 and 40,000 and Q_c varying between 2,000 and 6,000.

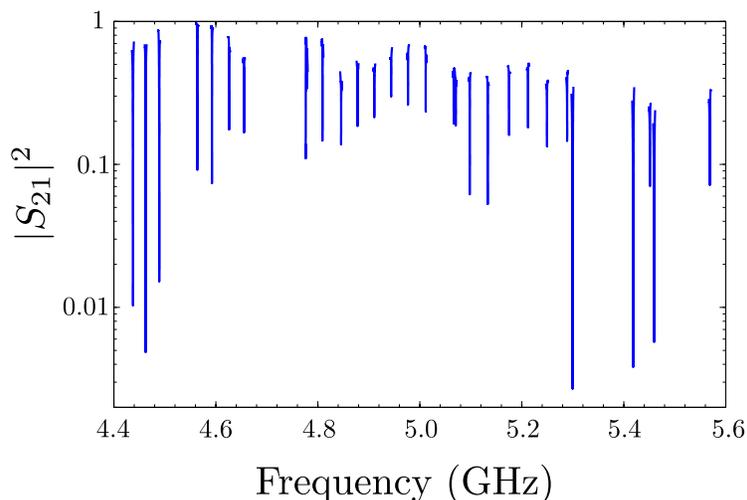


Figure 6.6: Survey of the μmux07a resonances.

6.3.2 Flux-variable Resonance Frequency

These microwave resonances shifted with magnetic flux in their respective SQUIDs in full agreement with the theory in Section 2.2.4. The transmission S_{21} traversed a circle in the complex plane, making the strongest response in the imaginary component of S_{21} and with the excitation frequency roughly halfway between the resonance frequency extrema.

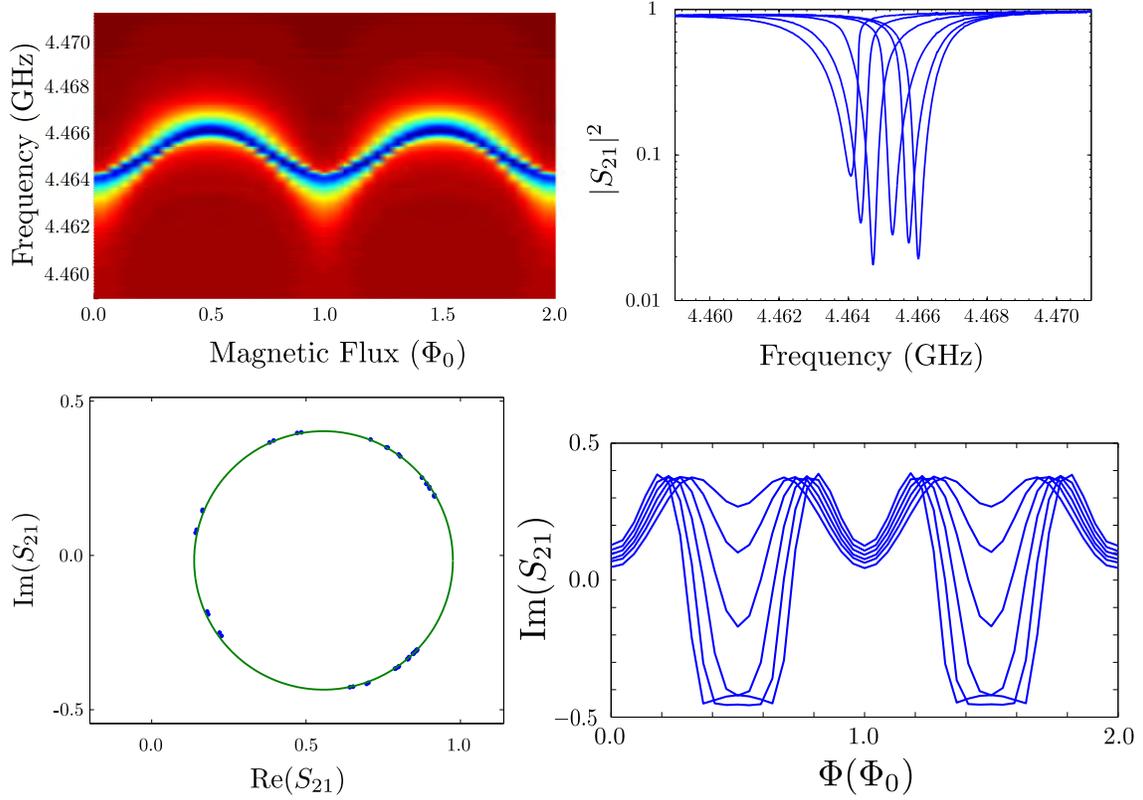


Figure 6.7: . Clockwise from the color map: $|S_{21}|$ in color (blue is no transmission, red is unity transmission) as a function of both frequency and magnetic flux, Lorentzian dips in transmitted power at different flux biases, $\text{Im}[S_{21}]$ as a function of flux at different excitation frequencies, S_{21} in the complex plane at one excitation frequency and multiple flux bias values. These data were obtained at excessive microwave power, which distorts the resonance shape.

Figure 6.7 shows the first SQUID/resonator pair we examined in μmux07a [63]. It is strongly overcou-

pled, so that variation in flux in the SQUID can shift the resonance with respect to a stationary excitation to make the excitation on resonance or almost completely off resonance.

This coupling strength means that a strong microwave excitation can drive the resonator to bifurcation[76]. When the resonance frequency shifts dramatically over one period of the microwave excitation, the resonator can occupy either of two modes which differ in the phase relationship between the microwave excitation on the feedline and the standing wave in the resonator and in internal power. The switching between these bistable modes disrupts any flux-locked loop or flux-ramp modulation scheme.

6.3.3 Flux Noise

To measure open-loop flux noise we bias to a steep slope of the $\text{Im}[S_{21}]$ vs. Φ curve, measure output noise, and divide it by the slope of the curve. The results indicate the flux noise we would obtain operating the SQUID in a flux-locked loop. The flux noise with flux-ramp modulation is worse by at least a factor of $\sqrt{2}$ than the open-loop flux noise at the modulation frequency. This penalty is even worse for overcoupled SQUIDs because their response is not sinusoidal.

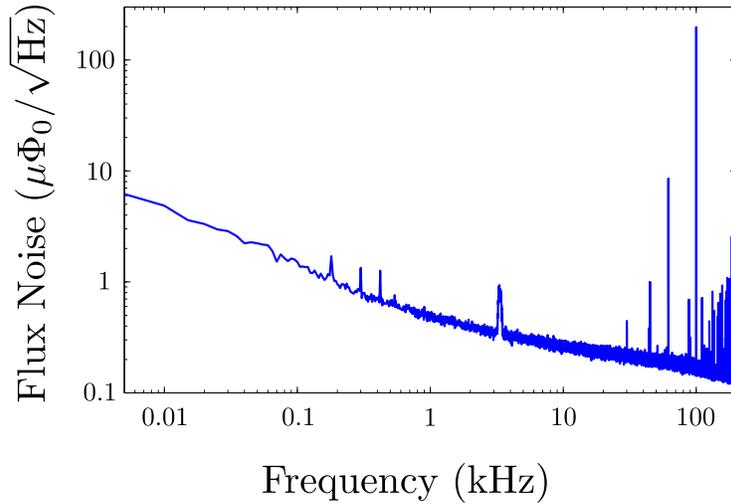


Figure 6.8: . The noise falls to $0.17 \mu\Phi_0/\sqrt{\text{Hz}}$ at 100 kHz. The 100 kHz tone comes from a known input flux that calibrated the measurement.

Using the homodyne measurement setup from Figure 4.2 to read out the SQUID, we observed open-loop flux noise falling from $5 \mu\Phi_0/\sqrt{\text{Hz}}$ at 10 Hz to $0.17 \mu\Phi_0/\sqrt{\text{Hz}}$ at 100 kHz (Figure 6.8). Few SQUIDS have demonstrated noise this low[88].

The low self-inductance of the SQUID limits the input coupling due to the fact that $M_{in} \leq \sqrt{L_s L_{in}}$ and the practical limit on the number of turns on the input coil in a $20 \mu\text{m} \times 20 \mu\text{m}$ area. The input coupling for this SQUID was only $M_{in} \approx 5 \text{ pH}$ so the open-loop input current noise was roughly $100 \text{ pA}/\sqrt{\text{Hz}}$. This is comparable to the output current noise of many TES designs.

6.3.4 Flux-ramp Modulation

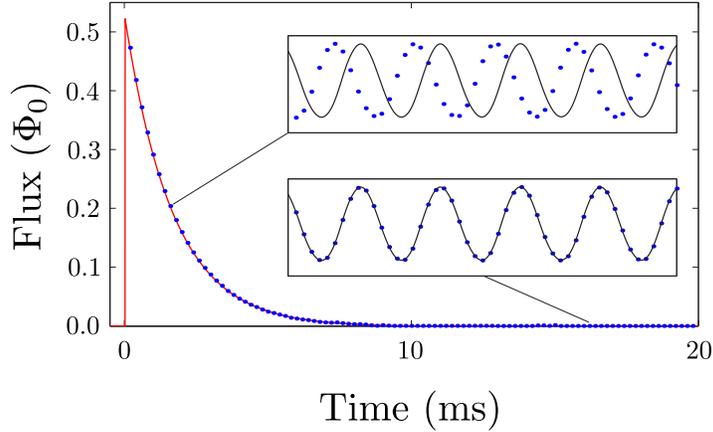


Figure 6.9: performed with μmux07a reproducing an input pulse and exponential decay.

We did not fully implement flux-ramp modulation because these SQUIDS had only a single input coil. Instead, we drove the SQUIDS with the sum of a 5 kHz, $10 \Phi_0$ sawtooth with a variety of slow functions, e.g. an exponential decay with a $\tau \approx 1.6 \text{ ms}$ time constant.

We began by Nyquist sampling the SQUID response and averaging the response to many ramps to form a Nyquist interpolated template function. We then performed a Wiener-optimal fit for the phase, shifting the template function against each measured time trace, filtering the traces according to the power

spectrum of the template.

We were able to reconstruct the slow input functions (Figure 6.9). The demodulated data matched the input functions without any additional fitting parameters because readout with flux-ramp modulation is linear according to $\phi = 2\pi \frac{\Phi}{\Phi_0}$.

6.3.5 Summary

μmux07a demonstrated readout of a dissipationless rf SQUID coupled to a superconducting microwave resonator. It showed we could achieve exceptionally low flux noise. It demonstrated flux-ramp demodulation to linearize the SQUID response. These results were promising and suggested the development of a Microwave SQUID Multiplexer with stronger input coupling to the SQUIDs for lower input current noise.

Chapter 7

μ mux09a: Low Input Current Noise

7.1 Design

μ mux09a was our first attempt to make a dissipationless rf SQUID for TES readout, meaning a SQUID matched to the bandwidth of its resonator, with low input current noise, low environmental pickup, and two flux bias lines. We therefore appropriated the design of the NIST dc-SQUID for time-division multiplexing and adapted it for microwave operation.

7.1.1 Resonator Design

The resonator CPW expanded to a μm center conductor with $13\ \mu\text{m}$ gaps to reduce the two-level system frequency noise. We again coupled 33 resonators to the feedline with a target spacing of roughly 30 MHz.

7.1.2 Resonator-Feedline Coupling

μ mux09a included several design splits, with coupling capacitors tuned for $Q_c \approx 400, 900, 10,000,$ and $40,000$. We will focus on results for the design for $Q_c \approx 400$.

7.1.3 Critical Current

The NIST dc-SQUID has a self-inductance of roughly 20 pH, requiring ($\lambda \approx 1/3$) a junction with a critical current of roughly $I_c = 5\ \mu\text{A}$. The fabrication team at NIST developed a $J_c = 0.5\ \mu\text{A}/\mu\text{m}^2$ trilayer

process that allowed us to use a $10 \mu\text{m}^2$ junction area. From this critical current Equation 2.163 predicts a flux noise due to the HEMT amplifier of roughly $0.6 \mu\Phi_0/\sqrt{\text{Hz}}$.

7.1.4 Coil Geometry

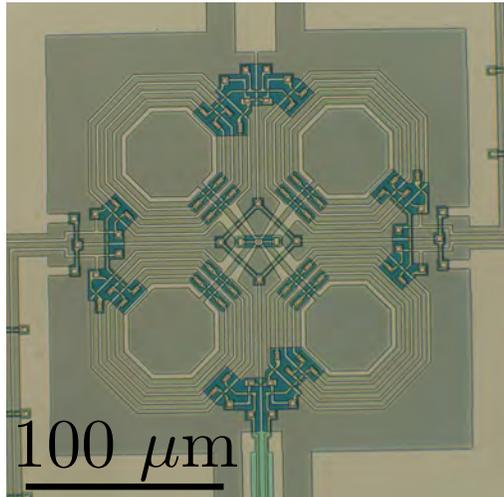


Figure 7.1: . (μmux09a)

This SQUID loop is a second-order gradiometer consisting of four parallel lobes arranged like a clover-leaf (Figure 7.1). Each lobe uses a slotted washer design with the input coil and feedback coil running in the slots to minimize their capacitive coupling to the SQUID. The orientation of the lobes alternates around the SQUID, making the SQUID insensitive to first-order gradients as well as uniform fields .

The flux bias coils alternate orientation from lobe to lobe and therefore generate fields that couple strongly to the gradiometer. The flux-ramp coil makes a half-turn around each lobe and the input coil makes one and a half turns around each lobe. The size of these lobes enables input coupling of $M_{in} \approx 85 \text{ pH}$.

Since metal traces on the chip can re-broadcast uniform fields from the environment as higher-order fields, we took care to symmetrize metal around the chip, either by 180° around the junction, or by reflection across the axis of the resonator.

7.1.5 SQUID-Resonator Coupling

There are several ways to couple microwave flux into the SQUID. Each design has a different inductance matrix between the SQUID, resonator, flux-ramp coil, and input coil. We simulated different designs with FastHenry[89] so that we could match the peak-to-peak shift in resonance frequency to the resonator bandwidth.

7.1.5.1 Direct Coupling

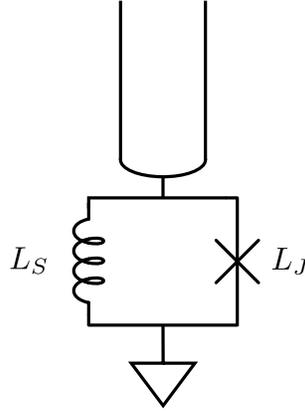


Figure 7.2: Circuit diagram for an rf SQUID directly coupling to the current anti-node of a resonator.

The theory we developed in Section 2.1 considers inductively coupled rf SQUIDs, but anti-node current flowing directly through the rf SQUID (Figure 7.2) produces a similar effect. Instead of a peak-to-peak change in termination inductance of

$$\Delta L_{pp} = \frac{2M_c^2}{L_J(1 - \lambda^2)} \quad (7.1)$$

direct coupling gives a peak-to-peak change of

$$\Delta L_{pp} = \frac{2\lambda^2 L_J}{1 - \lambda^2} \quad (7.2)$$

For $L_J \approx 60$ pH, $\lambda \approx 1/3$, $Z_1 \approx 50 \Omega$, and $f_0 \approx 6$ GHz the peak-to-peak frequency shift in response to magnetic flux is

$$\Delta f_0 = \frac{4f_0^2 L_{pp}}{Z_1} \quad (7.3)$$

$$= \frac{4f_0^2 2\lambda^2 L_J}{Z_1 1 - \lambda^2} \quad (7.4)$$

$$= \frac{4(6 \text{ GHz})^2 2\frac{1}{9}(60 \text{ pH})}{50 \Omega 1 - \frac{1}{9}} \quad (7.5)$$

$$\approx 30 \text{ MHz} \quad (7.6)$$

Direct coupling is therefore ideal for very high-bandwidth resonances. We tested directly coupled SQUIDS in μmux07a , but have since focused on narrower band resonances.

7.1.5.2 Weak Inductive Coupling

Antinode current that splits around the SQUID before it returns to ground (Figure 7.3) weakly couples flux into the SQUID. The current that wraps around the SQUID is closer than the return current on the ground plane and therefore creates a non-zero magnetic field in the lobes of the SQUID. The flux is stronger in the two lobes closer to the resonator and changes sign from left to right, therefore coupling to the gradiometric SQUID. This design does not require the resonator wiring to cross over the SQUID wiring.

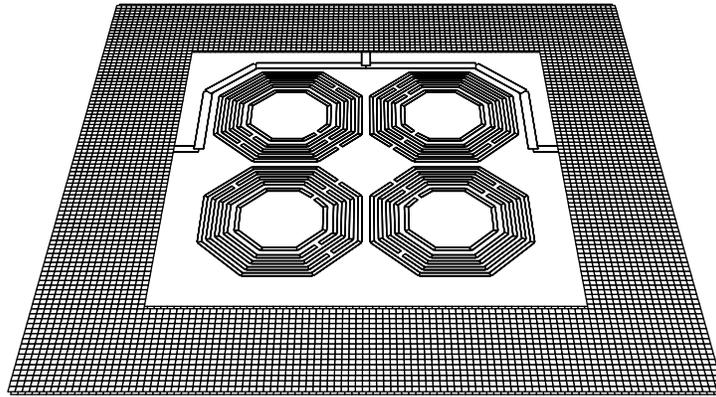


Figure 7.3: . Crossovers are implemented with virtual connections.

M_{ij} (pH)	SQUID	Ramp	Input	Resonator
SQUID	18.9	30.9	87.5	1.65
Ramp		312	115	3.64
Input			1044	8.81
Resonator				77.6

Table 7.1: Simulated inductance values for a weak inductive coupling between the resonator and the SQUID.

The simulation (Figure 7.3 and Table 7.1) gives inductance values that determine the maximum frequency shift in response to flux:

$$\Delta f_0 = \frac{4f_0^2}{Z_1} \frac{2\lambda M_c^2}{L_S(1-\lambda^2)} \quad (7.7)$$

$$= \frac{4(6 \text{ GHz})^2}{50 \Omega} \frac{(\frac{2}{3})(1.65 \text{ pH})^2}{(18.9 \text{ pH})(1-\frac{1}{9})} \quad (7.8)$$

$$\approx 310 \text{ kHz} \quad (7.9)$$

This weak inductive coupling is ideal for the narrowest bandwidth resonances that we can reliably space by ten times their bandwidth. We use this coupler design for the high- Q splits of μmux09a and for μmux10b .

7.1.5.3 Strong Inductive Coupling

Antinode current that wraps completely around two lobes strongly couples flux into the SQUID. Again this design generates a field with the appropriate second-order gradients to couple magnetic flux into the SQUID gradiometer. For symmetry, current can continue directly down to the ground plane along the line of symmetry, but it does not couple flux into the gradiometer. This design requires the resonator wiring to cross over the SQUID wiring.

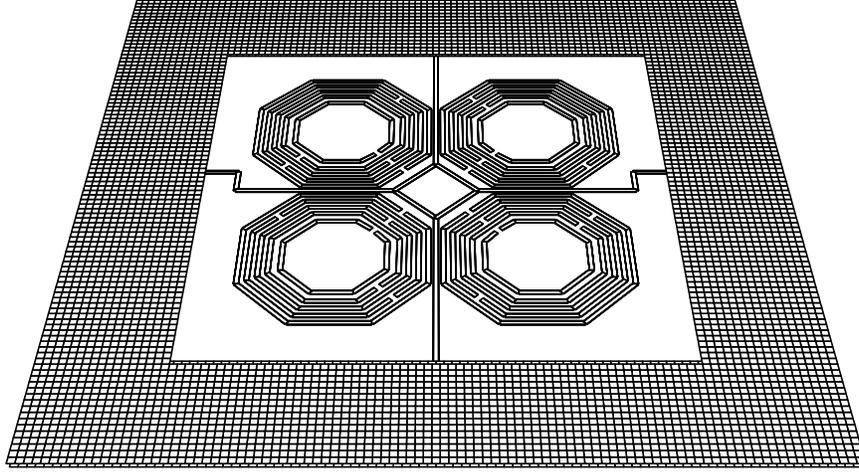


Figure 7.4: . Crossovers are implemented with virtual connections.

M_{ij} (pH)	SQUID	Ramp	Input	Resonator
SQUID	17.9	28.9	82.5	9.42
Ramp		308	104	20.2
Input			1017	51.9
Resonator				145

Table 7.2: Simulated inductance values for a strong inductive coupling between the resonator and the SQUID.

The simulation (Figure 7.4 and Table 7.2) gives inductance values that determine the maximum frequency shift in response to flux:

$$\Delta f_0 = \frac{4f_0^2}{Z_1} \frac{2\lambda M_c^2}{L_S(1 - \lambda^2)} \quad (7.10)$$

$$= \frac{4(6 \text{ GHz})^2}{50 \Omega} \frac{(\frac{2}{3})(9.42 \text{ pH})^2}{(17.9 \text{ pH})(1 - \frac{1}{9})} \quad (7.11)$$

$$\approx 11 \text{ MHz} \quad (7.12)$$

The stronger SQUID-resonator coupling causes a much larger frequency shift. We use this coupler design for the low- Q splits of μmux09a .

7.1.5.4 Adjustable Inductive Coupling

Since resonator current that flows to ground along the line of symmetry does not couple flux into the gradiometric SQUID, we can adjust the coupling of the strong inductive coupler by changing the distribution of current between the three branches to ground. We add inductance on the side branches to make a larger fraction of the current flow along the line of symmetry.

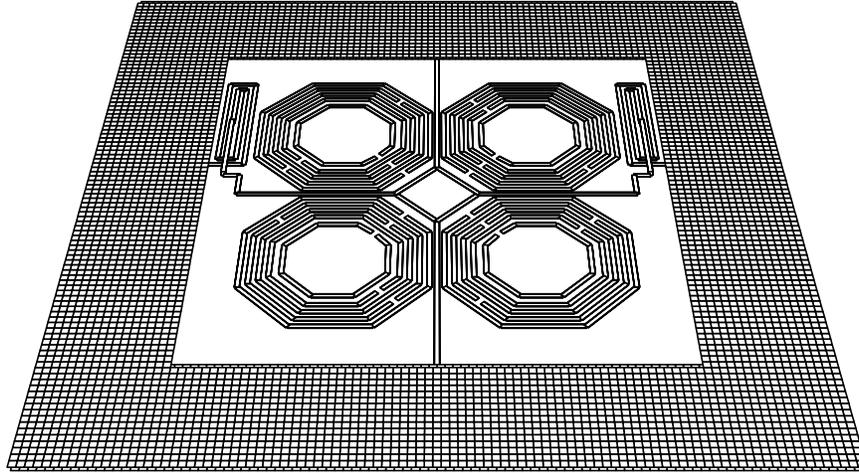


Figure 7.5: . Crossovers are implemented with virtual connections.

M_{ij} (pH)	SQUID	Ramp	Input	Resonator
SQUID	18.3	29.7	84.6	5.46
Ramp		310	108	11.7
Input			1028	30.3
Resonator				186

Table 7.3: Simulated inductance values for a variable inductive coupling between the resonator and the SQUID.

The simulation (Figure 7.5 and Table 7.3) gives inductance values that determine the maximum

frequency shift in response to flux:

$$\Delta f_0 = \frac{4f_0^2}{Z_1} \frac{2\lambda M_c^2}{L_S(1-\lambda^2)} \quad (7.13)$$

$$= \frac{4(6 \text{ GHz})^2}{50 \Omega} \frac{(\frac{2}{3})(5.46 \text{ pH})^2}{(18.3 \text{ pH})(1-\frac{1}{9})} \quad (7.14)$$

$$\approx 3.5 \text{ MHz} \quad (7.15)$$

The extra inductance on the coupling branches reduces the SQUID-resonator coupling so that the frequency shift is intermediate between the strong and weak inductive coupling designs. We have not used a coupler of this design, but it may be useful in the future.

7.2 Filter Design

The stronger input coupling requires stronger microwave blocking filters (Figure 7.6) on both the flux-ramp and input coils.

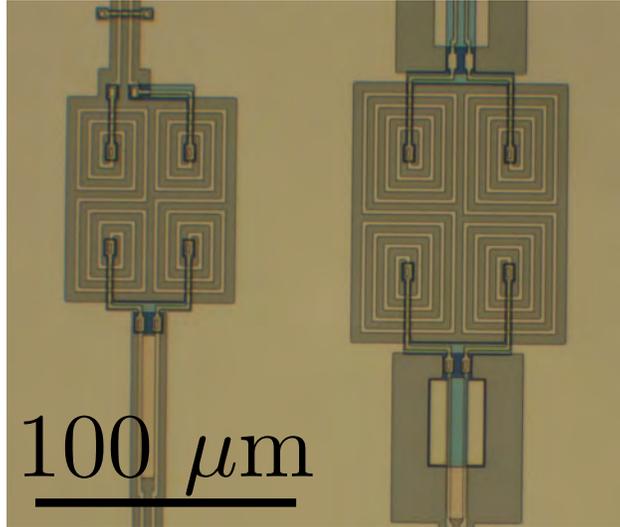


Figure 7.6: that block microwave power from reaching the input circuits.

With $L_F \approx 7 \text{ nH}$ and $R_F \approx 0.2 \Omega$, Equation 2.148 sets a limit on the internal quality factor due to losses in the filter on flux-ramp coil of $Q_f \approx 5 \times 10^5$. The limit on the quality factor due to the input coil filter is even higher. We have observed $Q_i > 2 \times 10^6$ in bare resonators, so it is possible for the filter to set

the Q_i of the resonator, but $Q_i \approx 5 \times 10^5$ is enough to make the power losses in a multiplexer insignificant in comparison to TES bias power.

7.3 Results

Measurements of μmux09a proved that complex SQUIDs work well in microwave resonant circuits. These SQUIDs demonstrated low input current noise. The flux-ramp modulation wire did not yield in this fabrication, so we could not demonstrate flux-ramp modulation.

7.3.1 Resonance Spacing

We surveyed S_{21} of a μmux09a chip designed for $Q_c \approx 400$ across wide frequency range. All resonances yielded, spaced by roughly 30 MHz, with one shifting from its design frequency to interfere with another (Figure 7.7).

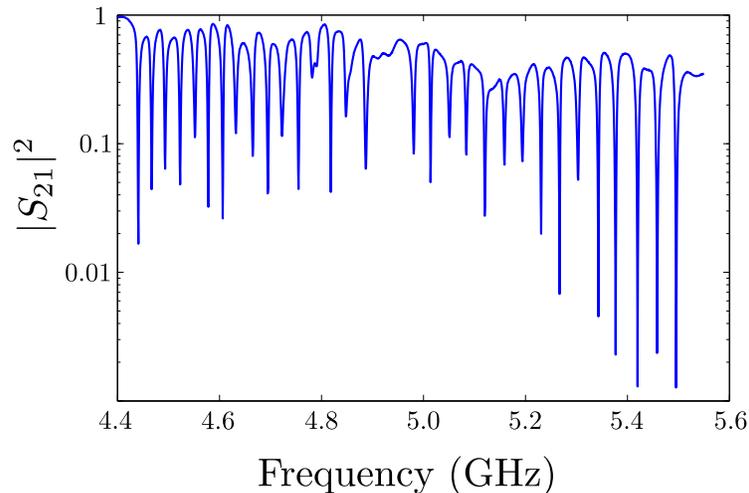


Figure 7.7: on a chip designed for $Q_c \approx 400$. The 4.8 GHz dip is actually two colliding resonances. One resonance is not shown.

7.3.2 Flux-variable Resonance Frequency

The microwave resonances shifted in agreement with theory in response to magnetic flux (Figure 7.8).

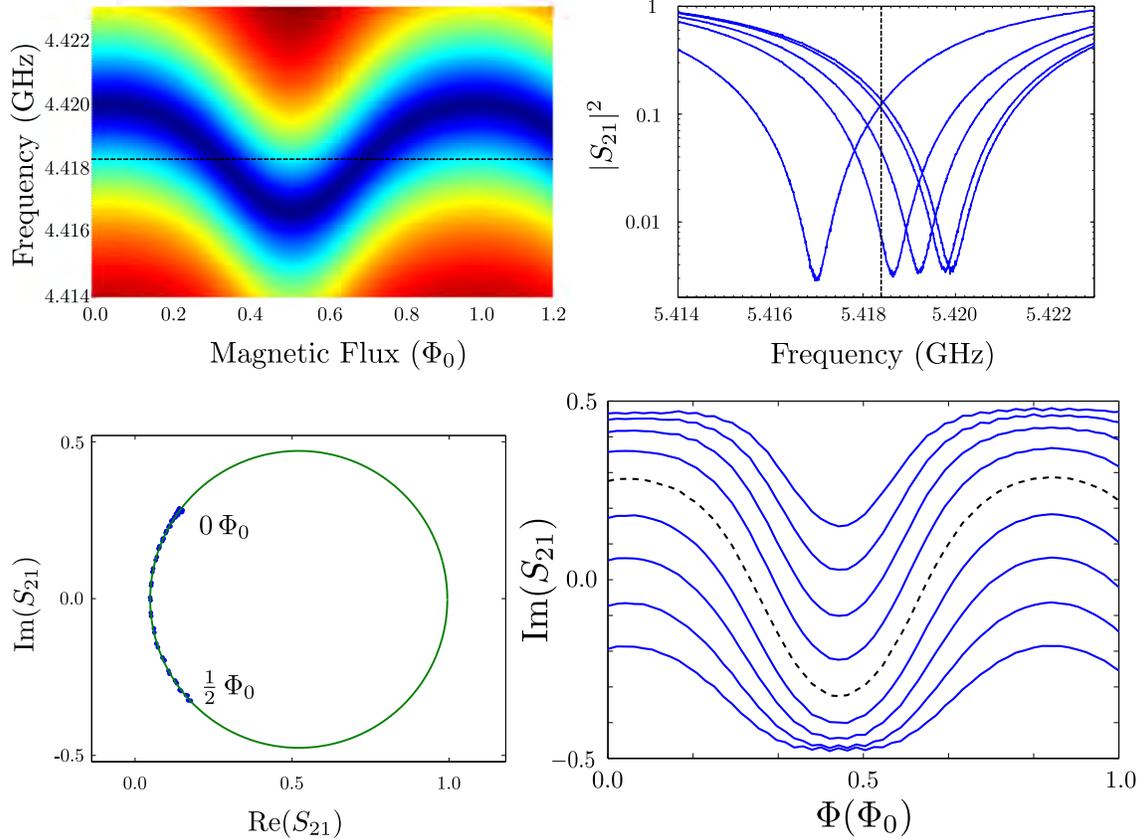


Figure 7.8: . Clockwise from the color map: $|S_{21}|$ in color (blue is no transmission, red is unity transmission) as a function of both frequency and magnetic flux, Lorentzian dips in transmitted power at different flux biases, $\text{Im}[S_{21}]$ as a function of flux at different excitation frequencies, S_{21} in the complex plane at several excitation frequencies as a function of flux bias.

The resonance in Figure 7.8 was slightly undercoupled. Since the peak-to-peak shift in resonance frequency of this resonance was less than the bandwidth of the resonance, we could find a microwave excitation frequency for which $\text{Im}[S_{21}]$ was almost sinusoidal, making it well suited to operation with flux-ramp modulation.

Unfortunately, several vias on the flux-ramp line failed to yield, preventing us from demonstrating true flux-ramp modulation and demodulation until μmux10b .

7.3.3 Flux Noise and Current Noise

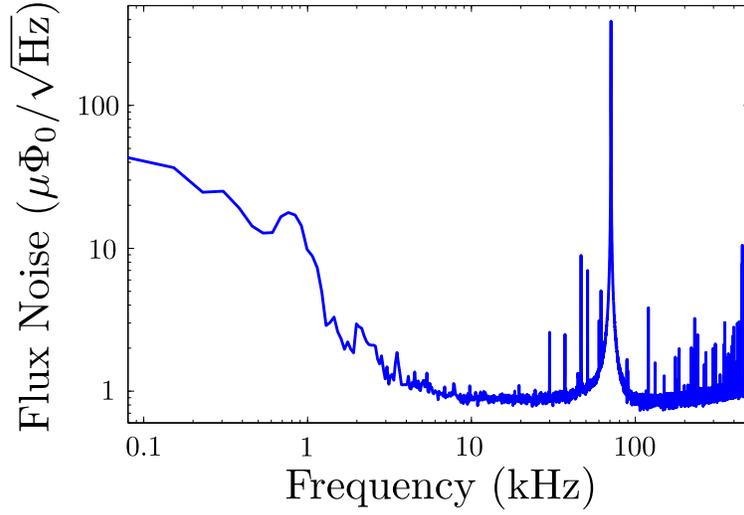


Figure 7.9: . The noise falls to roughly $0.8 \mu\Phi_0/\sqrt{\text{Hz}}$ at 100 kHz. The 71 kHz tone is due to an input flux oscillation that calibrates the measurement.

The $Q_c \approx 900$ resonances and wide CPWs caused the noise to reach the HEMT limit at much lower frequencies (Figure 7.9). The larger inductance of μmux09a also raised this noise floor. The flux noise reached roughly $0.8 \mu\Phi_0/\sqrt{\text{Hz}}$ at 100 kHz, which is higher than the flux noise of μmux07a but comparable to the flux noise of the NIST dc-SQUIDs.

The larger μmux09a SQUID design allowed stronger input coupling of $M_{in} = 87.5 \text{ pH}$. Therefore while flux noise increased, current noise fell to $19 \text{ pA}/\sqrt{\text{Hz}}$. With flux-ramp modulation we expect this current noise to degrade to roughly $30 \text{ pA}/\sqrt{\text{Hz}}$, which would allow readout of many Transition-Edge Sensors without degradation.

7.3.4 TES Readout

To demonstrate the utility of this multiplexer we obtained several TES chips designed for CMB polarimetry and interface chips with appropriate shunt resistors. We bonded one of these to the μmux09a (Figure 7.10) and measured its noise with the SQUID biased at a steep slope of its response curve.

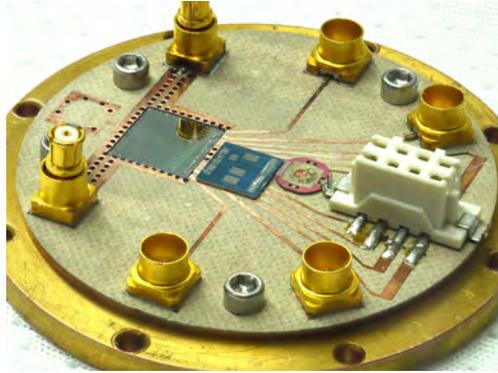


Figure 7.10: Photo of a μmux09a chip wired for readout of a CMB TES chip.

We measured a noise-equivalent power (Figure 7.11) with a plateau at $3 \times 10^{-17} \text{ W}/\sqrt{\text{Hz}}$, matching measurements of the same TES in a dc-SQUID system. The NEP rolled off at the thermal bandwidth of the TES. It showed significant excess noise at low frequencies that we attribute to pickup on the heater line.

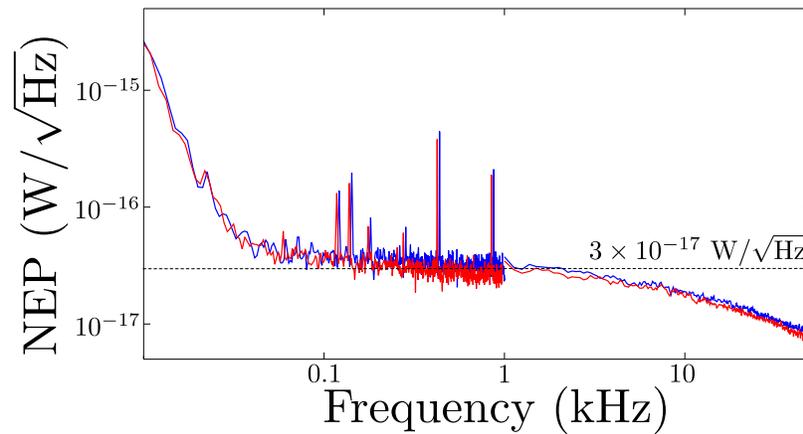


Figure 7.11: . The two curves were taken at two different TES bias points.

The excess noise did not depend on TES resistance, which indicated that it was a true power noise. Pickup on the heater lines is a common problem for TES readout with Time-Division Multiplexing as well and requires careful filtration to eliminate. See Section 8.5 for a measurement of NEP without heater lines.

7.3.5 Summary

μmux09a demonstrated microwave readout of a complex rf SQUID with high input mutual inductance. The strong input coupling of this SQUID produced low input current noise. We used this device to read out a CMB TES (Figure 7.10), but could not prove non-degrading readout at low frequencies because of pickup on the heater lines.

Chapter 8

μ mux10b: Multiplexer for CMB TESs

8.1 Design

μ mux10b was our first attempt to make a practical Microwave SQUID Multiplexer for arrays of TESs, specifically targeting the NIST CMB polarimeters. We packed the resonances tightly enough to fit 600 in an octave of bandwidth between 4 GHz and 8 GHz. We optimized the design to improve yield and enable flux-ramp modulation. We used a 20 mm \times 3 mm chip that matches the NIST time-division multiplexer with an identical layout of bond pads that allows us to easily couple to any detector chip that was designed for readout with the NIST time-division multiplexer.

8.1.1 Resonator Design

We wanted the μ mux10b resonances to fit in a 275 MHz band to match the open-source electronics described in Section 4.1. We chose to fill this band with 35 resonances, 33 to leverage the similarity with the NIST 1×33 time-division SQUID multiplexer and two for diagnostics. The number of resonances and the bandwidth indicate a spacing of roughly 6 MHz, which would allow more than 600 in the 4-8 GHz band. To avoid cross-coupling between resonances, the resonator bandwidth must be no more than 600 kHz (Section 2.3.9), so we chose to use the weak inductive coupler design, spacing the resonances by twenty times their bandwidth, with $BW \approx 300$ kHz and $Q_c \approx 16,000$.

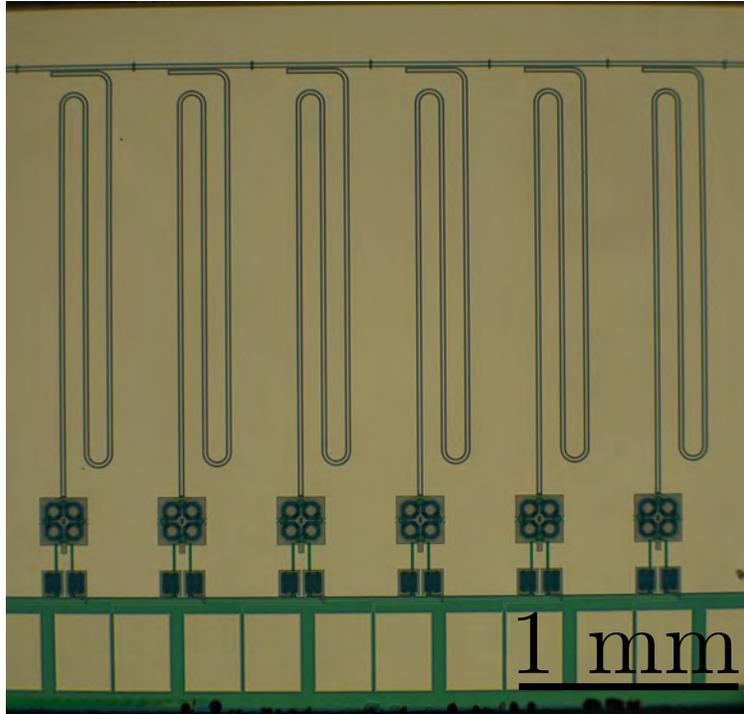


Figure 8.1: , each slightly different in length.

To fit the roughly 5 mm long resonators on the 3 mm wide chip we routed the resonators like trombones (Figure 8.1) changing the slider length by $3 \mu\text{m}$ between adjacent resonators so that $\Delta l = 6 \mu\text{m}$ and the frequency spacing is close to 6 MHz. We also reverted to CPWs with $10 \mu\text{m}$ center strips and $6 \mu\text{m}$ gaps.

8.1.2 Resonator-Feedline Coupling

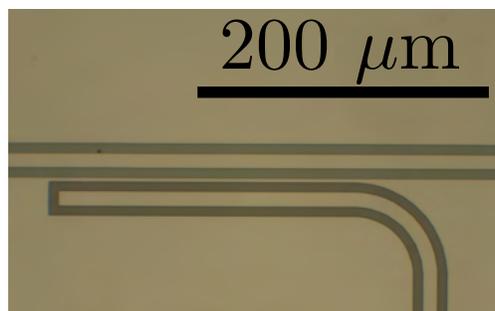


Figure 8.2: Photo of the elbow coupler in μmux10b .

μmux10b used an elbow coupler (Figure 8.2) designed for $Q_c \approx 16,000$. The elbow coupler simply brings the resonator close to the feedline for an extended length, with a narrow strip of ground plane between them.

This coupler is not compact, but allowed us to quickly fabricate and test a single-layer design (μmux10a). Measurements of μmux10a showed that low inductance connections between ground planes is critical for reliable resonator placement and Q 's.

8.1.3 Coil Geometry

We optimized the SQUID design and tuned the weak-inductive coupling to match the resonance frequency shift to the 300 kHz bandwidth (Figure 8.3).

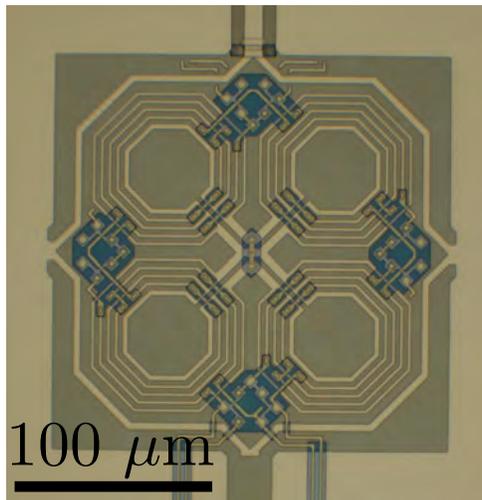


Figure 8.3: . (μmux10b)

We changed the design to comply with conservative design rules, particularly with regard to vias, to improve fabrication yield. We also reduced the amount of insulator in the SQUID. This dielectric can be a source of loss and two-level system noise, although the effect is weak at the current anti-node of the resonator.

8.1.4 Filter Design

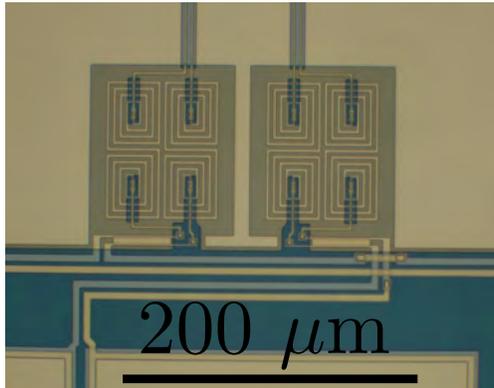


Figure 8.4: that block microwave power from reaching the input circuits.

We adapted the input filters slightly for the new SQUID design (Figure 8.4). Loss in the filters places a limit of $Q_i < 5 \times 10^6$. We also moved these filters farther away from the SQUID to make room for a low impedance connection between the ground planes on either side of the resonator.

8.2 Results

Measurements of μmux10b demonstrated accurate placement of resonators tightly spaced in frequency. We demonstrated linear readout of the SQUIDs with flux-ramp modulation. We demonstrated two-pixel multiplexing of SQUIDs and TESs. Finally we demonstrated measurement of TES devices for CMB polarimetry without degradation of the noise-equivalent power.

8.2.1 Resonance Spacing

We surveyed S_{21} of a μmux10b chip across a 250 MHz band (Figure 8.5). Almost all resonances yielded, with only one shifting from its design frequency. The remaining resonances were spaced by roughly 6 MHz.

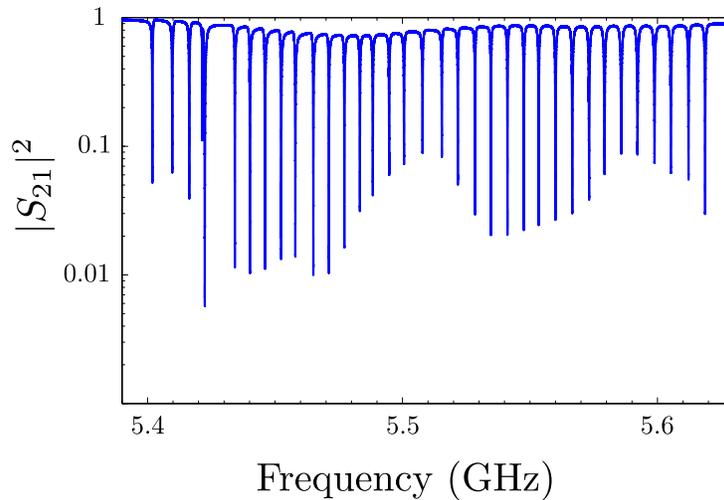


Figure 8.5: . The 5.42 GHz dip is the collision of two resonances.

The μmux10b couplers targeted $Q_c \approx 16,000$, but measured Q_c and Q_i varied significantly across the chip (Figure 8.6), although less than in previous designs. Time-domain reflectometry measurements showed significant reflection at the microwave launch. Reflections at the microwave launches could produce a standing wave on the feedline and change Q_c by changing the effective impedance the feedline presents to the coupling capacitor. The variation in Q is small enough that all resonances are useful for readout.

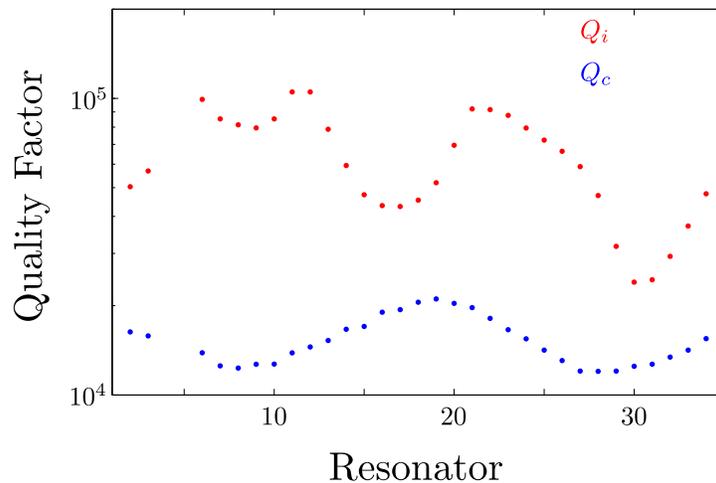


Figure 8.6: Internal and coupling quality factors for the resonances on a μmux10b chip.

Variation in Q_i is less important so long as Q_i is large enough to not limit the bandwidth or dissipate excessive power at the cold stage. We do not yet understand why the Q_i of a chip that has been through the full SQUID fabrication process is lower than that of a chip with bare niobium resonators.

8.2.2 Flux-variable Resonance Frequency

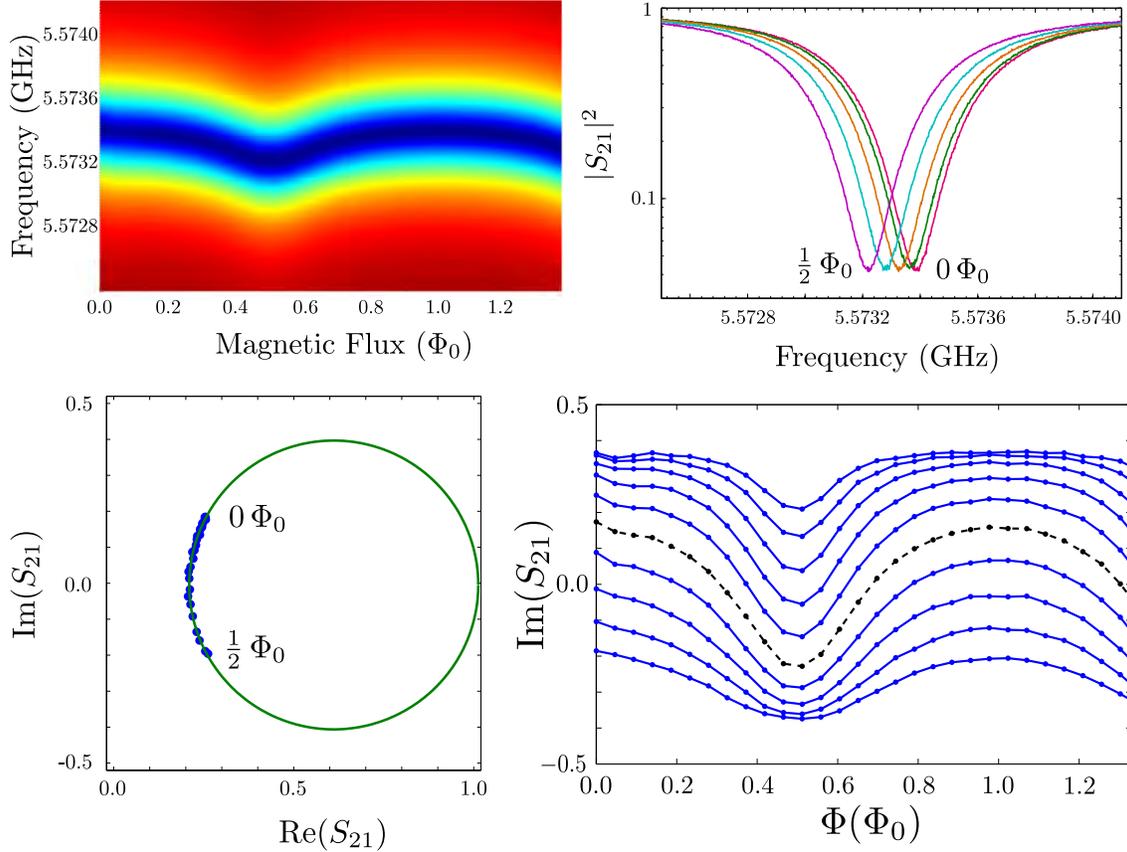


Figure 8.7: . Clockwise from the color map: $|S_{21}|$ in color (blue is no transmission, red is unity transmission) as a function of both frequency and magnetic flux, Lorentzian dips in transmitted power at different flux biases, $\text{Im}[S_{21}]$ as a function of flux at different excitation frequencies, S_{21} in the complex plane at several excitation frequencies as a function of flux bias.

This SQUID/resonator pair from μmux10b is slightly undercoupled (Figure 8.7) and at the appropriate measurement frequency its response in the imaginary component of S_{21} is approximately sinusoidal with flux,

with the deviation matching $\lambda \approx 1/3$. Current on the flux-ramp line shifts all resonances together and current on the individual input lines shifts individual resonances (Figure 8.8).

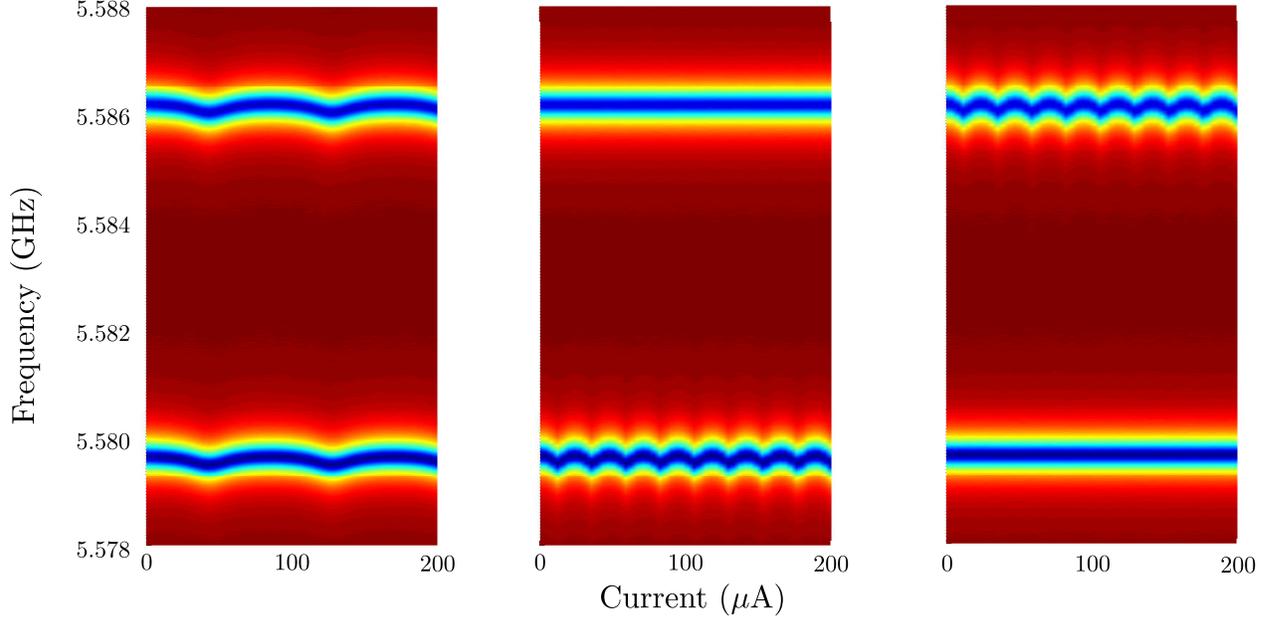


Figure 8.8: through their common (left) or individual (center and right) flux bias lines. The color plots show $|S_{21}|$ in color (blue is low, red is high) as a function of both frequency and current

The periodicity of the μmux10b response implies coupling inductances of $M_{fr} \approx 24$ pH and $M_{in} \approx 88$ pH, which are quite close to the simulated values.

8.2.3 Flux Noise and Frequency Noise

The increase in Q and reversion to a narrower CPW resulted in the two-level system noise dominating the HEMT noise even at 100 kHz, as shown by the difference in noise spectrum of the two quadratures (Figure 8.9). HEMT noise should appear equally in both quadratures, but two-level system, being a resonance frequency noise, should appear only in the imaginary quadrature.

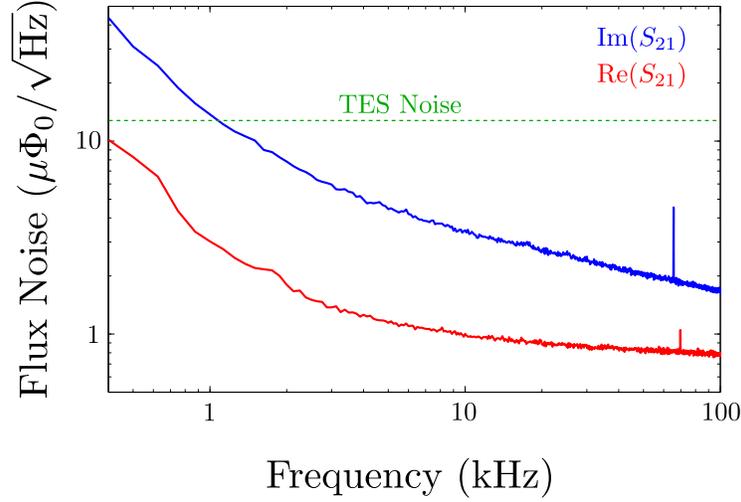


Figure 8.9: referred to flux noise. The $\text{Re}[S_{21}]$ noise has been scaled by the same factor as the $\text{Im}[S_{21}]$ noise, even though there is no SQUID response in the amplitude direction, to show what a reduction in two-level system noise can achieve. The dashed line shows a representative TES current noise of $300 \text{ pA}/\sqrt{\text{Hz}}$.

The noise curves in Figure 8.9 show output noise in both $\text{Re}[S_{21}]$ and $\text{Im}[S_{21}]$ referred through the gain in $\text{Im}[S_{21}]$. There is no gain in $\text{Re}[S_{21}]$ and so the noise in that real quadrature does not refer to actual flux noise, but rather indicates what the flux noise would be without two-level systems, roughly $0.8 \mu\Phi_0/\sqrt{\text{Hz}}$ at 100 kHz. The noise of the imaginary quadrature is the actual open-loop flux noise, including two-level systems, which falls to roughly $1.6 \mu\Phi_0/\sqrt{\text{Hz}}$ at 100 kHz. These noise levels show that a roughly factor of two reduction in two-level system noise should restore HEMT-limited operation. Without flux-ramp modulation the added noise of the Microwave SQUID Multiplexer would dominate the noise of a CMB TES below 1 kHz.

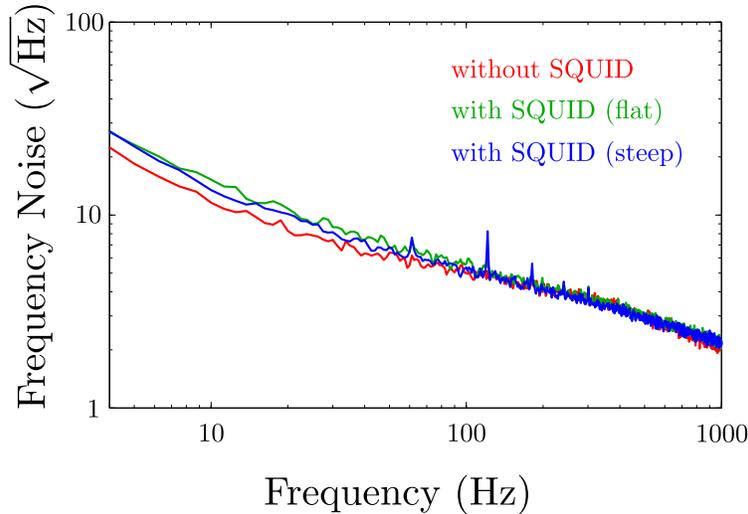


Figure 8.10: . The green and blue curves show noise for the same SQUID/resonator biased to be sensitive or insensitive to magnetic flux. The red curve shows noise for a resonator without a SQUID. The overlap of the green and blue curves shows that low-frequency flux noise in the SQUID is insignificant compared to TLS noise in the resonator. The other resonator could have a different density of two-level systems and was measured at a different internal power, so we expect a slight deviation.

Both fundamental noise mechanisms of an rf SQUID, flipping of magnetic dipoles on the loop[81] and critical current fluctuations in the junction[79][80], have $1/f$ frequency dependence. To investigate the contribution of these noise mechanisms we measured the frequency noise at different SQUID bias points. Output noise due to flipping magnetic dipoles should increase on the steep slopes of the SQUID response. Output noise due to fluctuations of the critical current should increase on the flat slopes of the SQUID response. The frequency noise at different bias points appears identical (Figure 8.10), indicating that the two-level-system noise of the resonator dominates.

8.2.4 Flux Ramp Modulation

To circumvent the low-frequency noise in the resonator we modulate the SQUID signal up to higher frequencies, ideally to where the two-level system noise falls below the HEMT noise.

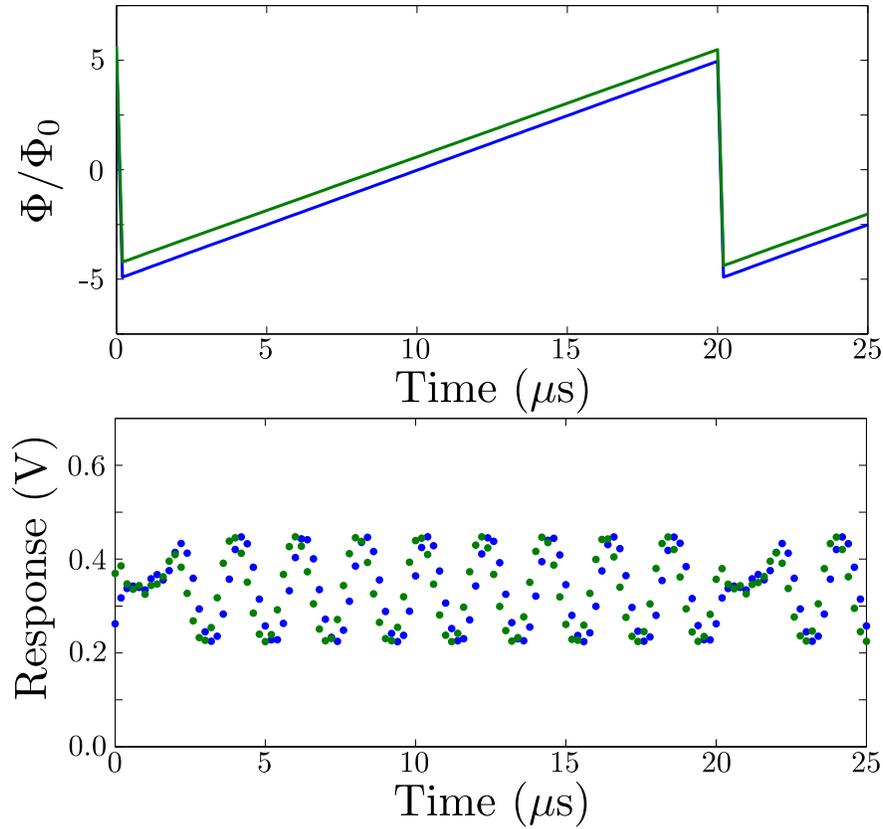


Figure 8.11: . An offset of the flux ramp causes a phase shift of the SQUID response.

We drive the flux-ramp line with a sawtooth (Figure 8.11) that sweeps through multiple flux quanta in the SQUIDs. The slew rate of this ramp must exceed that of any input signal. Therefore any input signal looks like a flux offset during the duration of the ramp, which produces a phase shift in the SQUID response to the ramp. This phase-modulation applies to all SQUIDs on a chip because the flux-ramp line couples to all SQUIDs.

Low-frequency signals shift the phase of the SQUID response while low-frequency noise from I_c fluctuations vary the amplitude of the SQUID response and low-frequency noise from two-level systems in the resonator vary the offset. We can therefore reject these sources of noise by extracting the phase of the SQUID response for each ramp.

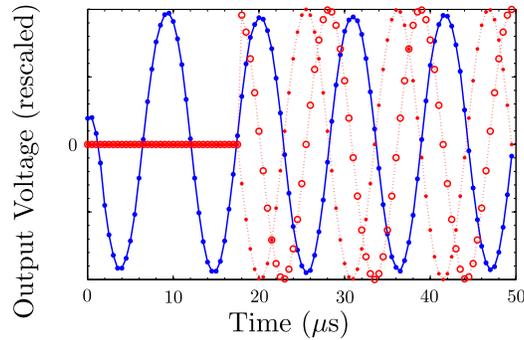


Figure 8.12: . The phase is the arctangent of the ratio of the data (blue) multiplied by both red curves. We discard part of the response to let the ramp reset transient settle.

We had previously used a Wiener optimal fitting algorithm to extract the phase of the ramp response, but have since settled on the simple Fourier solution: knowing the frequency of modulation, we multiply the SQUID response by that frequency sine and cosine (Figure 8.12) and extract the phase angle:

$$\phi = \arctan \left(\frac{\sum x_t \sin \omega_m t}{\sum x_t \cos \omega_m t} \right) \quad (8.1)$$

This solution is computationally efficient and requires no knowledge of the precise shape of the SQUID response. It discards the information in higher harmonics of the SQUID response, but for matched coupling the power in the higher harmonics is low and this has little impact on the readout noise.

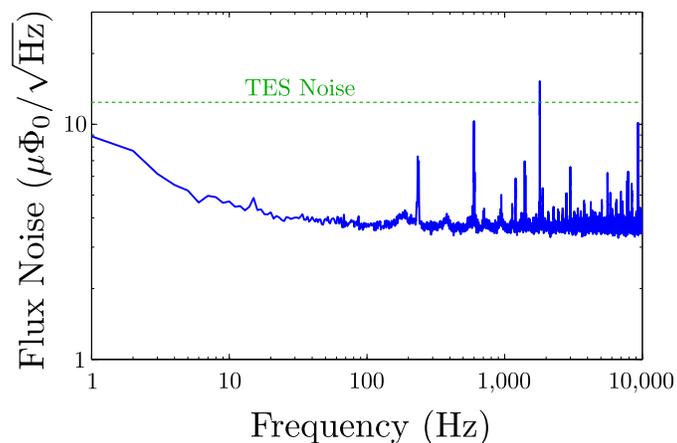


Figure 8.13: . The dashed line shows a representative TES current noise of $300 \text{ pA}/\sqrt{\text{Hz}}$.

For purely sinusoidal SQUID response, flux-ramp modulation imposes a $\sqrt{2}$ penalty (Equation 2.170) in flux noise compared with operation of the SQUID in a flux-locked loop because a substantial fraction of each ramp is spent measuring the SQUID at an extremum of its response curve where it is insensitive to changes in input flux. The noise further increases because we discard the ramp reset transient from the beginning of each ramp response and discard information in the higher harmonics. These factors combine to roughly double the flux noise compared to the open-loop noise at the modulation frequency (Figure 8.13). The noise after flux-ramp modulation/demodulation is flat down to roughly 10 Hz and less than the noise of a CMB TES.

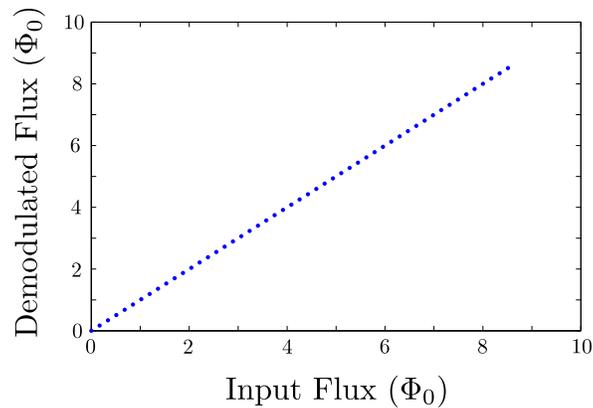


Figure 8.14: showing linear response.

An important benefit of flux-ramp modulation is the fact that it linearizes the response of all SQUIDs on a chip using a single twisted-pair. The phase shift of the ramp response is simply $\phi = 2\pi\Phi/\Phi_0$. To verify this we measured readout linearity with flux-ramp modulation (Figure 8.14).

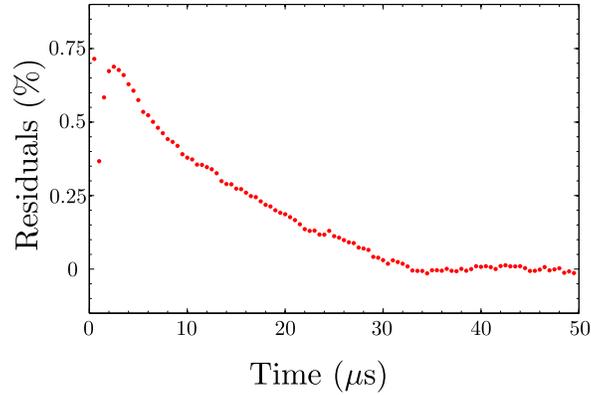


Figure 8.15: Difference between applied flux ramp and a perfectly linear ramp.

Since we do not compensate for the ramp reset transient, the flux ramp is not perfectly linear during its rise (Figure 8.15). This distortion affects the Fourier measurement differently at different phase shifts and therefore creates a readout non-linearity (Figure 8.16) that is periodic with flux in the SQUID. The current flux-ramp measurements are linear to roughly one part in 1,000, and should improve with better control of the flux ramp. This linearity is sufficient for most applications.

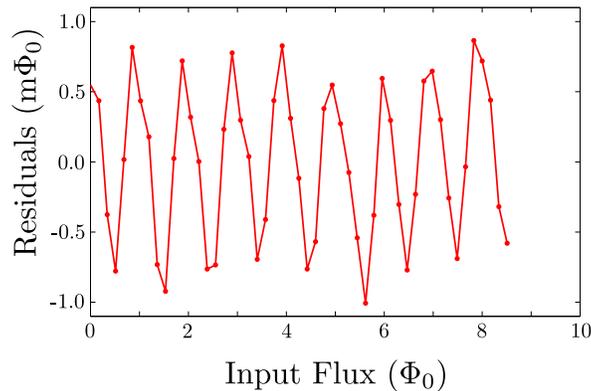


Figure 8.16: . The deviation from linearity is periodic with flux.

8.3 SQUID Multiplexing Demonstration

As a demonstration, we performed multiplexed readout of a pair of synthesized flux signals:

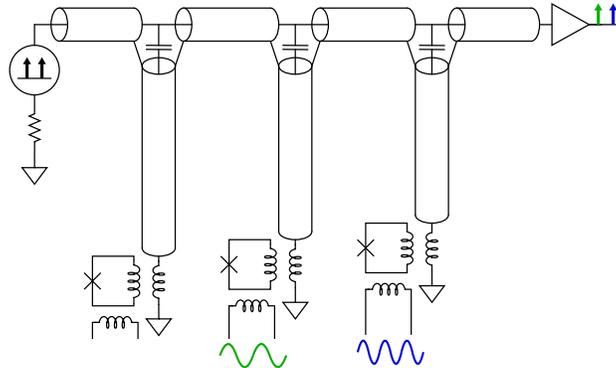


Figure 8.17: and performed multiplexed readout.

The signals from the two microwave resonators separated cleanly and reproduced the synthesized input signals (Figure 8.18). We detected no significant ac crosstalk between these synthesized flux signals.

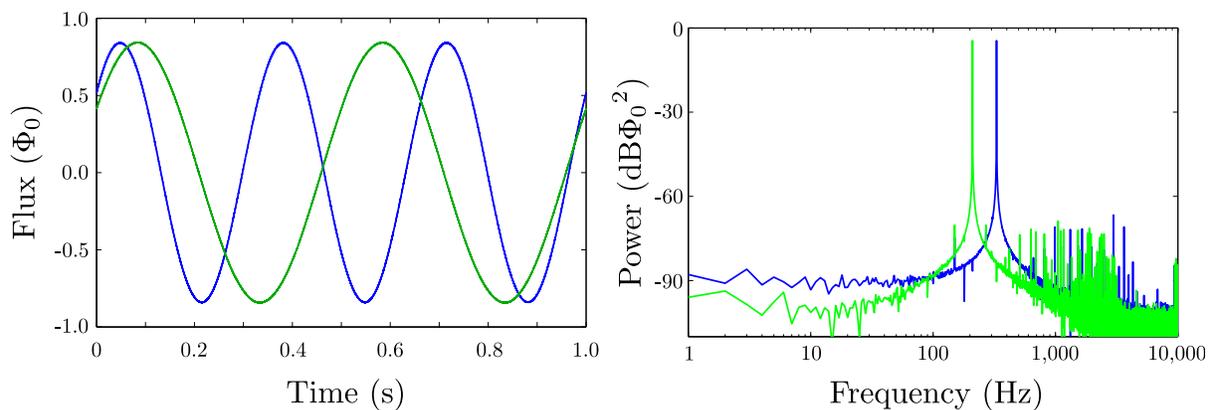


Figure 8.18: , at 2 Hz and 3 Hz (left). Power spectrum of two channels measuring synthesized signals at 210 Hz and 330 Hz (right).

A measurement of one SQUID with a slow sweep of flux in another indicates crosstalk at the part per 1,000 level (Figure 8.19). The periodicity of the crosstalk suggests that it occurs after the SQUID gain. It depends on the relative microwave power between the resonators and not on their separation in frequency, which suggest that it occurs in the HEMT or the IQ-mixer, for example through load on the HEMT bias circuit affecting the gain for all microwave signals. This level of crosstalk is sufficient for most applications.

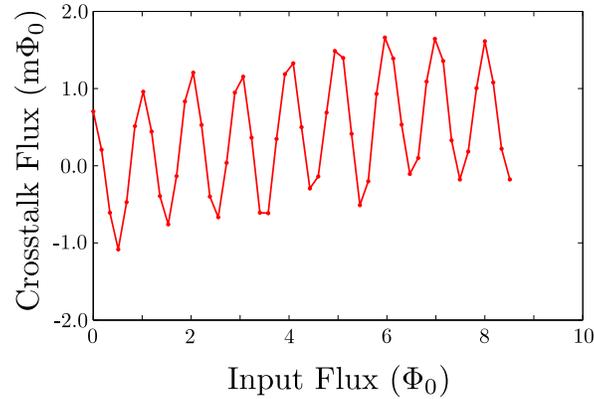


Figure 8.19: Crosstalk from one channel into the other. The periodicity of the crosstalk indicates that it occurs on the output side of the SQUID, e.g. through the HEMT amplifier.

8.4 TES Multiplexing

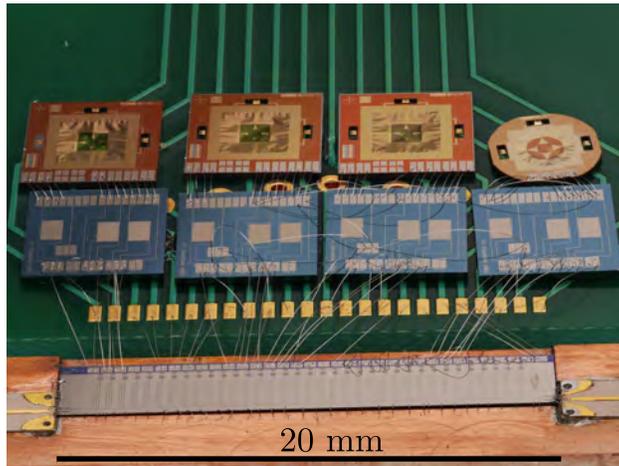


Figure 8.20: . In a real array the detector chips would be mounted on the multiplexer chip and use short wire bonds, or be integrated with the multiplexer chip.

We obtained several NIST CMB TESs (Figure 8.20) intended for future measurements of the polarization B-mode signature of the cosmic microwave background [72], wired them to a $\mu\text{mux}10\text{b}$ chip, and used them to demonstrate multiplexed TES readout.

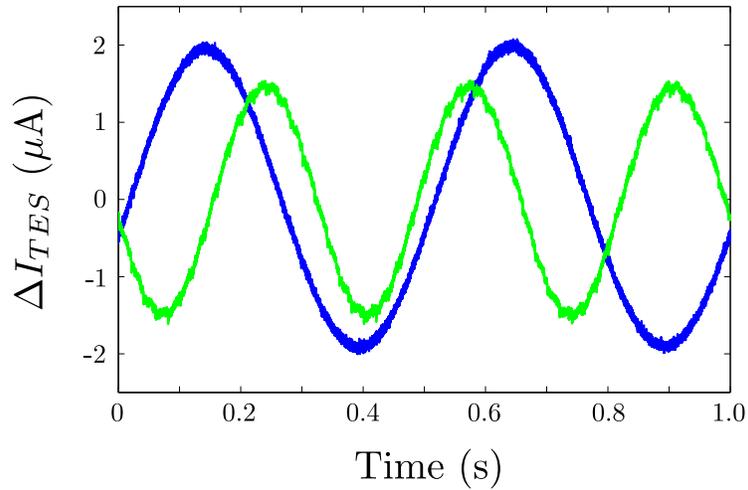


Figure 8.21: responding to sinusoidal heater signals. Pickup on the heater lines accounts for the noisy signal.

These devices have heaters on the TES membranes that we used to inject power into the TESs. We synthesized detector signals and performed multiplexed readout of the TESs. The signals separated cleanly (Figure 8.21), constituting the first full demonstration of microwave frequency multiplexing of TESs. Unfortunately, pickup on the heater lines made the readout appear noisy. Better filtration on the heater lines, or measurement of optical signals without heaters, should eliminate this effect in the future.

8.5 TES Readout

In order for this device to be considered for large scale TES arrays in the future, it is important to show that it does not degrade the TES sensitivity. We therefore removed the heater lines from the CMB pixels, biased them in their transition, and measured their noise-equivalent power using the Microwave SQUID Multiplexer (Figure 8.22).

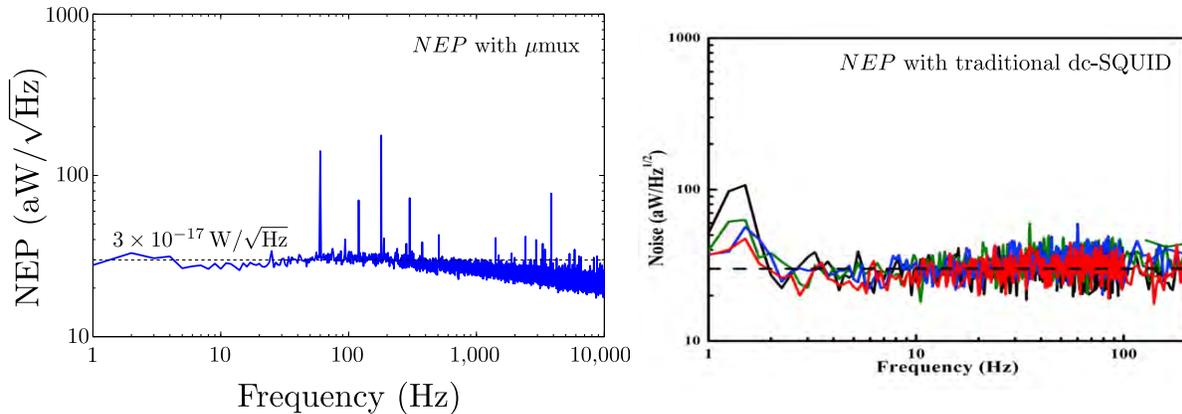


Figure 8.22: in its transition taken with the Microwave SQUID Multiplexer (left) and a traditional dc-SQUID (right).

We used full flux-ramp modulation to read out the TES current and referred it to incident power through the TES voltage bias. We measured NEP of $3 \times 10^{-17} \text{ W}/\sqrt{\text{Hz}}$, precisely matching previous measurements of the same TES using a dc-SQUID readout. The NEP is flat down to 1 Hz in contrast to open-loop measurements like in Figure 8.10. This measurement, performed with two coaxial cables and one twisted-pair for the flux ramp exactly as we would multiplex a large array, constitutes proof that the Microwave SQUID Multiplexer does not degrade the sensitivity of the detector.

8.5.1 Summary

$\mu\text{mux}10\text{b}$ demonstrated full flux-ramp modulation and demodulation, multiplexed SQUIDs with low-noise and low-crosstalk, and measured NIST TESs intended for polarimetry of the cosmic microwave background without any degradation of the noise-equivalent power. These results show that the Microwave SQUID Multiplexer is a real option for multiplexed readout of low-temperature detector arrays.

Chapter 9

Future Work

9.1 Multiplexer Re-design

We have already designed the next generation of the Microwave SQUID Multiplexer, incorporating the lessons of previous designs and attempting to improve input current noise to make the device suitable for TES detectors with higher operating resistance.

9.1.1 Surface State Reduction

We have seen evidence that the CF_4 sloped sidewall etch reduces Q_i , perhaps because of a suppression of superconductivity in the thinned edges of the niobium traces. Microwave currents flow mainly at the edges of the wires and some fraction of the resonator current could therefore pass through resistive niobium and dissipate power. We also worry about a possible increase in two-level systems on unprotected niobium surfaces during the insulator etch.

In the future, we intend to use a fabrication process that defines the resonator and most of the SQUID wiring in both layers of niobium, without insulator between them. This process hides the niobium surface that gets exposed to the insulator etch between the layers, where there is no electric field and therefore no sensitivity to two-level systems. The bottom surface of the resonator will be deposited on clean, HF-dipped silicon and the top surface will be protected by photoresist. This process will also restrict the CF_4 etch to the area immediately around crossovers and Josephson junctions. We expect this fabrication process to produce resonators with higher Q_i and lower two-level system noise, although we cannot predict the magnitude of improvement.

9.1.2 SQUID Design

The new SQUID design adds an extra loop inside each lobe of the gradiometer (Figure 9.1), reducing the self-inductance of the SQUID, and increases junction critical current to maintain $\lambda \approx 1/3$. These changes increase optimal microwave power on the feedline and reduce flux noise due to the HEMT. They also allow stronger input coupling: one full turn for the flux-ramp and two full turns for the input, which will further reduce input current noise.

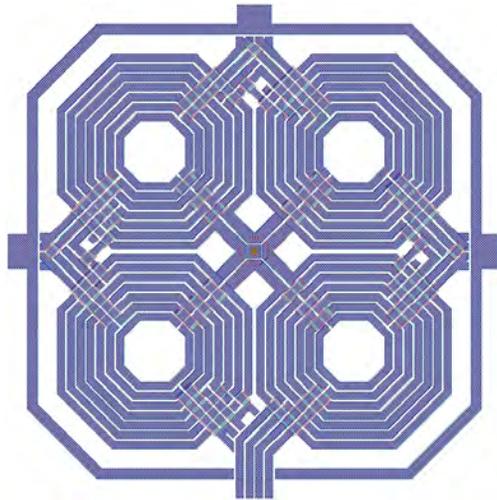


Figure 9.1: The layout for the rf SQUID in μmux11a .

9.1.3 Resonator Geometry

The new design widens the center strip of the coplanar waveguide to $s = 14 \mu\text{m}$ with $g = 8 \mu\text{m}$ gaps to reduce the two-level system noise[82]. The resonators will couple to the feedline with interdigitated capacitors for $Q_c \approx 10,000$ to further reduce the flux noise due to two-level systems. The SQUID-resonator coupling will increase to maintain the match between frequency shift and resonance bandwidth. We expect the combination of these changes with the process changes to make the flux noise HEMT-limited at 100 kHz.

Our goal is to make a multiplexer that adds only $30 \text{ pA}/\sqrt{\text{Hz}}$, capable of reading out many existing TES designs without degradation.

9.2 Microwave Launches

The variation of Q with position of the resonators on the chip suggests that there is a standing wave on the feedline, indicating reflections at the microwave launches. Time-domain reflectometry measurements support this theory. We intend to investigate the cause of these reflections and improve the microwave launch, for example by a flip-chip coupler, to eliminate them so that all input channels to the multiplexer require the same readout power and exhibit the same flux noise.

9.3 Room Temperature Electronics

The multiplexing demonstrations in this dissertation used two microwave signal generators to generate the tones for simultaneous readout of two pixels. In the future we plan to read out tens, hundreds, or thousands of pixels but we clearly cannot afford thousands of \$10,000 signal generators. Instead, we are collaborating with MKID groups and digital electronics engineers to create a digital solution: software defined radio (SDR) capable of digitally generating and demultiplexing hundreds of tones.



Figure 9.2: , designed and built by the Center for Astronomy Signal Processing and Electronics Research (CASPER).

We have received the first generation of open-source electronics for SDR from the collaboration (Figure 9.2). We intend to commission it and use it to demonstrate 33 pixel multiplexing of TESs. Although 33 pixels are not qualitatively different from 2 pixels, a successful demonstration should increase confidence in

the scalability of the Microwave SQUID Multiplexer.

9.4 Lumped-Element Resonators

Each resonator in μmux10b takes up a roughly $2.5 \text{ mm} \times 0.5 \text{ mm}$ footprint on the chip. We will need a more compact resonator design if we want to fit hundreds of resonators on a chip. A semi-lumped design, with a meander inductor and interdigitated capacitor can achieve the same resonance frequency using a $0.3 \text{ mm} \times 0.3 \text{ mm}$ area[90]. A true lumped-element design, with a spiral inductor and parallel-plate capacitor can achieve the same resonance frequency using even less area[91].

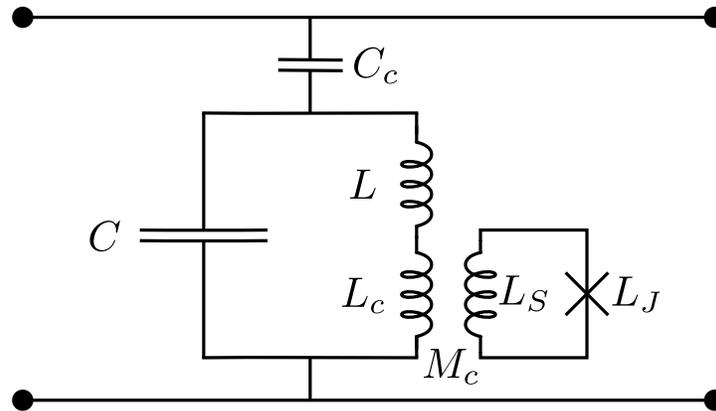


Figure 9.3: . Lumped-element resonators could significantly reduce the resonator footprint.

With multi-SQUID resonators (Section 9.5.1) or hybrid multiplexing techniques (Section 9.5.2), the footprint of the resonator may not be significant in future detector arrays.

9.5 Multiplexer Efficiency

Neither the Microwave SQUID Multiplexer nor the Microwave Kinetic Inductance Detector have high Shannon efficiency for the many applications that require very low bandwidth per pixel. The CMB polarimetry application we have been referring to requires only 100 Hz of bandwidth for each polarimeter, set primarily by the scan rate of the telescope on the sky. Efficient use of the microwave bandwidth would require a device with hundreds of thousands of resonators with $Q > 10^6$, spaced by roughly 10 kHz.

We cannot now fabricate devices with total Q that high and internal Q even higher. We cannot fabricate that many resonators on a chip. Most importantly, we cannot space the resonators that close together. We currently measure a roughly 1 MHz variation in resonance frequency placement, likely due to slight variations in over-etch, metal thickness, substrate thickness, etc. across the wafer. A similar scatter is observed with MKIDs. We must space the resonances at least 3 MHz apart to prevent a large number of collisions and excessive nearest-neighbor crosstalk (Section 2.3.9).

We would therefore use 3 MHz of bandwidth to measure a 100 Hz signal. We are pursuing more efficient use of the microwave bandwidth with multi-SQUID resonators and hybrid multiplexing schemes.

9.5.1 Multi-SQUID Resonators

One solution is to couple multiple SQUIDs to each microwave resonator and read out multiple input signals in its bandwidth. We can place multiple SQUIDs at the current anti-node and separate their signals by coupling them to the flux ramp with different mutual inductances (Figure 9.4), so that they modulate at different frequencies, e.g. 80 kHz, 100 kHz, and 120 kHz. Their input signals then appear in different sidebands of the microwave carrier (Figure 9.5).

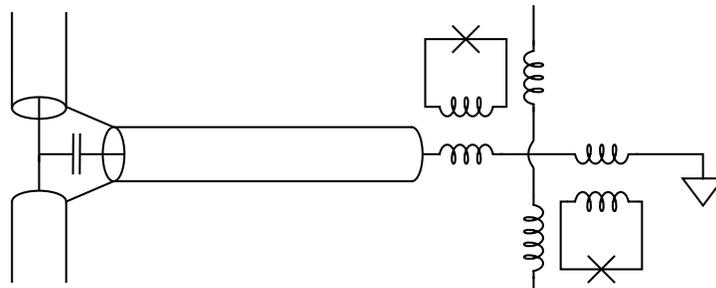


Figure 9.4: . Note that the flux-ramp line couples with different strength to each SQUID.

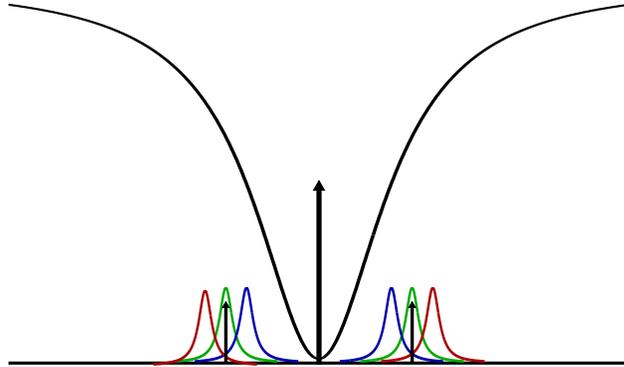


Figure 9.5: of the microwave carrier frequency.

The coupling between each rf SQUID and the resonator must be reduced by a factor of \sqrt{n} to maintain a match between total frequency shift and resonance bandwidth and prevent excessive coupling between the flux-ramp response of the different SQUIDs. The optimal readout power increases therefore by a factor of n and the voltage gain of each is reduced by \sqrt{n} . This strategy is therefore interesting but introduces a new mechanism for crosstalk and increases flux noise by \sqrt{n} .

9.5.2 Hybrid Multiplexing

Finally, let us consider hybrid multiplexing schemes. Hybrid multiplexing is a standard solution to reach higher Shannon efficiency in a variety of applications, for example 3G cell-phone communication[92].

Observe that the fundamental advantage of microwave multiplexing of SQUIDs is the output bandwidth of a coaxial cable, not the high Q that allows a large number of resonances per cable. Instead of multiplexing hundreds of thousands of narrow-band SQUIDs we can multiplex hundreds of wide-band SQUIDs. We can therefore use a Microwave SQUID Multiplexer to multiplex hundreds of high-bandwidth SQUIDs and low-frequency techniques to multiplex hundreds of low-bandwidth signals into each high-bandwidth SQUID.

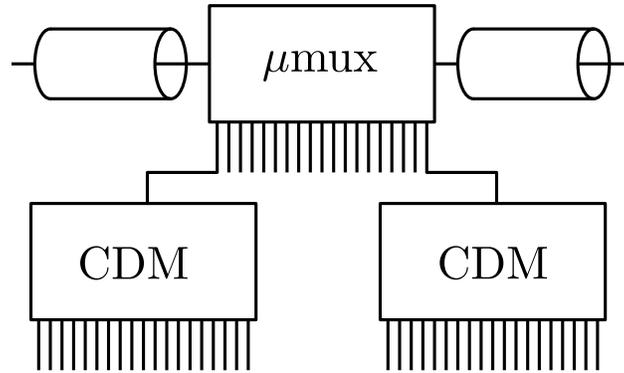


Figure 9.6: , e.g. a Code-Division Multiplexers.

Hybrid multiplexing of SQUIDs has been previously considered[93]. The complexity of combining two technologies is outweighed by the fact that we need not push the limits of either technology. The low-frequency multiplexing technology that seems to fit the Microwave SQUID Multiplexer best is a form of Code-Division Multiplexing with current-steering and binary addressing.

9.5.2.1 CDMA

This promising Code-Division Multiple Access (CDMA) scheme consists of SQUID switches steering TES current through inductors that couple with opposite polarity to the microwave SQUID. The SQUID switches allow rapid inversion of the flux signal from any TES. A Walsh code of inversions (Figure 9.7) provides a basis set that allows complete reconstruction of the input signals.

The code-division pre-multiplexer[53] can have very low power dissipation per pixel, on the order of the power dissipated in the TES bias circuit. It does not suffer the noise aliasing weakness of Time-Division Multiplexing, allowing larger multiplexing factors. Finally, because the switches are SQUIDs and therefore selected with flux modulo Φ_0 , it can use a binary addressing scheme to further reduce the wirecount.

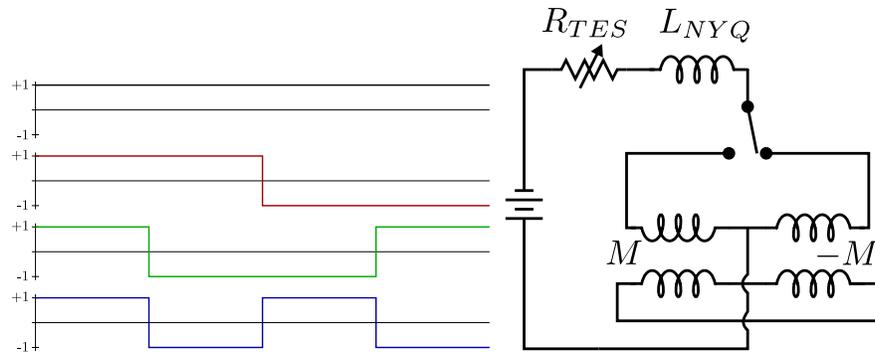


Figure 9.7: for CDM (left). Schematic of current-steering implementation of CDM (right).

A Microwave SQUID Multiplexer with 256 channels, each fed by a Code-Division Multiplexer with 256 channels can read out 65536 pixels with high spectral efficiency. This type of hybrid multiplexer could read out a megapixel array using only 40 coaxial cables and 24 twisted-pairs. We intend to pursue this technology to provide a real megapixel multiplexing solution in the next ten years.

Chapter 10

Conclusion

We developed a microwave frequency multiplexer of dissipationless rf SQUIDs for the readout of large-scale arrays of low-temperature detectors. This multiplexer needed to have scalability to large numbers of input channels, input current noise low enough to read out TESs without degradation, and low per-pixel power dissipation.

Using standard lithographic techniques we fabricated multiple superconducting microwave resonators on each multiplexer chip. These resonators had distinct resonance frequencies, similar bandwidths, and sufficiently low loss to limit the heat load to less than 5 pW per pixel.

Various designs exhibited flux noise as low as $0.17 \mu\Phi_0/\sqrt{\text{Hz}}$, input current noise as low as $20 \text{ pA}/\sqrt{\text{Hz}}$, and readout of a CMB TES pixel without degrading the noise-equivalent power of $3 \times 10^{-17} \text{ W}/\sqrt{\text{Hz}}$. Flux-ramp modulation avoided the low-frequency two-level system noise of the resonators and linearized the response of all SQUIDs using only a single twisted-pair for the entire multiplexer. Flux noise after modulation/demodulation remained flat to low frequencies.

Finally, we performed multiplexed readout of two SQUIDs driven by synthesized current signals and actual TES currents. The multiplexed signals separated cleanly with crosstalk of a part per 1,000.

These results indicate that the Microwave SQUID Multiplexer is suitable to read out arrays of thousands of low-temperature detectors. We are now working on hybrid multiplexing schemes to increase Shannon efficiency beyond what can be achieved with superconducting microwave resonators alone. This will enable the readout of tens of thousands of detectors using two coaxial cables and a handful of dc wires, providing the multiplexing factors necessary for the megapixel arrays of the future.

Bibliography

- [1] C. Enss (2005).
- [2] J. Bottomley and F. King. Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character **208**, 349 (1908).
- [3] R. Jones. JOSA **37**, 879 (1947).
- [4] D. H. Andrews, W. F. Brucksch, and W. T. Ziegler. Physical Review **59**, 1045 (1941).
- [5] D. H. Andrews, R. M. Milton, and W. Desorbo. Journal of the Optical Society of America **36**, 518 (1946).
- [6] W. Boyle, J. Rodgers, and Others. JOSA **49**, 66 (1959).
- [7] F. J. Low. Journal of the Optical Society of America **51**, 1300 (1961).
- [8] K. D. Irwin. Phonon-Mediated Particle Detection Using Superconducting Tungsten Transition-Edge Sensors. Ph.D. thesis (1995).
- [9] S. Bandler. SPIE Newsroom 2-4 (2010).
- [10] C. Enss, A. Fleischmann, K. Horst, J. Schonefeld, J. Sollner, J. S. Adams, Y. H. Huang, Y. H. Kim, and G. M. Seidel. Journal of Low Temperature Physics **121**, 137 (2000).
- [11] P. Day, H. LeDuc, B. Mazin, A. Vayonakis, and J. Zmuidzinas. Nature **425**, 817 (2003).
- [12] B. Mazin, P. Day, J. Zmuidzinas, and H. LeDuc. In AIP Conference Proceedings, 309-312. Citeseer (2002).
- [13] S. P. Langley. Proceedings of the American Academy of Arts and Sciences **14**, 106 (1878).
- [14] C. L. Hunt, J. J. Bock, P. K. Day, A. Goldin, A. E. Lange, H. G. LeDuc, A. Vayonakis, and J. Zmuidzinas. In Proceedings of SPIE, volume 4855, 318 (2003).
- [15] K. Irwin. Applied Physics Letters **66**, 1998 (1995).
- [16] K. D. Irwin, G. C. Hilton, D. a. Wollman, and J. M. Martinis. Applied Physics Letters **69**, 1945 (1996).
- [17] D. Schwan, F. Bertoldi, S. Cho, M. Dobbs, R. Guesten, N. W. Halverson, W. L. Holzapfel, E. Kreysa, T. M. Lanting, A. T. Lee, and Others. New Astronomy Reviews **47**, 933 (2003).
- [18] A. Kosowsky. New Astronomy Reviews **47**, 939 (2003).
- [19] D. Becker, J. Beall, H. M. Cho, W. Duncan, G. Hilton, R. Horansky, K. Irwin, P. Lowell, M. Niemack, N. Paulter, and Others (2010).

- [20] J. J. Bock, M. Kenyon, K. Irwin, M. Bradford, J. P. L. Caltech, M. Devlin, U. Penn, A. Lange, C. Jpl, A. Lee, U. C. Berkeley, and B. Mason. Jet Propulsion 1–11.
- [21] W. S. Holland, W. Duncan, B. D. Kelly, K. D. Irwin, A. J. Walton, P. A. R. Ade, and E. I. Robson. In Proceedings of SPIE, volume 4855, 1 (2003).
- [22] D. Morozov, P. D. Mauskopf, P. Ade, M. Bruijn, P. A. J. de Korte, H. Hoevers, M. Ridder, P. Khosropanah, B. Dirks, and J. R. Gao. In AIP Conference Proceedings, volume 1185, 48 (2009).
- [23] B. Cabrera, R. M. Clarke, P. Colling, A. J. Miller, S. Nam, and R. W. Romani. Applied Physics Letters **73**, 735 (1998).
- [24] N. E. White and H. Tananbaum. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment **436**, 201 (1999).
- [25] D. A. Wollman, K. D. Irwin, G. C. Hilton, L. L. Dulcie, D. E. Newbury, and J. M. Martinis. Journal of Microscopy **188**, 196 (1997).
- [26] W. B. Doriese, J. N. Ullom, J. a. Beall, W. D. Duncan, L. Ferreira, G. C. Hilton, R. D. Horansky, K. D. Irwin, J. a. B. Mates, C. D. Reintsema, L. R. Vale, Y. Xu, B. L. Zink, M. W. Rabin, a. S. Hoover, C. R. Rudy, and D. T. Vo. Applied Physics Letters **90**, 193508 (2007).
- [27] R. D. Horansky, J. N. Ullom, J. A. Beall, G. C. Hilton, K. D. Irwin, D. E. Dry, E. P. Hastings, S. P. Lamont, C. R. Rudy, and M. W. Rabin. Applied Physics Letters **93**, 123504 (2008).
- [28] J. C. Mather. Applied Optics **21**, 1125 (1982).
- [29] A. T. Lee, P. L. Richards, S. W. Nam, B. Cabrera, and K. D. Irwin. Applied Physics Letters **69**, 1801 (1996).
- [30] B. D. Josephson. Physics Letters **1**, 251 (1962).
- [31] B. D. Josephson. Reviews of Modern Physics **46**, 251 (1974).
- [32] J. Clarke and A. I. Braginski. The SQUID handbook, volume 1. Wiley-Vch (2006). ISBN 3527402292.
- [33] R. C. Jaklevic, J. Lambe, A. H. Silver, and J. E. Mercereau. Physical Review Letters **12**, 159 (1964).
- [34] A. H. Silver and J. E. Zimmerman. Physical Review **157**, 317 (1967).
- [35] Y. Aharonov and D. Bohm. Physical Review **115**, 485 (1959).
- [36] M. Tinkham. Introduction to superconductivity. Dover Pubns (2004). ISBN 0486435032.
- [37] T. Van Duzer and C. W. Turner. Principles of superconductive devices and circuits. Prentice Hall (1999). ISBN 0132627426.
- [38] W. Seidel, G. Forster, W. Christen, F. von Feilitzsch, H. G\`obel, F. Pr\`obst, and R. L. M\`oß bauer. Physics Letters B **236**, 483 (1990).
- [39] J. Zmuidzinas. historical trends. personal communication.
- [40] C. Shannon. Proceedings of the IEEE **86**, 447 (1998).
- [41] J. a. Chervenak, K. D. Irwin, E. N. Grossman, J. M. Martinis, C. D. Reintsema, and M. E. Huber. Applied Physics Letters **74**, 4043 (1999).
- [42] J. Chervenak, E. Grossman, K. Irwin, J. Martinis, C. Reintsema, C. Allen, D. Bergman, S. Moseley, and R. Shafer. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment **444**, 107 (2000).

- [43] D. J. Benford, T. A. Ames, J. A. Chervenak, E. N. Grossman, K. D. Irwin, S. A. Khan, B. Maffei, S. H. Moseley, F. Pajot, T. G. Phillips, and Others. In AIP Conference proceedings, 589–594. IOP INSTITUTE OF PHYSICS PUBLISHING LTD (2002).
- [44] D. Benford, C. Allen, J. Chervenak, M. Freund, A. Kuttyrev, S. Moseley, R. Shafer, J. Staguhn, E. Grossman, G. Hilton, and Others. International Journal of Infrared and Millimeter Waves **21**, 1909 (2000).
- [45] A. L. Woodcraft, P. a. R. Ade, D. Bintley, J. S. House, C. L. Hunt, R. V. Sudiwala, W. B. Doriese, W. D. Duncan, G. C. Hilton, K. D. Irwin, C. D. Reintsema, J. N. Ullom, M. D. Audley, M. a. Ellis, W. S. Holland, M. MacIntosh, C. C. Dunare, W. Parkes, A. J. Walton, J. B. Kycia, M. Halpern, and E. Schulte. Review of Scientific Instruments **78**, 024502 (2007).
- [46] T. Lanting, H. Cho, J. Clarke, M. Dobbs, A. Lee, P. Richards, A. Smith, and H. Spieler. Applied Superconductivity, IEEE Transactions on **13**, 626 (2003).
- [47] M. F. Cunningham, J. N. Ullom, T. Miyazaki, S. E. Labov, J. Clarke, T. M. Lanting, A. T. Lee, P. L. Richards, J. Yoon, and H. Spieler. Applied Physics Letters **81**, 159 (2002).
- [48] J. Yoon, J. Clarke, J. M. Gildemeister, A. T. Lee, M. J. Myers, P. L. Richards, and J. T. Skidmore. Applied Physics Letters **78**, 371 (2001).
- [49] M. Dobbs, N. W. Halverson, P. A. R. Ade, K. Basu, A. Beelen, F. Bertoldi, C. Cohalan, H. M. Cho, R. G\usten, W. L. Holzappel, and Others. New Astronomy Review **50**, 960 (2006).
- [50] J. Ruhl, P. A. Ade, J. E. Carlstrom, H. M. Cho, T. Crawford, M. Dobbs, C. H. Greer, N. Halverson, W. L. Holzappel, T. M. Lanting, and Others. In Proceedings of SPIE (2004). ISBN 0819454303.
- [51] P. Oxley, P. A. Ade, C. Baccigalupi, P. DeBernardis, H. M. Cho, M. J. Devlin, S. Hanany, B. R. Johnson, T. Jones, A. T. Lee, and Others. In Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, volume 5543, 320–331 (2004).
- [52] K. D. Irwin, M. D. Niemack, J. Beyer, H. M. Cho, W. B. Doriese, G. C. Hilton, C. D. Reintsema, D. R. Schmidt, J. N. Ullom, and L. R. Vale. Superconductor Science and Technology **23**, 34004 (2010).
- [53] M. D. Niemack, J. Beyer, H. M. Cho, W. B. Doriese, G. C. Hilton, K. D. Irwin, C. D. Reintsema, D. R. Schmidt, J. N. Ullom, and L. R. Vale. Applied Physics Letters **96**, 163509 (2010).
- [54] C. E. Shannon. ACM SIGMOBILE Mobile Computing and Communications Review **5**, 3 (2001).
- [55] K. Irwin. In AIP Conference Proceedings, volume 1185, 229 (2009).
- [56] B. a. Mazin, B. Bumble, P. K. Day, M. E. Eckart, S. Golwala, J. Zmuidzinas, and F. A. Harrison. Applied Physics Letters **89**, 222507 (2006).
- [57] P. K. Day, H. G. Leduc, A. Goldin, T. Vayonakis, B. A. Mazin, S. Kumar, J. Gao, and J. Zmuidzinas. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment **559**, 561 (2006).
- [58] J. Gao. The physics of superconducting microwave resonators. Ph.D. thesis, California Institute of Technology (2008).
- [59] K. Duh, M. Pospieszalski, W. Kopp, P. Ho, a.a. Jabra, P.-C. Chao, P. Smith, L. Lester, J. Ballingall, and S. Weinreb. IEEE Transactions on Electron Devices **35**, 249 (1988).
- [60] P. R. Maloney, N. G. Czakon, P. K. Day, T. P. Downes, R. Duan, J. Gao, J. Glenn, S. R. Golwala, M. I. Hollister, H. G. LeDuc, and Others. In Proceedings of SPIE, volume 7741, 77410F (2010).
- [61] J. Schlaerth, A. Vayonakis, P. Day, J. Glenn, J. Gao, S. Golwala, S. Kumar, H. LeDuc, B. Mazin, J. Vaillancourt, and Others. J Low Temp Phys **151**, 684 (2008).

- [62] K. Irwin and K. Lehnert. Applied Physics Letters **85**, 2107 (2004).
- [63] J. a. B. Mates, G. C. Hilton, K. D. Irwin, L. R. Vale, and K. W. Lehnert. Applied Physics Letters **92**, 023514 (2008).
- [64] P. Day, B. Bumble, and H. G. Leduc. Journal of Low Temperature Physics **151**, 934 (2008).
- [65] S. Corvaja. Nature **458** (2009).
- [66] L. Knox and Y.-S. Song. Physical Review Letters **89**, 1 (2002).
- [67] S. Dodelson, R. Easther, S. Hanany, L. McAllister, S. Meyer, L. Page, P. Ade, A. Amblard, A. Ashoorioon, C. Baccigalupi, and Others. In AGB Stars and Related Phenomena2010: The Astronomy and Astrophysics Decadal Survey, volume 2010, 67 (2009).
- [68] T. Essinger-Hileman, J. W. Appel, J. A. Beal, H. M. Cho, J. Fowler, M. Halpern, M. Hasselfield, K. D. Irwin, T. A. Marriage, M. D. Niemack, and Others. In AIP Conference Proceedings, volume 1185, 494 (2009).
- [69] J. J. McMahon, K. A. Aird, B. A. Benson, L. E. Bleem, J. Britton, J. E. Carlstrom, C. L. Chang, H. S. Cho, T. de Haan, T. M. Crawford, and Others. In American Institute of Physics Conference Series, volume 1185, 511–514 (2009).
- [70] M. C. Runyan, P. A. R. Ade, R. S. Bhatia, J. J. Bock, M. D. Daub, J. H. Goldstein, C. V. Haynes, W. L. Holzapfel, C. L. Kuo, A. E. Lange, and Others. The Astrophysical Journal Supplement Series **149**, 265 (2003).
- [71] J. Britton, K. Yoon, J. Beall, D. Becker, H. Cho, G. Hilton, M. Niemack, and K. Irwin. In American Institute of Physics Conference Series, B. Young, B. Cabrera, & A. Miller, ed., American Institute of Physics Conference Series 1185, 375378 (Dec, 375–378 (2009)).
- [72] K. W. Yoon, J. W. Appel, J. E. Austermann, J. A. Beall, D. Becker, B. A. Benson, L. E. Bleem, J. Britton, C. L. Chang, J. E. Carlstrom, and Others. In AIP Conference Proceedings, volume 1185, 515 (2009).
- [73] P. K. Hansma. Journal of Applied Physics **44**, 130 (1973).
- [74] K. K. Likharev. Dynamics of Josephson Junctions and Circuits. Gordon and Breach (1986). ISBN 2-88124-042-9.
- [75] D. M. Pozar. Microwave Engineering, 3rd (2005).
- [76] I. Siddiqi, R. Vijay, F. Pierre, C. M. Wilson, L. Frunzio, M. Metcalfe, C. Rigetti, R. J. Schoelkopf, M. H. Devoret, D. Vion, and Others. Physical review letters **94**, 27005 (2005).
- [77] A. Vayonakis and J. Zmuidzinis. Radiative losses from 2-d apertures.
- [78] J. Gao, M. Daal, A. Vayonakis, S. Kumar, J. Zmuidzinis, B. Sadoulet, B. A. Mazin, P. K. Day, and H. G. Leduc. Applied Physics Letters **92**, 152505 (2008).
- [79] F. C. Wellstood, C. Urbina, and J. Clarke. APPLIED PHYSICS LETTERS **85**, 22 (2004).
- [80] F. C. Wellstood, C. Urbina, and J. Clarke. Applied Physics Letters **50**, 772 (2009).
- [81] R. H. Koch, D. P. DiVincenzo, and J. Clarke. Physical review letters **98**, 267003 (2007).
- [82] J. Gao, J. Zmuidzinis, B. A. Mazin, H. G. LeDuc, and P. K. Day. Applied Physics Letters **90**, 102507 (2009).
- [83] H. G. Leduc, B. Bumble, P. K. Day, A. D. Turner, B. H. Eom, S. Golwala, D. C. Moore, O. Noroozian, J. Zmuidzinis, J. Gao, and Others .

- [84] B. A. Mazin, P. K. Day, K. D. Irwin, C. D. Reintsema, and J. Zmuidzinas. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment **559**, 799 (2006).
- [85] B. A. Mazin, J. B. Mates, and K. D. Irwin .
- [86] AWR. TX-Line.
- [87] J. Krupka, J. Breeze, A. Centeno, N. Alford, T. Claussen, and L. Jensen. Microwave Theory and Techniques, IEEE Transactions on **54**, 3995 (2006).
- [88] M. J. Hatridge. SQUID magnetometry from nanometer to centimeter length scales. Ph.D. thesis, Ernest Orlando Lawrence Berkeley National Laboratory, Berkeley, CA (US) (2010).
- [89] M. Kamon, M. J. Tsuk, and J. K. White. Microwave Theory and Techniques, IEEE Transactions on **42**, 1750 (2002).
- [90] S. Doyle, P. Mauskopf, J. Naylor, A. Porch, and C. Duncombe. Journal of Low Temperature Physics **151**, 530 (2008).
- [91] S. Weber, K. W. Murch, D. H. Slichter, R. Vijay, and I. Siddiqi. Arxiv preprint arXiv:1102.2917 (2011).
- [92] C. Huang. In Vehicular Technology Conference, 1999 IEEE 49th, volume 1, 342-345. IEEE (2002). ISBN 0780355652.
- [93] C. D. Reintsema, J. Beall, W. Doriese, W. Duncan, L. Ferreira, G. C. Hilton, K. D. Irwin, D. Schmidt, J. Ullom, L. Vale, and Others. J Low Temp Phys **151**, 927 (2008).
- [94] B. A. Mazin. Microwave kinetic inductance detectors (2005). ISBN 0496944924.
- [95] S. Kumar, J. Gao, J. Zmuidzinas, B. A. Mazin, H. G. LeDuc, and P. K. Day. Applied Physics Letters **92**, 123503 (2008).
- [96] J. D. Jackson. Classical Electrodynamics. John Wiley and Sons, Inc. (1999).
- [97] J. E. Zimmerman and A. H. Silver. Physical Review **141**, 367 (1966).
- [98] C. M. Caves. Physical Review D **26**, 1817 (1982).

Appendix A

Instruments and Components

The primary instruments used in this work were:

- Agilent E5071B Network Analyzer
- Agilent E4407B Spectrum Analyzer
- Agilent E8247C Signal Generator
- Anritsu 68369A/NV Signal Generator
- National Instruments PCI-6132 Digitizer
- HP 3562A Dynamic Signal Analyzer

The primary microwave components were:

- HEMT 0.5-11 GHz (Sander Weinreb, Caltech)
- Miteq AFS4-00101000-35-10P-4 Amplifier
- Marki Microwave IQ0307LXP IQ-Mixer
- Pamtech CTH1392K4 Circulator

We used coaxial cables from Coax-Co to minimize the heat load on our ADR.

- SC-086/50-SCN-CN
- SC-160/50-NbTi-NbTi

Appendix B

Transformer Coupling Calculations

Screening currents (Figure B.1) flow in the rf SQUID loop and change the effective load on the resonator.

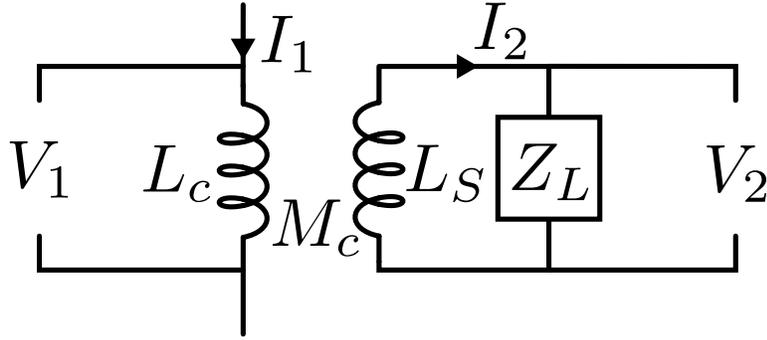


Figure B.1: Circuit diagram of an rf-SQUID screening an inductor.

To find the effective load Z_{eff} we solve a coupled set of linear equations:

$$V_1 = i\omega I_1 L_c - i\omega I_2 M_c \quad (\text{B.1})$$

$$V_2 = -i\omega I_2 L_S + i\omega I_1 M_c = I_2 Z_L \quad (\text{B.2})$$

We can write the screening current in terms of the primary current:

$$I_1 = I_2 \frac{i\omega L_S + Z_L}{i\omega M_c} \quad (\text{B.3})$$

and apply it back to the primary inductor to find the voltage drop:

$$V_1 = i\omega I_1 \left(L_c - \frac{M_c^2}{L_S + Z_L/i\omega} \right) \quad (\text{B.4})$$

which defines an effective impedance of

$$\boxed{Z_{eff} = i\omega \left(L_c - \frac{M_c^2}{L_S + Z_L/i\omega} \right)} \quad (\text{B.5})$$

B.1 Shunted Junction

Practical Josephson junctions have some capacitance C_J and leakage resistance R_{sg} which shunt the junction inductance (Figure B.2).

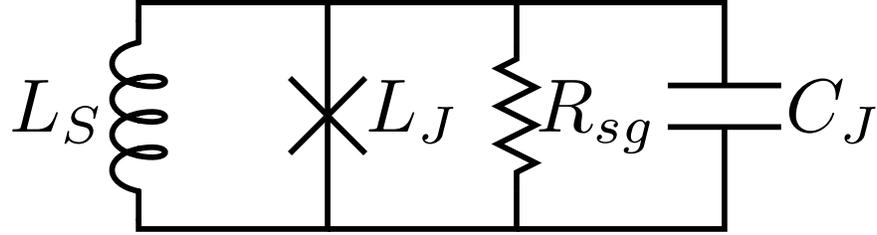


Figure B.2: SQUID circuit including leakage resistance and junction capacitance.

The effect of these shunts on the effective load impedance can be calculated exactly, but it is more elucidating to consider the perturbation theory. Let $y_s \equiv i\omega L_J Y_s$ be the admittance of the parallel shunts, normalized to the junction inductance.

$$Z_{eff} = i\omega \left(L_c - \frac{M_c^2}{L_S + \frac{L_J}{\cos \phi + y_s}} \right) \quad (\text{B.6})$$

We then consider y_s to be small:

$$Z_{eff} = i\omega \left(L_c - \frac{M_c^2}{L_S + L_J \sec \phi} \right) - i\omega \frac{M_c^2}{L_J} \left(\lambda + \frac{1}{\cos \phi} \right)^{-2} (\cos \phi)^{-2} y_s + \dots \quad (\text{B.7})$$

$$\approx i\omega \left(L_c - \frac{M_c^2}{L_S + L_J \sec \phi} \right) - i\omega \frac{M_c^2}{L_J} (1 + \lambda \cos \phi)^{-2} y_s \quad (\text{B.8})$$

$$\approx i\omega \left(L_c - \frac{M_c^2}{L_S + L_J \sec \phi} \right) + \frac{(\omega M_c)^2 Y_s}{(1 + \lambda \cos \phi)^2} \quad (\text{B.9})$$

Note that the denominator minimizes to $1 - \lambda$ so the greatest change in load impedance due to the junction shunts is

$$\boxed{\Delta Z_{eff} \approx \frac{(\omega M_c)^2 Y_s}{(1 + \lambda)^2}} \quad (\text{B.10})$$

B.2 Input Filter

The circuit that feeds a magnetic flux signal to the SQUID (Figure 2.17) may present an impedance with a real component. Microwave power in the resonator may therefore dissipate in the input circuit.

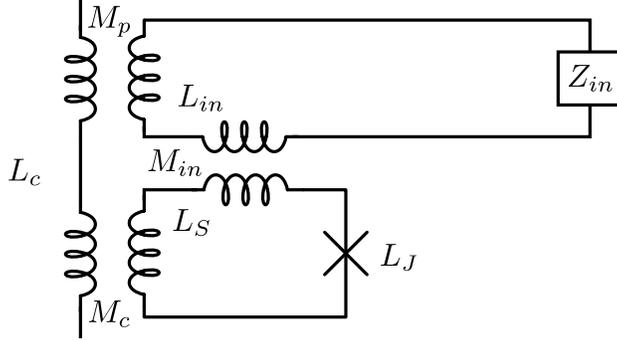


Figure B.3: Schematic of the rf-SQUID and input coil coupling to both the resonator and each other.

There are two ways for the microwave power to couple to the input circuit. First, there is an unavoidable parasitic coupling from the resonator termination into the input coil. Second, the resonator drives microwave currents in the SQUID loop which directly couple to the input coil. Maximum coupling occurs when $\phi = \pi$ and the SQUID effectively "anti-screens" flux from the resonator. Solving the coupled set of linear equations:

$$Z_L = i\omega \left(L_c - \frac{M_p^2 L_S (1 - \lambda^{-1}) + M_c^2 (L_{in} + Z_{in}/i\omega) - 2M_{in} M_c M_p}{L_S (1 - \lambda^{-1}) (L_{in} + Z_{in}/i\omega) - M_{in}^2} \right) \quad (\text{B.11})$$

$$= i\omega \left(L_c + \frac{M_c^2 (L_{in} + Z_{in}/i\omega) - 2L_S M_p^2 - 2M_{in} M_c M_p}{2L_S (L_{in} + Z_{in}/i\omega) + M_{in}^2} \right) \quad (\text{B.12})$$

Assume that the input circuit can be seen as a low-pass filter $Z_{in} = i\omega L_F + R_F$ with a cutoff frequency of $\omega_F = \frac{R_F}{L_F + L_{in}}$ far below the microwave resonance frequency. On resonance then:

$$\operatorname{Re}[Z_L] = -\omega_0 \operatorname{Im} \left[\frac{M_c^2(L_{in} + Z_{in}/i\omega) - 2L_S M_p^2 - 2M_{in} M_c M_p}{2L_S(L_{in} + Z_{in}/i\omega) + M_{in}^2} \right] \quad (\text{B.13})$$

$$= -\omega_0 \operatorname{Im} \left[\frac{M_c^2(L_{in} + L_F)(1 - i\frac{\omega_F}{\omega_0}) - 2L_S M_p^2 - 2M_{in} M_c M_p}{2L_S(L_{in} + L_F)(1 - i\frac{\omega_F}{\omega_0}) + M_{in}^2} \right] \quad (\text{B.14})$$

$$= \omega_F \frac{M_c^2(L_{in} + L_F)(2L_S(L_{in} + L_F) + M_{in}^2) - 2L_S(L_{in} + L_F)(M_c^2(L_{in} + L_F) - 2L_S M_p^2 - 2M_{in} M_c M_p)}{(2L_S(L_{in} + L_F) + M_{in}^2)^2 + O\left(\left(\frac{\omega_F}{\omega_0}\right)^2\right)} \quad (\text{B.15})$$

$$= R_F \frac{M_c^2(2L_S(L_{in} + L_F) + M_{in}^2) - 2L_S(M_c^2(L_{in} + L_F) - 2L_S M_p^2 - 2M_{in} M_c M_p)}{(2L_S(L_{in} + L_F) + M_{in}^2)^2 + O\left(\left(\frac{\omega_F}{\omega_0}\right)^2\right)} \quad (\text{B.16})$$

$$= R_F \frac{M_c^2 M_{in}^2 + 4L_S(L_S M_p^2 + M_{in} M_c M_p)}{(2L_S(L_{in} + L_F) + M_{in}^2)^2 + O\left(\left(\frac{\omega_F}{\omega_0}\right)^2\right)} \quad (\text{B.17})$$

$$\approx R_F \frac{M_c^2 M_{in}^2 + 4L_S(L_S M_p^2 + M_{in} M_c M_p)}{(2L_S(L_{in} + L_F) + M_{in}^2)^2} \quad (\text{B.18})$$