

Supplementary material: The Tidal-Thermal Evolution of the Pluto-Charon System

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ABSTRACT

Supplementary material: Tidal coefficients

This supplementary material summarizes all the tidal coefficients that are required in the development of the orbital theory as formulated in Appendix B of the main manuscript.

ORCID(s):

Table 1
 ϕ_j^{2i}

$j \setminus 2i$	ϕ_j^{2i}			
	0	2	4	6
-1	0	0	0	$\frac{1}{1536}$
1	0	$-\frac{3}{8}$	$\frac{3}{8}$	$-\frac{13}{135}$
2	-3	15	$-\frac{32}{189}$	$\frac{51^2}{135}$
3	0	$-\frac{441}{8}$	$\frac{7749}{8}$	$-\frac{12}{197775}$
4	0	0	$-\frac{32}{867}$	$\frac{512}{1955}$
5	0	0	0	$-\frac{3570125}{1536}$

$j \setminus 2i$	ϕ_j^{2i}					
	8	10	12	14	16	18
-7	0	0	0	0	0	$\frac{232630513987207}{469654673817600}$
-6	0	0	0	0	$\frac{4782969}{1594323}$	$\frac{20070400}{1619873046875}$
-5	0	0	0	$\frac{1220703125}{11098128384}$	$\frac{13427734375}{177570054144}$	$\frac{1619873046875}{14611478740992}$
-4	0	0	$\frac{32}{675}$	$\frac{32}{675}$	$\frac{92}{1575}$	$\frac{7808}{127575}$
-3	0	$\frac{59049}{3276800}$	$\frac{59049}{2621440}$	$\frac{38558997}{1468006400}$	$\frac{18482337}{671088640}$	$\frac{72655602021}{2630667468800}$
-2	$\frac{1}{192}$	$\frac{1}{619}$	$\frac{1}{64537}$	$\frac{1}{10193}$	$\frac{1}{127453}$	$\frac{1}{781813}$
-1	$\frac{12288}{113}$	$\frac{655360}{2639}$	$\frac{70778880}{129709}$	$\frac{39636172800}{73185503}$	$\frac{211392921600}{21004537637}$	$\frac{273965226393600}{5242619992367}$
1	$\frac{12288}{2881}$	$\frac{327680}{3481}$	$\frac{19660800}{12791}$	$\frac{13212057600}{288163}$	$\frac{4439251353600}{1190849}$	$\frac{1278504389836800}{535500317}$
2	$\frac{768}{1193715}$	$\frac{6400}{41889501}$	$\frac{92160}{237091671}$	$\frac{5644800}{1586248587}$	$\frac{24084480}{863843049}$	$\frac{12541132800}{220413818169}$
3	$\frac{4096}{83551}$	$\frac{327680}{131319}$	$\frac{6553600}{2459537}$	$\frac{209715200}{232033433}$	$\frac{939524096}{77502109}$	$\frac{751619276800}{1305096129}$
4	$\frac{24}{137418125}$	$\frac{40}{2936126875}$	$\frac{1280}{356500341875}$	$\frac{302400}{29029068521875}$	$\frac{345600}{79231092391625}$	$\frac{30481920}{5599532275549375}$
5	$\frac{12288}{2556801}$	$\frac{131072}{66328119}$	$\frac{14158776}{5938832817}$	$\frac{1585446912}{13527027819}$	$\frac{8455716864}{1497798429597}$	$\frac{1565515579392}{931614290435}$
6	$\frac{256}{0}$	$\frac{1280}{364996466863}$	$\frac{51200}{981779068235}$	$\frac{89600}{2921128205246293}$	$\frac{11468800}{2760195487312025}$	$\frac{11468800}{9940709866552998323}$
7	0	$\frac{9830400}{0}$	$\frac{4718592}{5383010161}$	$\frac{5662310400}{56914314263}$	$\frac{3623878656}{578658802849}$	$\frac{13045963161600}{77318558814617}$
8	0	0	$\frac{43200}{0}$	$\frac{75600}{3982050900860283}$	$\frac{282240}{82748209967119359}$	$\frac{22861440}{11230592195266666389}$
9	0	0	0	$\frac{10276044800}{0}$	$\frac{32883343360}{2010318936125}$	$\frac{1503238553600}{880939918004375}$
10	0	0	0	0	$\frac{1769472}{0}$	$\frac{111476736}{73137037043155488990059}$
11	0	0	0	0	0	$\frac{23013079017062400}{0}$

Table 2

$\hat{\phi}_j^{2i}$

		$\hat{\phi}_j^{2i}$			
$j \setminus 2i$	0	2	4	6	
1	0	$-\frac{9}{4}$	$-\frac{81}{16}$	$-\frac{2295}{63}$	
2	0	0	$-\frac{8}{81}$	$-\frac{4}{8427}$	
3	0	0	0	$-\frac{4}{256}$	

		$\hat{\phi}_j^{2i}$					
$j \setminus 2i$	8	10	12	14	16	18	
1	$-\frac{28403}{1661}$	$-\frac{3240741}{12027}$	$-\frac{87480539}{525521}$	$-\frac{76230921863}{13519469}$	$-\frac{143189681823}{729537581}$	$-\frac{3798168072196597}{56670971611}$	
2	$-\frac{64}{62487}$	$-\frac{320}{18059643}$	$-\frac{10240}{96458913}$	$-\frac{201600}{10219795947}$	$-\frac{8601600}{1455222699297}$	$-\frac{541900800}{28832512267869}$	
3	$-\frac{2048}{5929}$	$-\frac{327680}{9933}$	$-\frac{1310720}{708871}$	$-\frac{104857600}{1736021}$	$-\frac{11744051200}{27865091}$	$-\frac{187904819200}{2578148063}$	
4	$-\frac{64}{0}$	$-\frac{320}{15717645}$	$-\frac{6400}{14736585}$	$-\frac{14400}{67620980465}$	$-\frac{172032}{212874343975}$	$-\frac{12902400}{13595131130225}$	
5	0	$-\frac{65536}{0}$	$-\frac{262144}{30089667}$	$-\frac{264241152}{20778687}$	$-\frac{1409286144}{592612065}$	$-\frac{52613349376}{124700799}$	
6	0	0	$-\frac{51200}{0}$	$-\frac{44800}{1306918425967}$	$-\frac{802816}{9297259269991}$	$-\frac{20070400}{6999935835286093}$	
7	0	0	0	$-\frac{943718400}{0}$	$-\frac{5033164800}{31801943561}$	$-\frac{2899102924800}{1606626956771}$	
8	0	0	0	0	$-\frac{10035200}{0}$	$-\frac{270950400}{37292291277118641}$	
9	0	0	0	0	0	$-\frac{5261334937600}{0}$	

Supplementary material: Evolution of the Pluto-Charon Binary

Table 3
 λ_j^{2i-1}

$j \setminus i-1$	λ_j^{2i-1}			
	1	3	5	7
-2	0	0	0	$\frac{1}{192}$
-1	0	0	$\frac{1}{1024}$	$\frac{24576}{175}$
1	$\frac{3}{16}$	$-\frac{3}{64}$	$-\frac{1024}{35}$	$-\frac{24576}{3299}$
2	$\frac{4}{147}$	$-\frac{16}{3759}$	$-\frac{4}{150933}$	$-\frac{768}{1525953}$
3	$\frac{16}{16}$	$-\frac{64}{867}$	$-\frac{1024}{10421}$	$-\frac{8192}{310463}$
4	0	$-\frac{8}{714025}$	$-\frac{16}{1024}$	$-\frac{192}{105299675}$
5	0	0	0	$-\frac{24576}{852267}$
6	0	0	0	$\frac{256}{256}$

$j \setminus i-1$	λ_j^{2i-1}				
	9	11	13	15	17
-7	0	0	0	0	$\frac{33232930569601}{104367705292800}$
-6	0	0	0	$\frac{1594323}{4638671875}$	$-\frac{6908733}{80281600}$
-5	0	0	$\frac{244140625}{3170893824}$	$-\frac{355140108288}{4}$	$\frac{861572265625}{29222957481984}$
-4	0	$\frac{8}{225}$	$\frac{675}{17970579}$	$\frac{315}{193818501}$	$\frac{8505}{15029958483}$
-3	$\frac{19683}{1310720}$	$\frac{177147}{26214400}$	$\frac{2936012800}{3943}$	$\frac{46976204800}{2951}$	$\frac{5261334937600}{1904023}$
-2	$\frac{3840}{577}$	$\frac{230400}{12769}$	$\frac{2419200}{12959141}$	$\frac{2764800}{40473781}$	$\frac{2786918400}{9530091053}$
-1	$\frac{1310720}{5201}$	$\frac{47185920}{237091}$	$\frac{79272345600}{130286017}$	$\frac{422785843200}{7343600351}$	$\frac{182643484262400}{9052063323217}$
1	$\frac{655360}{1289}$	$\frac{39321600}{25999}$	$\frac{26424115200}{9569}$	$\frac{1775700541440}{250247}$	$\frac{2557008779673600}{818693}$
2	$\frac{1024}{85340127}$	$\frac{102400}{725981637}$	$\frac{921600}{6616127097}$	$\frac{15052800}{148062766923}$	$\frac{68812800}{151996417911}$
3	$\frac{655360}{1395551}$	$\frac{13107200}{430553}$	$\frac{419430400}{4725763337}$	$\frac{46976204800}{7265523173}$	$\frac{300647710720}{10430507833}$
4	$\frac{640}{2949543325}$	$\frac{240}{158224764775}$	$\frac{4838400}{50749964351125}$	$\frac{19353600}{25510094869225}$	$\frac{97542144}{37167304047148325}$
5	$\frac{262144}{109744167}$	$\frac{9437184}{6191465853}$	$\frac{3170893824}{71414520309}$	$\frac{2415919104}{124297728921}$	$\frac{7305739370496}{3840730800633}$
6	$\frac{5120}{52142352409}$	$\frac{102400}{21286018449073}$	$\frac{716800}{310385854107391}$	$\frac{1146880}{89963543887907011}$	$\frac{45875200}{15814983098946990889}$
7	$\frac{3932160}{0}$	$\frac{235929600}{5383010161}$	$\frac{11324620800}{1629711040201}$	$\frac{181193932800}{49681058130967}$	$\frac{26091926323200}{5306385354261451}$
8	0	$\frac{115200}{0}$	$\frac{4838400}{442450100095587}$	$\frac{45158400}{378432207128810421}$	$\frac{2438553600}{12113553017690673723}$
9	0	0	$\frac{2936012800}{0}$	$\frac{328833433600}{402063787225}$	$\frac{3006477107200}{1637474036163575}$
10	0	0	0	$\frac{884736}{0}$	$\frac{445906944}{664882154937771726369}$
11	0	0	0	0	$\frac{5114017559347200}{5114017559347200}$

Table 4

$\hat{\lambda}_j^{2i-1}$

$j \setminus 2i-1$	$\hat{\lambda}_j^{2i-1}$			
	1	3	5	7
1	$-\frac{9}{8}$	$-\frac{45}{31}$	$-\frac{999}{45}$	$-\frac{10043}{4096}$
2	0	$-\frac{16}{81}$	$-\frac{16}{45}$	$-\frac{128}{653}$
3	0	0	$-\frac{8427}{512}$	$-\frac{4929}{4096}$
4	0	0	0	$-\frac{5929}{128}$

$j \setminus 2i-1$	$\hat{\lambda}_j^{2i-1}$				
	9	11	13	15	17
1	968501	22665719	3488799931	2201362888069	705270944819797
2	$\frac{327680}{1861}$	$\frac{6553600}{140657}$	$\frac{880803840}{30772389}$	$\frac{493250150400}{458120711}$	$\frac{142056043315200}{1338763001}$
3	$-\frac{320}{8061723}$	$-\frac{70480}{24220341}$	$-\frac{6451200}{2503082907}$	$-\frac{51609600}{310605353233}$	$-\frac{135475200}{5548949079117}$
4	$\frac{655360}{154}$	$\frac{2621440}{510211}$	$\frac{209715200}{112849}$	$\frac{23488102400}{534407009}$	$\frac{375809638400}{244133119}$
5	$-\frac{15717645}{131072}$	$\frac{12800}{77607165}$	$-\frac{23040}{82475458145}$	$\frac{25804800}{443312655515}$	$-\frac{12902400}{16937466865475}$
6	0	$\frac{524288}{30089667}$	$\frac{528482304}{75371433}$	$\frac{8455716864}{24124153401}$	$\frac{315680096256}{7345300413}$
7	0	102400	$\frac{143360}{1306918425967}$	$\frac{40140800}{9760494525089}$	$\frac{20070400}{12355157174800909}$
8	0	0	1887436800	$\frac{6039797760}{31801945561}$	$\frac{5798205849600}{1232639743459}$
9	0	0	0	$\frac{20070400}{0}$	$\frac{270950400}{37292291277118641}$
					10522669875200

Table 5

ξ_j^{2i}

$j \setminus 2i$	ξ_j^{2i}				
	0	2	4	6	8
-3	0	0	0	0	$\frac{19683}{6553600}$
-2	0	0	0	0	$\frac{1}{7}$
-1	0	0	0	$\frac{1}{3072}$	$\frac{768}{11}$
1	0	$\frac{3}{15}$	$-\frac{3}{189}$	$\frac{1024}{155}$	$\frac{24576}{2881}$
2	$\frac{3}{4}$	$-\frac{16}{15}$	$\frac{64}{189}$	$\frac{1024}{155}$	$\frac{24576}{2881}$
3	0	$\frac{4}{147}$	$-\frac{32}{2583}$	$\frac{48}{65925}$	$-\frac{3072}{397905}$
4	0	0	$\frac{64}{867}$	$\frac{1024}{1955}$	$\frac{8192}{83551}$
5	0	0	0	$\frac{16}{714025}$	$-\frac{8}{27483625}$
6	0	0	0	0	$\frac{192}{852267}$
7	0	0	0	0	$\frac{1024}{0}$
					$\frac{5120}{19660800}$

Supplementary material: Evolution of the Pluto-Charon Binary

Table 6

ξ_j^{2i}

$j \setminus 2i$	ξ_j^{2i}									
	0	2	4	6	8	10	12	14	16	18
1	0	$\frac{9}{8}$	$\frac{81}{32}$	$\frac{2295}{65}$	$\frac{28403}{1661}$	$\frac{3240741}{12027}$	$\frac{87480539}{525521}$	$\frac{76230921863}{13519469}$	$\frac{143189681823}{729537381}$	$\frac{3798168072196597}{56670971611}$
2	0	0	$\frac{32}{81}$	$\frac{16}{65}$	$\frac{256}{20829}$	$\frac{1280}{6019881}$	$\frac{40960}{32152971}$	$\frac{806400}{3406598649}$	$\frac{34406400}{485074233099}$	$\frac{2167603200}{9610837422623}$
3	0	0	0	$\frac{512}{2809}$	$\frac{4096}{5929}$	$\frac{655360}{9933}$	$\frac{2621440}{708871}$	$\frac{209715200}{1736021}$	$\frac{23488102400}{27865091}$	$\frac{375809638400}{2578148063}$
4	0	0	0	0	$\frac{512}{3143529}$	$\frac{2560}{2947317}$	$\frac{51200}{115200}$	$\frac{2947317}{13524196093}$	$\frac{1376256}{42574868795}$	$\frac{103219200}{2718626226045}$
5	0	0	0	0	0	$\frac{131072}{204800}$	$\frac{524288}{10029889}$	$\frac{528482304}{6926229}$	$\frac{2818572288}{197537355}$	$\frac{105226698752}{41566933}$
6	0	0	0	0	0	0	$\frac{204800}{179200}$	$\frac{186702632281}{1887436800}$	$\frac{3211264}{1328179895713}$	$\frac{80281600}{999990833612299}$
7	0	0	0	0	0	0	0	0	$\frac{10066329600}{31801945561}$	$\frac{5798205849600}{1606626956771}$
8	0	0	0	0	0	0	0	0	$\frac{160563200}{0}$	$\frac{4335206400}{4143587919679849}$
9	0	0	0	0	0	0	0	0	0	$\frac{10522669875200}{0}$

$j \setminus 2i$	ξ_j^{2i}			
	12	14	16	18
-7	0	0	0	$\frac{33232930569601}{939309347635200}$
-6	0	0	$\frac{1594323}{80281600}$	$\frac{531441}{323974609375}$
-5	0	$\frac{244140625}{22196256768}$	$\frac{2685546875}{355140108288}$	$\frac{80281600}{29222957481984}$
-4	$\frac{4}{19683}$	$\frac{4}{12852999}$	$\frac{23}{6160779}$	$\frac{976}{24218534007}$
-3	$\frac{675}{5242880}$	$\frac{675}{2936012800}$	$\frac{3150}{1342177280}$	$\frac{127575}{5261334937600}$
-2	$\frac{19683}{949}$	$\frac{12852999}{10193}$	$\frac{6160779}{127453}$	$\frac{24218534007}{781813}$
-1	$\frac{460800}{64537}$	$\frac{4838400}{33683687}$	$\frac{61931520}{165074183}$	$\frac{398131200}{195618433037}$
1	$\frac{141557760}{129709}$	$\frac{79272345600}{73185503}$	$\frac{422785843200}{21004537637}$	$\frac{547930452787200}{5242619992367}$
2	$\frac{39321600}{12791}$	$\frac{26424115200}{288163}$	$\frac{8878502707200}{1190849}$	$\frac{2557008779673600}{535500317}$
3	$\frac{368640}{79030557}$	$\frac{22579200}{528749529}$	$\frac{96337920}{287947683}$	$\frac{50164531200}{73471272723}$
4	$\frac{13107200}{2459537}$	$\frac{419430400}{232033433}$	$\frac{1879048192}{77502109}$	$\frac{1503238553600}{1505096129}$
5	$\frac{10240}{71300068375}$	$\frac{2419200}{5805813704375}$	$\frac{2764800}{15846218478325}$	$\frac{243855360}{1119906455109875}$
6	$\frac{28311552}{1979610939}$	$\frac{3170893824}{4509009273}$	$\frac{16911433728}{499266143199}$	$\frac{3131031158784}{310538096811}$
7	$\frac{204800}{140254152605}$	$\frac{358400}{417304029320899}$	$\frac{45875200}{394313641044575}$	$\frac{45875200}{1420101409507571189}$
8	$\frac{9437184}{5383010161}$	$\frac{11324620800}{56914314263}$	$\frac{7247757312}{578658802849}$	$\frac{26091926323200}{77318558814617}$
9	$\frac{691200}{0}$	$\frac{1209600}{442450100095587}$	$\frac{4515840}{9194245551902151}$	$\frac{365783040}{1247843577251851821}$
10	0	0	$\frac{65766686720}{402063787225}$	$\frac{3006477107200}{176187983600875}$
11	0	0	0	$\frac{445906944}{6648821549377771726369}$
				$\frac{46026158034124800}{0}$

Table 7

χ_j^{2i}

$j \setminus 2i$	χ_j^{2i}					
	0	2	4	6	8	10
-3	0	0	0	0	0	$-\frac{19683}{3276800}$
-2	0	0	0	0	$-\frac{1}{7}$	$-\frac{1920}{619}$
-1	0	0	0	$-\frac{1}{1536}$	$-\frac{384}{113}$	$-\frac{655360}{2639}$
1	0	$-\frac{3}{15}$	$\frac{3}{32}$	$-\frac{1536}{13}$	$-\frac{12288}{113}$	$-\frac{327680}{3481}$
2	$-\frac{3}{2}$	$\frac{15}{8}$	$-\frac{32}{189}$	$-\frac{512}{135}$	$-\frac{12288}{2881}$	$-\frac{12800}{131319}$
3	0	$-\frac{2}{147}$	$\frac{16}{2583}$	$-\frac{24}{65925}$	$-\frac{1536}{397905}$	$-\frac{131072}{15963167}$
4	0	0	$-\frac{8}{32}$	$-\frac{512}{1955}$	$-\frac{4096}{83551}$	$-\frac{327680}{131319}$
5	0	0	0	$-\frac{4}{714025}$	$-\frac{96}{27483625}$	$-\frac{160}{587225375}$
6	0	0	0	$-\frac{1536}{1536}$	$-\frac{12288}{852267}$	$-\frac{22109373}{131072}$
7	0	0	0	0	0	$-\frac{52142352409}{9830400}$

$j \setminus 2i$	χ_j^{2i}			
	12	14	16	18
-7	0	0	0	$-\frac{33232930569601}{469654673817600}$
-6	0	0	$-\frac{1594323}{40140800}$	$-\frac{531441}{323974609375}$
-5	0	$-\frac{244140625}{11098128384}$	$-\frac{40140800}{2685546875}$	$-\frac{40140800}{14611478740992}$
-4	$-\frac{8}{675}$	$-\frac{675}{12852999}$	$-\frac{1575}{6160779}$	$-\frac{127575}{24218534007}$
-3	$-\frac{2621440}{949}$	$-\frac{1468006400}{10193}$	$-\frac{671088640}{127453}$	$-\frac{2630667468800}{781813}$
-2	$-\frac{230400}{64537}$	$-\frac{2419200}{33683687}$	$-\frac{30965760}{165074183}$	$-\frac{199065600}{195618433037}$
-1	$-\frac{70778880}{129709}$	$-\frac{39636172800}{73185503}$	$-\frac{211392921600}{21004537637}$	$-\frac{273965226393600}{5242619992367}$
1	$-\frac{19660800}{12791}$	$-\frac{13212057600}{288163}$	$-\frac{4439251353600}{1190849}$	$-\frac{1278504389836800}{535500317}$
2	$-\frac{184320}{79030557}$	$-\frac{11289600}{528749529}$	$-\frac{48168960}{287947683}$	$-\frac{25082265600}{73471272723}$
3	$-\frac{6553600}{2489537}$	$-\frac{209715200}{232033333}$	$-\frac{939524096}{77502109}$	$-\frac{751619276800}{1306096199}$
4	$-\frac{5120}{71300068375}$	$-\frac{1209600}{5805813704375}$	$-\frac{1382400}{15846218478325}$	$-\frac{121927680}{1119906455109875}$
5	$-\frac{14155776}{1979610939}$	$-\frac{1585446912}{4509009273}$	$-\frac{8455716864}{499266143199}$	$-\frac{1565515579392}{310538096811}$
6	$-\frac{102400}{140254152605}$	$-\frac{179200}{417304029320899}$	$-\frac{22937600}{394313641044575}$	$-\frac{22937600}{1420101409507571189}$
7	$-\frac{4718592}{5383010161}$	$-\frac{5662310400}{56914314263}$	$-\frac{3623878656}{578658802849}$	$-\frac{13045963161600}{77318558814617}$
8	$-\frac{345600}{0}$	$-\frac{604800}{442430100095587}$	$-\frac{2257920}{9194245551902151}$	$-\frac{182891520}{1247843577251851821}$
9	0	$-\frac{10276044800}{0}$	$-\frac{32883343360}{402063787225}$	$-\frac{1503238553600}{176187983600875}$
10	0	0	$-\frac{3538944}{0}$	$-\frac{222953472}{664882154937771726369}$
11	0	0	0	$-\frac{23013079017062400}{0}$