The influence of curvature on convection in a temperature-dependent viscosity fluid: implications for the 2D and 3D modeling of moons
Supplementary Material

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1 Data Tables

2 Resolution Test

The effect of resolution on solutions is presented for two geometries, f = 0.3 and f = 0.9. In the first case we investigate solutions for viscosity contrasts that yield transition regime as well as stagnant-lid regime convection and in the second we examine the effect of resolution on a transition regime case. Table 6 summarizes the temporally averaged statistics from these calcualtions and shows that mobility agrees well for the resolution used in our 2D study and cases with higher resolution. A reduction in radial resolution amplifies mobility in the cases examined and reduces heat flux. In Figure 1 we plot surface and basal heat flux for the three cases analyzed.

Table 1. 2D spherical annulus small core size results ($0.2 \leq f \leq 0.4$). Spherical annulus grid size denotes ($n_r \times n_\phi$).

f	$\Delta\eta_T$	Grid Size	$\frac{T_m\text{-}T_{surf}}{\Delta T}$	F_{top}	F_{bot}	Nu	V_{rms}	M	Ra_m
0.2	1	128×1024	0.062	1.261	31.478	6.307	186.1	1.060	3.20×10^5
0.2	1.00×10^2	128×1024	0.079	0.990	24.789	4.949	74.9	0.961	4.60×10^4
0.2	1.00×10^3	128×1024	0.096	0.875	21.762	4.376	46.5	0.886	1.93×10^4
0.2	1.00×10^5	128×1024	0.134	0.685	16.814	3.427	31.4	0.505	4.69×10^3
0.2	1.00×10^6	128×1024	0.197	0.682	17.582	3.431	38.6	0.240	4.89×10^3
0.2	1.00×10^7	128×1024	0.281	0.713	17.576	3.564	61.7	0.083	9.27×10^3
0.2	1.00×10^8	128×1024	0.522	0.933	24.861	4.663	103.3	0.012	1.68×10^5
0.3	1	128×1024	0.082	1.691	18.805	5.636	206.7	1.144	3.20×10^5
0.3	1.00×10^1	128×1024	0.123	1.603	17.825	5.344	149.7	1.162	1.34×10^5
0.3	1.00×10^2	128×1024	0.191	1.583	17.603	5.278	113.5	1.166	7.72×10^4
0.3	1.00×10^3	128×1024	0.152	1.628	18.054	5.427	73.2	0.942	2.90×10^4
0.3	1.00×10^4	128×1024	0.141	1.154	12.827	3.848	55.9	0.830	1.17×10^4
0.3	1.00×10^5	128×1024	0.207	1.267	14.077	4.224	62.1	0.547	1.10×10^4
0.3	1.00×10^6	128×1024	0.351	1.360	15.145	4.533	93.7	0.235	4.08×10^4
0.3	1.78×10^6	128×1024	0.383	1.352	15.096	4.507	108.0	0.173	5.95×10^4
0.3	3.20×10^6	128×1024	0.539	1.606	17.731	5.352	225.0	0.077	5.73×10^5
0.3	5.62×10^6	128×1024	0.605	1.653	18.352	5.511	315.8	0.044	1.63×10^6
0.3	1.00×10^7	128×1024	0.715	1.932	21.441	6.440	404.1	0.008	1.02×10^7
0.3	1.78×10^7	128×1024	0.712	2.112	23.479	7.039	455.7	0.007	1.09×10^7
0.3	3.20×10^7	128×1024	0.728	2.164	24.053	7.213	566.8	0.003	1.63×10^7
0.3	5.62×10^7	128×1024	0.749	2.211	24.629	7.371	743.6	0.001	2.71×10^7
0.4	1	128×1280	0.145	3.305	20.708	8.263	267.0	1.082	3.20×10^5
0.4	1.00×10^1	128×1280	0.170	2.931	18.379	7.327	189.1	1.084	1.50×10^5
0.4	1.00×10^2	128×1280	0.193	2.631	16.440	6.576	135.2	1.055	7.79×10^4
0.4	1.00×10^3	128×1280	0.233	2.481	15.491	6.203	105.5	0.972	5.06×10^4
0.4	1.00×10^4	128×1280	0.349	2.470	15.437	6.175	123.8	0.983	7.96×10^4
0.4	1.00×10^5	128×1280	0.502	2.626	16.444	6.566	154.3	0.585	3.26×10^5
0.4	3.20×10^5	128×1280	0.609	2.730	17.060	6.826	272.1	0.247	1.27×10^6
0.4	1.00×10^6	128×1280	0.701	2.559	15.965	6.397	484.0	0.084	5.14×10^6
0.4	3.20×10^6	128×1280	0.778	2.986	18.667	7.465	789.3	0.005	2.04×10^7
0.4	1.00×10^7	128×1280	0.802	3.165	19.755	7.912	1200.3	0.002	4.13×10^7
0.4	1.78×10^7	128×1280	0.806	3.242	20.312	8.105	1339.2	0.001	5.32×10^7
0.4	3.20×10^7	128×1280	0.797	3.253	20.550	8.132	1502.0	9.00×10^{-4}	5.4×10^7

Table 2. 2D spherical annulus large core size results ($0.5 \leq f \leq 0.7$). Spherical annulus grid size denotes ($n_r \times n_\phi$).

f	$\Delta\eta_T$	Grid Size	$\frac{T_m\text{-}T_{surf}}{\Delta T}$	F_{top}	F_{bot}	Nu	V_{rms}	M	Ra_m
0.5	1	128×1664	0.220	5.233	20.904	10.466	319.8	1.086	3.20×10^5
0.5	1.00×10^1	128×1664	0.244	3.961	15.846	7.922	223.4	1.137	1.78×10^5
0.5	1.00×10^2	128×1664	0.296	3.747	14.981	7.494	183.9	1.122	1.25×10^5
0.5	1.00×10^3	128×1664	0.385	3.848	15.395	7.695	176.5	1.096	1.45×10^5
0.5	1.00×10^4	128×1664	0.474	3.835	15.358	7.671	166.5	0.820	2.51×10^5
0.5	1.00×10^5	128×1664	0.641	3.753	15.024	7.506	340.1	0.432	1.62×10^6
0.5	1.78×10^5	128×1664	0.695	3.663	14.670	7.326	450.4	0.262	3.40×10^6
0.5	3.20×10^5	128×1664	0.741	3.515	14.055	7.029	603.8	0.149	6.76×10^6
0.5	5.62×10^5	128×1664	0.814	3.489	14.005	6.978	868.2	0.066	2.03×10^7
0.5	1.00×10^6	128×1664	0.818	3.771	15.135	7.543	1007.6	0.015	2.58×10^7
0.5	3.20×10^6	128×1664	0.830	4.056	16.273	8.112	1389.6	0.005	4.44×10^7
0.5	1.00×10^7	128×1664	0.834	4.457	17.881	8.913	1806.6	7.00×10^{-4}	6.92×10^7
0.5	1.78×10^7	128×1664	0.833	4.660	18.680	9.320	2118.3	2.00×10^{-4}	8.31×10^7
0.55	1	128×1664	0.227	5.613	18.562	10.206	318.2	1.129	3.20×10^5
0.55	1.00×10^1	128×1664	0.272	5.192	17.169	9.440	248.9	1.090	1.89×10^5
0.55	1.00×10^2	128×1664	0.310	4.765	15.749	8.663	196.7	1.045	1.34×10^5
0.55	1.00×10^3	128×1664	0.360	4.531	14.974	8.238	166.4	0.946	1.22×10^5
0.55	1.00×10^4	128×1664	0.530	4.517	14.936	8.213	293.4	0.706	4.21×10^5
0.55	1.00×10^5	128×1664	0.696	4.277	14.121	7.776	472.8	0.298	3.05×10^6
0.55	3.20×10^5	128×1664	0.801	3.919	12.920	7.125	873.1	0.116	1.45×10^7
0.55	5.62×10^5	128×1664	0.829	4.014	13.264	7.298	1020.4	0.058	2.50×10^7
0.55	1.00×10^6	128×1664	0.836	4.246	14.044	7.720	1187.1	0.017	3.32×10^7
0.55	3.20×10^6	128×1664	0.830	4.454	14.726	8.099	1464.8	0.001	4.47×10^7
0.55	1.00×10^7	128×1664	0.843	4.947	16.411	8.994	2340.1	0.001	8.11×10^7
0.55	1.78×10^7	128×1664	0.851	5.196	17.141	9.448	2515.7	2.10×10^{-4}	1.12×10^8
0.7	1	128×2560	0.326	7.876	16.083	11.251	352.5	1.164	3.20×10^5
0.7	1.00×10^1	128×2560	0.371	7.355	15.010	10.507	288.9	1.108	2.38×10^5
0.7	1.00×10^2	128×2560	0.437	6.941	14.189	9.915	255.8	1.021	2.39×10^5
0.7	1.00×10^3	128×2560	0.575	6.541	13.345	9.344	245.5	0.752	5.36×10^5
0.7	1.00×10^4	128×2560	0.680	6.229	12.711	8.898	509.9	0.926	1.69×10^6
0.7	1.00×10^5	128×2560	0.804	5.500	11.273	7.858	983.8	0.333	1.06×10^7
0.7	3.20×10^5	128×2560	0.867	5.070	10.358	7.243	1263.1	0.131	3.35×10^7
0.7	5.68×10^5	128×2560	0.870	5.241	10.701	7.487	1586.5	0.044	4.29×10^7
0.7	1.00×10^6	128×2560	0.880	5.393	11.034	7.704	1896.5	0.014	6.10×10^7
0.7	3.20×10^6	128×2560	0.881	5.354	10.945	7.649	1866.3	0.011	9.59×10^7
0.7	1.00×10^7	128×2560	0.880	6.449	13.163	9.213	3370.3	0.002	1.47×10^8
0.7	1.78×10^7	128×2560	0.882	6.734	13.743	9.621	3758.5	2.60×10^{-4}	1.88×10^8

Table 3. 2D spherical annulus thin-shelled results ($f = 0.9$ and plane-layer). Spherical annulus grid size denotes ($n_r \times n_\phi$).

f	$\Delta\eta_T$	Grid Size	$\frac{T_m\text{-}T_{surf}}{\Delta T}$	F_{top}	F_{bot}	Nu	V_{rms}	M	Ra_m
0.9	1	128×2560	0.447	11.441	14.125	12.712	381.3	1.195	3.20×10^5
0.9	1.00×10^1	128×2560	0.510	10.490	12.974	11.656	350.1	1.114	3.27×10^5
0.9	1.00×10^2	128×2560	0.609	10.290	12.700	11.433	341.8	0.951	5.28×10^5
0.9	1.00×10^3	128×2560	0.722	8.916	11.008	9.907	535.0	1.123	1.48×10^6
0.9	1.00×10^4	128×2560	0.784	8.375	10.333	9.306	951.5	0.978	4.38×10^6
0.9	1.00×10^5	128×2560	0.881	6.283	7.755	6.981	1413.9	0.475	2.57×10^7
0.9	1.58×10^5	128×2560	0.889	6.000	7.406	6.667	1620.1	0.153	3.36×10^7
0.9	3.20×10^5	128×2560	0.902	6.143	7.591	6.826	1639.1	0.101	5.17×10^7
0.9	5.62×10^5	128×2560	0.887	6.460	7.977	7.178	1823.5	0.012	5.38×10^7
0.9	1.00×10^6	128×2560	0.910	6.270	7.738	6.966	2538.1	0.014	9.23×10^7
0.9	3.20×10^6	128×2560	0.917	7.017	8.664	7.797	3379.4	0.013	1.64×10^8
0.9	1.00×10^7	128×2560	0.918	7.714	9.519	8.572	4657.4	0.002	2.70×10^8
1	1	128×1280	0.500	13.054	13.062	13.054	398.7	1.188	3.20×10^5
1	1.00×10^1	128×1280	0.572	12.637	12.634	12.637	385.0	1.043	3.78×10^5
1	1.00×10^2	128×1280	0.676	11.173	11.175	11.173	356.5	0.769	7.21×10^5
1	1.00×10^3	128×1280	0.790	9.680	9.673	9.680	476.9	0.733	2.38×10^6
1	1.00×10^4	128×1280	0.872	7.985	8.002	7.985	786.6	0.369	9.83×10^6
1	1.78×10^4	128×1280	0.887	7.516	7.490	7.516	903.8	0.239	1.41×10^7
1	3.20×10^4	128×1280	0.900	7.011	7.040	7.011	1068.0	0.172	2.03×10^7
1	5.62×10^4	128×1280	0.911	6.525	6.564	6.525	1265.2	0.099	2.87×10^7
1	1.00×10^5	128×1280	0.913	6.440	6.451	6.440	1410.7	0.041	3.70×10^7
1	1.58×10^5	128×1280	0.915	6.551	6.477	6.551	1687.3	0.006	4.60×10^7
1	3.20×10^5	128×1280	0.918	6.897	6.884	6.897	1941.7	0.005	6.39×10^7
1	1.00×10^6	128×1280	0.923	7.244	7.248	7.244	2512.8	6.00×10^{-4}	1.10×10^8
1	3.20×10^6	128×1280	0.928	7.736	7.762	7.736	3354.7	2.00×10^{-4}	1.92×10^8
1	1.00×10^7	128×1280	0.928	8.626	8.635	8.626	4555.7	1.00×10^{-4}	3.18×10^8

Table 4. 2D Spherical annulus calculations emulating the results presented by Yao et al., (2014). Spherical annulus grid size denotes ($n_r \times n_\phi$)

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f	Ra	$\Delta\eta_T$	Grid Size	$\frac{T_m\text{-}T_{surf}}{\Delta T}$	F_{top}	F_{bot}	Nu	V_{rms}	M	Ra_m
0.5	3.20×10^4	1.00×10^5	128×1664	0.526	1.474	5.896	2.947	93.9	0.166	4.32×10^4
0.5	1.00×10^4	3.20×10^5	128×1664	0.771	1.351	5.407	2.703	64.2	0.004	3.11×10^5
0.5	1.00×10^4	1.00×10^6	128×1664	0.793	1.484	5.931	2.968	97.5	0.002	5.74×10^5
0.7	2.00×10^4	1.00×10^6	128×2560	0.880	2.445	4.994	3.493	233.4	0.004	3.81×10^6
0.7	1.00×10^4	3.20×10^4	128×2560	0.658	1.490	3.041	2.129	54.5	0.206	1.80×10^4
0.7	1.00×10^4	3.20×10^5	128×2560	0.858	2.029	4.141	2.899	119.9	0.004	9.37×10^5
0.7	1.00×10^4	1.00×10^5	128×2560	0.829	1.957	3.991	2.796	89.7	0.009	4.40×10^5
0.7	1.00×10^4	1.00×10^6	128×2560	0.838	1.939	3.958	2.771	138.7	0.004	1.07×10^6

Table 5. 3D Results. For spherical shell calculations the grid size denotes ($2 \times (n_\theta \times n_\phi \times n_r)$) and for the plane-layer case grid size denotes ($n_x \times n_y \times n_z$)

f	$\Delta\eta_T$	Grid Size	$\frac{T_m\text{-}T_{surf}}{\Delta T}$	F_{top}	F_{bot}	Nu	V_{rms}	M	Ra_m
0.2	1.00×10^8	$2 \times (128 \times 384 \times 64)$	0.219	0.520	13.020	2.602	55.8	0.014	1.81×10^3
0.3	1.00×10^7	$2 \times (128 \times 384 \times 64)$	0.480	1.848	20.569	6.161	164.6	0.017	2.33×10^5
0.4	3.20×10^6	$2 \times (128 \times 384 \times 64)$	0.546	2.882	18.138	7.205	308.4	0.017	6.41×10^5
0.9	1.00×10^5	$2 \times (192 \times 1152 \times 96)$	0.757	8.365	11.376	9.295	1825.4	0.449	6.16×10^6
1	1.00×10^5	$512 \times 512 \times 128$	0.878	7.817	7.822	7.817	1394.5	0.005	2.49×10^7

Table 6. Resolution test for $f = 0.3$ and $f = 0.9$ at the transition to stagnant-lid convection. Spherical annulus grid size denotes ($n_r \times n_\phi$)

f	$\Delta\eta_T$	Grid Size	$\frac{T_m\text{-}T_{surf}}{\Delta T}$	F_{top}	F_{bot}	M
0.3	5.62×10^6	64×576	0.575	1.557	17.451	0.049
0.3	5.62×10^6	128×1024	0.605	1.653	18.352	0.044
0.3	5.62×10^6	128×1152	0.614	1.672	18.761	0.042
0.3	1.00×10^7	64×576	0.635	1.762	19.587	0.020
0.3	1.00×10^7	128×1024	0.715	1.932	21.441	0.008
0.3	1.00×10^7	128×1152	0.692	2.035	22.609	0.011
0.9	5.62×10^5	64×4096	0.899	6.236	7.698	0.039
0.9	5.62×10^5	128×2560	0.887	6.460	7.977	0.012
0.9	5.62×10^5	128×4096	0.907	6.393	7.893	0.023

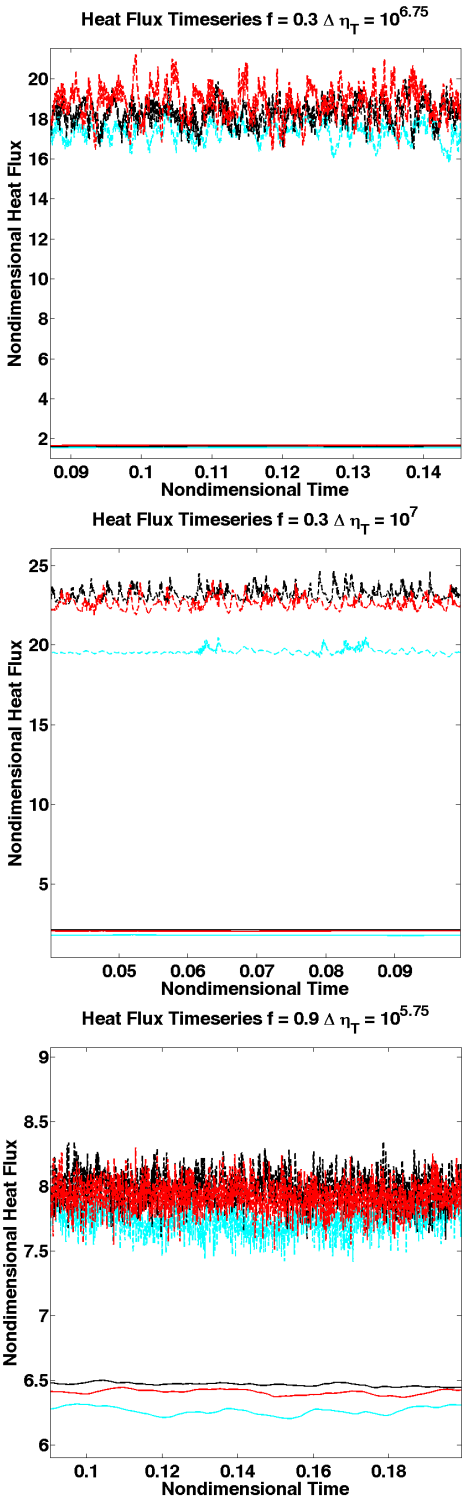

Figure 1. Time series for surface and basal heat flux for resolution test cases. Color corresponds to the resolution where low resolution is in cyan, resolution from this study is in black and high resolution is in red. Basal heat flux is indicated by a solid curve and surface heat flux is indicated by a dashed curve.

Table 7. 2D spherical annulus results (f = 0.3 and 0.7) using initial conditions with decreasing viscosity contrast (see Figure 3 and 7 of the main text). Spherical annulus grid size denotes ($n_r \times n_\phi$).

f	$\Delta\nu_T$	Grid Size	$\frac{T_m T_{surf}}{\Delta T}$	F_{top}	F_{bot}	Nu	V_{rms}	M	Ra_m
0.3	1.0×10^1	128×1024	0.125	2.115	23.360	7.049	162.0	1.054	1.35×10^5
0.3	1.0×10^2	128×1024	0.132	1.832	20.291	6.106	107.6	1.016	5.88×10^4
0.3	1.0×10^3	128×1024	0.153	1.677	18.622	5.589	74.8	0.941	2.91×10^4
0.3	1.0×10^4	128×1024	0.198	1.674	18.599	5.580	59.3	0.769	1.99×10^4
0.3	1.0×10^5	128×1024	0.214	1.305	14.470	4.343	63.8	0.322	1.18×10^4
0.3	1.0×10^6	128×1024	0.359	1.407	15.635	4.690	98.5	0.231	4.56×10^4
0.3	1.78×10^6	128×1024	0.533	1.651	18.387	5.504	205.9	0.103	5.17×10^5
0.3	3.20×10^6	128×1024	0.602	1.829	20.321	6.096	200.7	0.044	1.47×10^6
0.3	5.62×10^6	128×1024	0.683	1.879	20.888	6.264	326.2	0.025	5.47×10^6
0.3	1.00×10^7	128×1024	0.692	2.040	22.660	6.800	359.9	0.011	7.02×10^6
0.3	1.78×10^7	128×1024	0.712	2.140	23.817	7.134	467.5	0.005	1.11×10^7
0.3	3.20×10^7	128×1024	0.734	2.134	23.823	7.114	591.8	0.002	1.83×10^7
0.7	1	128×2560	0.354	8.344	16.593	11.920	359.5	1.122	3.20×10^5
0.7	1.0×10^1	128×2560	0.373	7.511	15.329	10.730	292.2	1.105	2.39×10^5
0.7	1.0×10^2	128×2560	0.485	6.866	14.038	9.809	269.5	1.036	2.99×10^5
0.7	1.0×10^3	128×2560	0.589	6.398	13.050	9.140	354.8	1.149	5.90×10^5
0.7	1.0×10^4	128×2560	0.660	6.484	13.230	9.263	604.1	1.074	1.40×10^6
0.7	1.0×10^5	128×2560	0.804	5.488	11.194	7.841	979.5	0.334	1.05×10^7
0.7	3.20×10^5	128×2560	0.869	5.067	10.338	7.239	1422.7	0.103	3.43×10^7
0.7	5.62×10^5	128×2560	0.877	5.155	10.535	7.365	1615.2	0.045	4.68×10^7
0.7	1.0×10^6	128×2560	0.878	5.521	11.261	7.887	1881.1	0.003	5.95×10^7
0.7	3.20×10^6	128×2560	0.880	5.918	12.088	8.455	2497.3	0.001	9.50×10^7

Table 8. 2D spherical annulus results using a shifted reference temperature (see Figure 6 main body). Spherical annulus grid size denotes ($n_r \times n_\phi$).

f	T^*	$\frac{T_m T_{surf}}{\Delta T}$	F_{top}	F_{bot}	Nu	V_{rms}	M	Ra_m
0.3	0.17	0.370	5.183	57.406	17.275	698.1	0.805	5.06×10^6
0.4	0.23	0.578	7.717	48.427	19.293	1858.0	0.347	3.89×10^7
0.5	0.29	0.770	8.395	33.856	16.791	6093.7	0.074	2.43×10^8
0.7	0.38	0.867	8.706	17.775	12.438	5860.8	0.044	2.66×10^8
0.9	0.46	0.908	7.803	9.604	8.670	3652.9	0.023	1.56×10^8